

Обратные гиперболические функции — Производные

$$728.1. \quad \frac{d}{dx} \operatorname{Arsh} \frac{x}{a} = \frac{1}{\sqrt{x^2 + a^2}}.$$

$$728.2. \quad \frac{d}{dx} \operatorname{Arch} \frac{x}{a} = \frac{1}{\sqrt{x^2 - a^2}} \quad \left[\operatorname{Arch} \frac{x}{a} > 0, \frac{x}{a} > 1 \right].$$

$$728.3. \quad \frac{d}{dx} \operatorname{Arch} \frac{x}{a} = \frac{-1}{\sqrt{x^2 - a^2}} \quad \left[\operatorname{Arch} \frac{x}{a} < 0, \frac{x}{a} > 1 \right].$$

$$728.4. \quad \frac{d}{dx} \operatorname{Arth} \frac{x}{a} = \frac{a}{a^2 - x^2} \quad [x^2 < a^2].$$

$$728.5. \quad \frac{d}{dx} \operatorname{Arcth} \frac{x}{a} = \frac{a}{a^2 - x^2} \quad [x^2 > a^2].$$

$$728.6. \quad \frac{d}{dx} \operatorname{Arsech} \frac{x}{a} = \frac{-a}{x \sqrt{a^2 - x^2}} \quad \left[\operatorname{Arsech} \frac{x}{a} > 0, 0 < \frac{x}{a} < 1 \right].$$

$$728.7. \quad \frac{d}{dx} \operatorname{Arsech} \frac{x}{a} = \frac{a}{x \sqrt{a^2 - x^2}} \quad \left[\operatorname{Arsech} \frac{x}{a} < 0, 0 < \frac{x}{a} < 1 \right].$$

$$728.8. \quad \frac{d}{dx} \operatorname{Arcsch} \frac{x}{a} = \frac{-a}{x \sqrt{x^2 + a^2}}.$$

(Всюду, кроме 728.4 и 728.5, a должно быть положительным.)

Обратные гиперболические функции — Интегралы

Здесь всюду $a > 0$

$$730. \quad \int \operatorname{Arsh} \frac{x}{a} dx = x \operatorname{Arsh} \frac{x}{a} - \sqrt{x^2 + a^2}.$$

$$730.1. \quad \int x \operatorname{Arsh} \frac{x}{a} dx = \left(\frac{x^2}{2} + \frac{a^2}{4} \right) \operatorname{Arsh} \frac{x}{a} - \frac{x}{4} \sqrt{x^2 + a^2}.$$

$$730.2. \quad \int x^2 \operatorname{Arsh} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{Arsh} \frac{x}{a} + \frac{2a^2 - x^2}{9} \sqrt{x^2 + a^2}.$$

$$730.3. \quad \int x^3 \operatorname{Arsh} \frac{x}{a} dx = \\ = \left(\frac{x^4}{4} - \frac{3a^4}{32} \right) \operatorname{Arsh} \frac{x}{a} + \frac{3a^2x - 2x^3}{32} \sqrt{x^2 + a^2}.$$

$$730.4. \quad \int x^4 \operatorname{Arsh} \frac{x}{a} dx = \\ = \frac{x^5}{5} \operatorname{Arsh} \frac{x}{a} - \frac{8a^4 - 4a^2x^2 + 3x^4}{75} \sqrt{x^2 + a^2}.$$

[См. 625 — 625.4.]

$$730.9. \quad \int x^p \operatorname{Arsh} \frac{x}{a} dx = \frac{x^{p+1}}{p+1} \operatorname{Arsh} \frac{x}{a} - \frac{1}{p+1} \int \frac{x^{p+1} dx}{\sqrt{x^2 + a^2}} \quad [p \neq -1].$$

[См. 201.01 — 207.01 и 625.9.]

$$731.1. \quad \int \frac{1}{x} \operatorname{Arsh} \frac{x}{a} dx = \\ = -\frac{1}{2} \left(\ln \left| \frac{2x}{a} \right| \right)^2 + \frac{1}{2^3} \frac{a^2}{x^2} - \frac{1 \cdot 3}{2 \cdot 4^3} \frac{a^4}{x^4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6^3} \frac{a^6}{x^6} - \dots$$

