

Интегралы, содержащие $\operatorname{sh} x$ и $\operatorname{ch} x$

$$685.11. \quad \int \operatorname{sh} x \operatorname{ch} x dx = \frac{\operatorname{sh}^2 x}{2} = \frac{\operatorname{ch}^2 x}{2} + \operatorname{const} = \frac{\operatorname{ch} 2x}{4} + \operatorname{const}.$$

$$685.12. \quad \int \operatorname{sh} x \operatorname{ch}^2 x dx = \frac{\operatorname{ch}^3 x}{3}.$$

$$685.13. \quad \int \operatorname{sh} x \operatorname{ch}^3 x dx = \frac{\operatorname{ch}^4 x}{4}.$$

$$685.19. \quad \int \operatorname{sh} x \operatorname{ch}^p x dx = \frac{\operatorname{ch}^{p+1} x}{p+1} \quad [p \neq -1].$$

$$685.21. \quad \int \operatorname{sh}^2 x \operatorname{ch} x dx = \frac{\operatorname{sh}^3 x}{3}.$$

$$685.22. \quad \int \operatorname{sh}^2 x \operatorname{ch}^2 x dx = \frac{\operatorname{sh} 4x}{32} - \frac{x}{8}.$$

$$685.31. \quad \int \operatorname{sh}^3 x \operatorname{ch} x \, dx = \frac{\operatorname{sh}^4 x}{4}.$$

$$685.91. \quad \int \operatorname{sh}^p x \operatorname{ch} x \, dx = \frac{\operatorname{sh}^{p+1} x}{p+1} \quad [p \neq -1].$$

$$686.11. \quad \int \frac{dx}{\operatorname{sh} x \operatorname{ch} x} = \ln |\operatorname{th} x|.$$

$$686.12. \quad \int \frac{dx}{\operatorname{sh} x \operatorname{ch}^2 x} = \frac{1}{\operatorname{ch} x} + \ln \left| \operatorname{th} \frac{x}{2} \right|.$$

$$686.13. \quad \int \frac{dx}{\operatorname{sh} x \operatorname{ch}^3 x} = \frac{1}{2 \operatorname{ch}^2 x} + \ln |\operatorname{th} x|.$$

$$686.19. \quad \int \frac{dx}{\operatorname{sh} x \operatorname{ch}^p x} = \frac{1}{(p-1) \operatorname{ch}^{p-1} x} + \int \frac{dx}{\operatorname{sh} x \operatorname{ch}^{p-2} x} \quad [p \neq 1].$$

$$686.21. \quad \int \frac{dx}{\operatorname{sh}^2 x \operatorname{ch} x} = -\frac{1}{\operatorname{sh} x} - \operatorname{arctg}(\operatorname{sh} x).$$

$$686.22. \quad \int \frac{dx}{\operatorname{sh}^2 x \operatorname{ch}^2 x} = -2 \operatorname{cth} 2x.$$

$$686.31. \quad \int \frac{dx}{\operatorname{sh}^3 x \operatorname{ch} x} = -\frac{1}{2 \operatorname{sh}^2 x} - \ln |\operatorname{th} x|.$$

$$686.91. \quad \int \frac{dx}{\operatorname{sh}^p x \operatorname{ch} x} = -\frac{1}{(p-1) \operatorname{sh}^{p-1} x} - \int \frac{dx}{\operatorname{sh}^{p-2} x \operatorname{ch} x} \quad [p \neq 1].$$

$$687.11. \quad \int \frac{\operatorname{sh} x \, dx}{\operatorname{ch} x} = \int \operatorname{th} x \, dx = \ln \operatorname{ch} x. \quad [\text{См. } 691.01.]$$

$$687.12. \quad \int \frac{\operatorname{sh} x \, dx}{\operatorname{ch}^2 x} = -\frac{1}{\operatorname{ch} x} = -\operatorname{sech} x.$$

$$687.13. \quad \int \frac{\operatorname{sh} x \, dx}{\operatorname{ch}^3 x} = -\frac{1}{2 \operatorname{ch}^2 x} = \frac{\operatorname{th}^2 x}{2} + \operatorname{const}.$$

$$687.19. \quad \int \frac{\operatorname{sh} x \, dx}{\operatorname{ch}^p x} = -\frac{1}{(p-1) \operatorname{ch}^{p-1} x} \quad [p \neq 1].$$

$$687.21. \quad \int \frac{\operatorname{sh}^2 x}{\operatorname{ch} x} \, dx = \operatorname{sh} x - \operatorname{arctg}(\operatorname{sh} x).$$

$$687.22. \quad \int \frac{\operatorname{sh}^2 x}{\operatorname{ch}^2 x} \, dx = \int \operatorname{th}^2 x \, dx = x - \operatorname{th} x. \quad [\text{См. } 691.02.]$$

$$687.29. \quad \int \frac{\operatorname{sh}^2 x}{\operatorname{ch}^p x} \, dx = -\frac{\operatorname{sh} x}{(p-1) \operatorname{ch}^{p-1} x} + \frac{1}{p-1} \int \frac{dx}{\operatorname{ch}^{p-2} x} \quad [p \neq 1].$$

$$687.31. \quad \int \frac{\operatorname{sh}^3 x}{\operatorname{ch} x} \, dx = \frac{\operatorname{sh}^2 x}{2} - \ln \operatorname{ch} x.$$

$$687.32. \quad \int \frac{\operatorname{sh}^3 x}{\operatorname{ch}^2 x} \, dx = \operatorname{ch} x + \operatorname{sech} x.$$