$\left[\frac{x}{a} < -1 \right],$

$$= \frac{x}{a} - \frac{1}{2 \cdot 3 \cdot 3} \frac{x^3}{a^3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5 \cdot 5} \frac{x^5}{a^5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} \frac{x^7}{a^7} + \dots \quad [x^2 < a^2].$$

$$= \frac{1}{2} \left(\ln \frac{2x}{a} \right)^2 - \frac{1}{2^3} \frac{a^2}{x^2} + \frac{1 \cdot 3}{2 \cdot 4^3} \frac{a^4}{x^4} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6^3} \frac{a^6}{x^6} + \dots \quad \left[\frac{x}{a} > 1 \right],$$

$$731.2. \quad \int \frac{1}{x^2} \operatorname{Arsh} \frac{x}{a} dx = -\frac{1}{x} \operatorname{Arsh} \frac{x}{a} - \frac{1}{a} \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|.$$

$$731.3. \quad \int \frac{1}{x^3} \operatorname{Arsh} \frac{x}{a} dx = -\frac{1}{2x^2} \operatorname{Arsh} \frac{x}{a} - \frac{\sqrt{x^2 + a^2}}{2a^2 x}.$$

[См. 626.1—626.3.]

$$731.9. \quad \int \frac{1}{x^p} \operatorname{Arsh} \frac{x}{a} dx = -\frac{1}{(p-1)x^{p-1}} \operatorname{Arsh} \frac{x}{a} + \frac{1}{p-1} \int \frac{dx}{x^{p-1} \sqrt{x^2 + a^2}} \quad [p \neq 1].$$

[См. 221.01—226.01 и 626.9.]

$$732. \quad \int \operatorname{Arch} \frac{x}{a} dx = x \operatorname{Arch} \frac{x}{a} - \sqrt{x^2 - a^2} \quad \left[\operatorname{Arch} \frac{x}{a} > 0 \right],$$

$$= x \operatorname{Arch} \frac{x}{a} + \sqrt{x^2 - a^2} \quad \left[\operatorname{Arch} \frac{x}{a} < 0 \right].$$

$$732.1. \quad \int x \operatorname{Arch} \frac{x}{a} dx = \left(\frac{x^2}{2} - \frac{a^2}{4} \right) \operatorname{Arch} \frac{x}{a} - \frac{x}{4} \sqrt{x^2 - a^2} \quad \left[\operatorname{Arch} \frac{x}{a} > 0 \right],$$

$$= \left(\frac{x^2}{2} - \frac{a^2}{4} \right) \operatorname{Arch} \frac{x}{a} + \frac{x}{4} \sqrt{x^2 - a^2} \quad \left[\operatorname{Arch} \frac{x}{a} < 0 \right].$$

$$732.2. \quad \int x^2 \operatorname{Arch} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{Arch} \frac{x}{a} - \frac{2a^2 + x^2}{9} \sqrt{x^2 - a^2} \quad \left[\operatorname{Arch} \frac{x}{a} > 0 \right],$$

$$= \frac{x^3}{3} \operatorname{Arch} \frac{x}{a} + \frac{2a^2 + x^2}{9} \sqrt{x^2 - a^2} \quad \left[\operatorname{Arch} \frac{x}{a} < 0 \right].$$

$$732.3. \quad \int x^3 \operatorname{Arch} \frac{x}{a} dx = \left(\frac{x^4}{4} - \frac{3a^4}{32} \right) \operatorname{Arch} \frac{x}{a} - \frac{3a^2 x + 2x^3}{32} \sqrt{x^2 - a^2} \quad \left[\operatorname{Arch} \frac{x}{a} > 0 \right],$$

$$= \left(\frac{x^4}{4} - \frac{3a^4}{32} \right) \operatorname{Arch} \frac{x}{a} + \frac{3a^2 x + 2x^3}{32} \sqrt{x^2 - a^2} \quad \left[\operatorname{Arch} \frac{x}{a} < 0 \right].$$

$$\begin{aligned}
 732.4. \quad \int x^4 \operatorname{Arch} \frac{x}{a} dx &= \\
 &= \frac{x^5}{5} \operatorname{Arch} \frac{x}{a} - \frac{8a^4 + 4a^2x^2 + 3x^4}{75} \sqrt{x^2 - a^2} \\
 &\quad \left[\operatorname{Arch} \frac{x}{a} > 0 \right], \\
 &= \frac{x^5}{5} \operatorname{Arch} \frac{x}{a} + \frac{8a^4 + 4a^2x^2 + 3x^4}{75} \sqrt{x^2 - a^2} \\
 &\quad \left[\operatorname{Arch} \frac{x}{a} < 0 \right]. \quad [\text{См. } 627-627.4.]
 \end{aligned}$$

$$\begin{aligned}
 732.9. \quad \int x^p \operatorname{Arch} \frac{x}{a} dx &= \\
 &= \frac{x^{p+1}}{p+1} \operatorname{Arch} \frac{x}{a} - \frac{1}{p+1} \int \frac{x^{p+1} dx}{\sqrt{x^2 - a^2}} \\
 &\quad \left[\operatorname{Arch} \frac{x}{a} > 0, \quad p \neq -1 \right], \\
 &= \frac{x^{p+1}}{p+1} \operatorname{Arch} \frac{x}{a} + \frac{1}{p+1} \int \frac{x^{p+1} dx}{\sqrt{x^2 - a^2}} \\
 &\quad \left[\operatorname{Arch} \frac{x}{a} < 0, \quad p \neq -1 \right]. \quad [\text{См. } 261.01-267.01 \text{ и } 627.9.]
 \end{aligned}$$

$$\begin{aligned}
 733.1. \quad \int \frac{1}{x} \operatorname{Arch} \frac{x}{a} dx &= \\
 &= \frac{1}{2} \left(\ln \frac{2x}{a} \right)^2 + \frac{1}{2^3} \frac{a^2}{x^2} + \frac{1 \cdot 3}{2 \cdot 4^3} \frac{a^4}{x^4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6^3} \frac{a^6}{x^6} + \dots \\
 &\quad \left[\operatorname{Arch} \frac{x}{a} > 0 \right], \\
 &= - \left[\frac{1}{2} \left(\ln \frac{2x}{a} \right)^2 + \frac{1}{2^3} \frac{a^2}{x^2} + \frac{1 \cdot 3}{2 \cdot 4^3} \frac{a^4}{x^4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6^3} \frac{a^6}{x^6} + \dots \right] \\
 &\quad \left[\operatorname{Arch} \frac{x}{a} < 0 \right].
 \end{aligned}$$

$$\begin{aligned}
 733.2. \quad \int \frac{1}{x^2} \operatorname{Arch} \frac{x}{a} dx &= -\frac{1}{x} \operatorname{Arch} \frac{x}{a} + \frac{1}{a} \operatorname{arcsec} \left| \frac{x}{a} \right| \\
 &\quad \left[\operatorname{Arch} \frac{x}{a} > 0, \quad 0 < \operatorname{arcsec} \left| \frac{x}{a} \right| < \frac{\pi}{2} \right], \\
 &= -\frac{1}{x} \operatorname{Arch} \frac{x}{a} - \frac{1}{a} \operatorname{arcsec} \left| \frac{x}{a} \right| \\
 &\quad \left[\operatorname{Arch} \frac{x}{a} < 0, \quad 0 < \operatorname{arcsec} \left| \frac{x}{a} \right| < \frac{\pi}{2} \right].
 \end{aligned}$$

$$\begin{aligned}
 733.3. \quad \int \frac{1}{x^3} \operatorname{Arch} \frac{x}{a} dx &= -\frac{1}{2x^2} \operatorname{Arch} \frac{x}{a} + \frac{\sqrt{x^2-a^2}}{2a^2x} \left[\operatorname{Arch} \frac{x}{a} > 0 \right], \\
 &= -\frac{1}{2x^2} \operatorname{Arch} \frac{x}{a} - \frac{\sqrt{x^2-a^2}}{2a^2x} \left[\operatorname{Arch} \frac{x}{a} < 0 \right]. \quad [\text{См. 628.1—628.3.}]
 \end{aligned}$$