- 687.33. $\int \frac{\text{sh}^3 x}{\text{ch}^3 x} dx = \int \text{th}^3 x dx = -\frac{\text{th}^2 x}{2} + \ln \text{ch } x.$ [См. 691.03.]
- 687.34. $\int \frac{\text{sh}^3 x}{\text{ch}^4 x} dx = \frac{1}{3\text{ch}^3 x} - \frac{1}{\text{ch } x}.$
- 687.39. $\int \frac{\text{sh}^3 x}{\text{ch}^p x} dx = \frac{1}{(p-1)\text{ch}^{p-1} x} - \frac{1}{(p-3)\text{ch}^{p-3} x}$ [$p \neq 1$ или 3].
- 687.7. $\int \frac{\text{sh}^{p-2} x}{\text{ch}^p x} dx = \frac{\text{th}^{p-1} x}{p-1}$ [$p \neq 1$].
- 688.11. $\int \frac{\text{ch } x}{\text{sh } x} dx = \int \text{cth } x dx = \ln |\text{sh } x|.$ [См. 692.01.]
- 688.12. $\int \frac{\text{ch } x}{\text{sh}^2 x} dx = -\frac{1}{\text{sh } x} = -\text{csch } x.$
- 688.13. $\int \frac{\text{ch } x}{\text{sh}^3 x} dx = -\frac{1}{2\text{sh}^2 x} = -\frac{\text{cth}^2 x}{2} + \text{const.}$
- 688.19. $\int \frac{\text{ch } x}{\text{sh}^p x} dx = -\frac{1}{(p-1)\text{sh}^{p-1} x}$ [$p \neq 1$].
- 688.21. $\int \frac{\text{ch}^2 x}{\text{sh } x} dx = \text{ch } x + \ln \left| \text{th } \frac{x}{2} \right|.$
- 688.22. $\int \frac{\text{ch}^2 x}{\text{sh}^2 x} dx = \int \text{cth}^2 x dx = x - \text{cth } x.$ [См. 692.02.]
- 688.29. $\int \frac{\text{ch}^2 x}{\text{sh}^p x} dx = -\frac{\text{ch } x}{(p-1)\text{sh}^{p-1} x} + \frac{1}{p-1} \int \frac{dx}{\text{sh}^{p-2} x}$ [$p \neq 1$].
- 688.31. $\int \frac{\text{ch}^3 x}{\text{sh } x} dx = \frac{\text{ch}^2 x}{2} + \ln |\text{sh } x|.$
- 688.32. $\int \frac{\text{ch}^3 x}{\text{sh}^2 x} dx = \text{sh } x - \text{csch } x.$
- 688.33. $\int \frac{\text{ch}^3 x}{\text{sh}^3 x} dx = \int \text{cth}^3 x dx = -\frac{\text{cth}^2 x}{2} + \ln |\text{sh } x|.$ [См. 692.03.]
- 688.34. $\int \frac{\text{ch}^3 x}{\text{sh}^4 x} dx = -\frac{1}{3\text{sh}^3 x} - \frac{1}{\text{sh } x}.$
- 688.39. $\int \frac{\text{ch}^3 x}{\text{sh}^p x} dx = -\frac{1}{(p-1)\text{sh}^{p-1} x} - \frac{1}{(p-3)\text{sh}^{p-3} x}$ [$p \neq 1$ или 3].
- 688.7. $\int \frac{\text{ch}^{p-2} x}{\text{sh}^p x} dx = -\frac{\text{cth}^{p-1} x}{p-1}$ [$p \neq 1$].
- 689.01. $\int \frac{\text{sh } x dx}{\text{ch } x + 1} = \ln (\text{ch } x + 1).$
- 689.02. $\int \frac{\text{sh } x dx}{\text{ch } x - 1} = \ln (\text{ch } x - 1).$

$$689.03. \quad \int \frac{dx}{\operatorname{sh} x (\operatorname{ch} x + 1)} = -\frac{1}{2(\operatorname{ch} x + 1)} + \frac{1}{2} \ln \left| \operatorname{th} \frac{x}{2} \right|.$$

$$689.04. \quad \int \frac{dx}{\operatorname{sh} x (\operatorname{ch} x - 1)} = \frac{1}{2(\operatorname{ch} x - 1)} - \frac{1}{2} \ln \left| \operatorname{th} \frac{x}{2} \right|.$$

$$689.05. \quad \int \frac{\operatorname{sh} x dx}{\operatorname{ch} x (\operatorname{ch} x + 1)} = \ln \left(\frac{\operatorname{ch} x}{\operatorname{ch} x + 1} \right).$$

$$689.06. \quad \int \frac{\operatorname{sh} x dx}{\operatorname{ch} x (\operatorname{ch} x - 1)} = \ln \left(\frac{\operatorname{ch} x - 1}{\operatorname{ch} x} \right).$$

$$689.07. \quad \int \operatorname{sh} mx \operatorname{ch} nx dx = \frac{\operatorname{ch} (m+n)x}{2(m+n)} + \frac{\operatorname{ch} (m-n)x}{2(m-n)}$$

[$m^2 \neq n^2$; при $m^2 = n^2$ см. 685.11].