$$\begin{aligned}
 733.9. \quad \int \frac{1}{x^p} \operatorname{Arch} \frac{x}{a} dx &= \\
 &= -\frac{1}{(p-1)x^{p-1}} \operatorname{Arch} \frac{x}{a} + \frac{1}{p-1} \int \frac{dx}{x^{p-1} \sqrt{x^2-a^2}} \left[\operatorname{Arch} \frac{x}{a} > 0, \quad p \neq 1 \right], \\
 &= -\frac{1}{(p-1)x^{p-1}} \operatorname{Arch} \frac{x}{a} - \frac{1}{p-1} \int \frac{dx}{x^{p-1} \sqrt{x^2-a^2}} \left[\operatorname{Arch} \frac{x}{a} < 0, \quad p \neq 1 \right]. \quad [\text{См. 281.01—284.01 и 628.9.}]
 \end{aligned}$$

От 732 до 733.9 всюду $\frac{x}{a} > 1$.

$$734. \quad \int \operatorname{Arth} \frac{x}{a} dx = x \operatorname{Arth} \frac{x}{a} + \frac{a}{2} \ln(a^2 - x^2).$$

$$734.1. \quad \int x \operatorname{Arth} \frac{x}{a} dx = \frac{x^2-a^2}{2} \operatorname{Arth} \frac{x}{a} + \frac{ax}{2}.$$

$$734.2. \quad \int x^2 \operatorname{Arth} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{Arth} \frac{x}{a} + \frac{ax^2}{6} + \frac{a^3}{6} \ln(a^2 - x^2).$$

$$734.3. \quad \int x^3 \operatorname{Arth} \frac{x}{a} dx = \frac{x^4-a^4}{4} \operatorname{Arth} \frac{x}{a} + \frac{ax^3}{12} + \frac{a^3x}{4}.$$

$$\begin{aligned}
 734.9. \quad \int x^p \operatorname{Arth} \frac{x}{a} dx &= \frac{x^{p+1}}{p+1} \operatorname{Arth} \frac{x}{a} - \frac{a}{p+1} \int \frac{x^{p+1} dx}{a^2-x^2} \\
 & \quad [p \neq -1]. \quad [\text{См. 141.1—148.1.}]
 \end{aligned}$$

$$735.1. \quad \int \frac{1}{x} \operatorname{Arth} \frac{x}{a} dx = \frac{x}{a} + \frac{x^3}{3^2 a^3} + \frac{x^5}{5^2 a^5} + \frac{x^7}{7^2 a^7} + \dots$$

$$735.2. \quad \int \frac{1}{x^2} \operatorname{Arth} \frac{x}{a} dx = -\frac{1}{x} \operatorname{Arth} \frac{x}{a} - \frac{1}{2a} \ln \left(\frac{a^2-x^2}{x^2} \right).$$

$$735.3. \quad \int \frac{1}{x^3} \operatorname{Arth} \frac{x}{a} dx = \frac{1}{2} \left(\frac{1}{a^2} - \frac{1}{x^2} \right) \operatorname{Arth} \frac{x}{a} - \frac{1}{2ax}.$$

$$\begin{aligned}
 735.4. \quad \int \frac{1}{x^4} \operatorname{Arth} \frac{x}{a} dx &= \\
 &= -\frac{1}{3x^3} \operatorname{Arth} \frac{x}{a} - \frac{1}{6ax^2} - \frac{1}{6a^3} \ln \left(\frac{a^2-x^2}{x^2} \right).
 \end{aligned}$$

$$735.5. \quad \int \frac{1}{x^5} \operatorname{Arth} \frac{x}{a} dx = \frac{1}{4} \left(\frac{1}{a^4} - \frac{1}{x^4} \right) \operatorname{Arth} \frac{x}{a} - \frac{1}{12ax^3} - \frac{1}{4a^3x}.$$

$$735.9. \quad \int \frac{1}{x^p} \operatorname{Arth} \frac{x}{a} dx = -\frac{1}{(p-1)x^{p-1}} \operatorname{Arth} \frac{x}{a} + \frac{a}{p-1} \int \frac{dx}{x^{p-1}(a^2-x^2)} \quad [p \neq 1].$$

[См. 151.1—155.1.]

В 734—735.9 всюду $x^2 < a^2$.

$$736. \quad \int \operatorname{Arcth} \frac{x}{a} dx = x \operatorname{Arcth} \frac{x}{a} + \frac{a}{2} \ln(x^2 - a^2).$$

$$736.1. \quad \int x \operatorname{Arcth} \frac{x}{a} dx = \frac{x^2 - a^2}{2} \operatorname{Arcth} \frac{x}{a} + \frac{ax}{2}.$$

$$736.2. \quad \int x^2 \operatorname{Arcth} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{Arcth} \frac{x}{a} + \frac{ax^2}{6} + \frac{a^3}{6} \ln(x^2 - a^2).$$

$$736.3. \quad \int x^3 \operatorname{Arcth} \frac{x}{a} dx = \frac{x^4 - a^4}{4} \operatorname{Arcth} \frac{x}{a} + \frac{ax^3}{12} + \frac{a^3x}{4}.$$

$$736.9. \quad \int x^p \operatorname{Arcth} \frac{x}{a} dx = \frac{x^{p+1}}{p+1} \operatorname{Arcth} \frac{x}{a} - \frac{a}{p+1} \int \frac{x^{p+1} dx}{a^2 - x^2}$$

[$p \neq -1$]. [См. 141.1—148.1.]

$$737.1. \quad \int \frac{1}{x} \operatorname{Arcth} \frac{x}{a} dx = -\frac{a}{x} - \frac{a^3}{3^2x^3} - \frac{a^5}{5^2x^5} - \frac{a^7}{7^2x^7} - \dots$$

$$737.2. \quad \int \frac{1}{x^2} \operatorname{Arcth} \frac{x}{a} dx = -\frac{1}{x} \operatorname{Arcth} \frac{x}{a} - \frac{1}{2a} \ln \left(\frac{x^2 - a^2}{x^2} \right).$$

$$737.3. \quad \int \frac{1}{x^3} \operatorname{Arcth} \frac{x}{a} dx = \frac{1}{2} \left(\frac{1}{a^2} - \frac{1}{x^2} \right) \operatorname{Arcth} \frac{x}{a} - \frac{1}{2ax}.$$

$$737.4. \quad \int \frac{1}{x^4} \operatorname{Arcth} \frac{x}{a} dx =$$

$$= -\frac{1}{3x^3} \operatorname{Arcth} \frac{x}{a} - \frac{1}{6ax^2} - \frac{1}{6a^3} \ln \left(\frac{x^2 - a^2}{x^2} \right).$$

$$737.5. \quad \int \frac{1}{x^5} \operatorname{Arcth} \frac{x}{a} dx = \frac{1}{4} \left(\frac{1}{a^4} - \frac{1}{x^4} \right) \operatorname{Arcth} \frac{x}{a} - \frac{1}{12ax^3} - \frac{1}{4a^3x}.$$

$$737.9. \quad \int \frac{1}{x^p} \operatorname{Arcth} \frac{x}{a} dx = -\frac{1}{(p-1)x^{p-1}} \operatorname{Arcth} \frac{x}{a} + \frac{a}{p-1} \int \frac{dx}{x^{p-1}(a^2-x^2)} \quad [p \neq 1].$$

[См. 151.1—155.1.]

В 736—737.9 всюду $x^2 > a^2$.

$$\begin{aligned}
 738. \quad \int \operatorname{Arsech} \frac{x}{a} dx &= x \operatorname{Arsech} \frac{x}{a} + a \arcsin \frac{x}{a} \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} > 0 \right], \\
 &= x \operatorname{Arsech} \frac{x}{a} - a \arcsin \frac{x}{a} \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} < 0 \right].
 \end{aligned}$$

$$\begin{aligned}
 738.1. \quad \int x \operatorname{Arsech} \frac{x}{a} dx &= \frac{x^2}{2} \operatorname{Arsech} \frac{x}{a} - \frac{a}{2} \sqrt{a^2 - x^2} \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} > 0 \right], \\
 &= \frac{x^2}{2} \operatorname{Arsech} \frac{x}{a} + \frac{a}{2} \sqrt{a^2 - x^2} \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} < 0 \right].
 \end{aligned}$$

$$\begin{aligned}
 738.2. \quad \int x^2 \operatorname{Arsech} \frac{x}{a} dx &= \frac{x^3}{3} \operatorname{Arsech} \frac{x}{a} - \frac{ax}{6} \sqrt{a^2 - x^2} + \frac{a^3}{6} \arcsin \frac{x}{a} \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} > 0 \right], \\
 &= \frac{x^3}{3} \operatorname{Arsech} \frac{x}{a} + \frac{ax}{6} \sqrt{a^2 - x^2} - \frac{a^3}{6} \arcsin \frac{x}{a} \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} < 0 \right].
 \end{aligned}$$

$$\begin{aligned}
 738.9. \quad \int x^p \operatorname{Arsech} \frac{x}{a} dx &= \frac{x^{p+1}}{p+1} \operatorname{Arsech} \frac{x}{a} + \frac{a}{p+1} \int \frac{x^p dx}{\sqrt{a^2 - x^2}} \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} > 0 \quad p \neq -1 \right], \\
 &= \frac{x^{p+1}}{p+1} \operatorname{Arsech} \frac{x}{a} - \frac{a}{p+1} \int \frac{x^p dx}{\sqrt{a^2 - x^2}} \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} < 0, \quad p \neq -1 \right]. \\
 &\quad \text{[См. 320.01—327.01.]}
 \end{aligned}$$

$$\begin{aligned}
 739.1. \quad \int \frac{1}{x} \operatorname{Arsech} \frac{x}{a} dx &= -\frac{1}{2} \left(\ln \frac{a}{x} \right) \ln \frac{4a}{x} - \frac{1}{2^3} \frac{x^2}{a^2} - \frac{1 \cdot 3}{2 \cdot 4^3} \frac{x^4}{a^4} - \\
 &\quad - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6^3} \frac{x^6}{a^6} - \dots \quad \left[\operatorname{Arsech} \frac{x}{a} > 0 \right], \\
 &= \frac{1}{2} \left(\ln \frac{a}{x} \right) \ln \frac{4a}{x} + \frac{1}{2^3} \frac{x^2}{a^2} + \frac{1 \cdot 3}{2 \cdot 4^3} \frac{x^4}{a^4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6^3} \frac{x^6}{a^6} + \dots \\
 &\quad \left[\operatorname{Arsech} \frac{x}{a} < 0 \right].
 \end{aligned}$$

$$739.2. \quad \int \frac{1}{x^2} \operatorname{Arsech} \frac{x}{a} dx = -\frac{1}{x} \operatorname{Arsech} \frac{x}{a} + \frac{\sqrt{a^2-x^2}}{ax} \\ \left[\operatorname{Arsech} \frac{x}{a} > 0 \right], \\ = -\frac{1}{x} \operatorname{Arsech} \frac{x}{a} - \frac{\sqrt{a^2-x^2}}{ax} \left[\operatorname{Arsech} \frac{x}{a} < 0 \right].$$

$$739.9. \quad \int \frac{1}{x^p} \operatorname{Arsech} \frac{x}{a} dx = \\ = -\frac{1}{(p-1)x^{p-1}} \operatorname{Arsech} \frac{x}{a} - \frac{a}{p-1} \int \frac{dx}{x^p \sqrt{a^2-x^2}} \\ \left[\operatorname{Arsech} \frac{x}{a} > 0, p \neq 1 \right], \\ = -\frac{1}{(p-1)x^{p-1}} \operatorname{Arsech} \frac{x}{a} + \frac{a}{p-1} \int \frac{dx}{x^p \sqrt{a^2-x^2}} \\ \left[\operatorname{Arsech} \frac{x}{a} < 0, p \neq 1 \right]. \quad [\text{См. } 342.01-346.01.]$$

В 738—739.9 всюду $0 < x < a$.

$$740. \quad \int \operatorname{Arcsch} \frac{x}{a} dx = x \operatorname{Arcsch} \frac{x}{a} + a \operatorname{Arsh} \frac{x}{a}.$$

$$740.1. \quad \int x \operatorname{Arcsch} \frac{x}{a} dx = \frac{x^2}{2} \operatorname{Arcsch} \frac{x}{a} + \frac{a}{2} \sqrt{x^2+a^2}.$$

$$740.9. \quad \int x^p \operatorname{Arcsch} \frac{x}{a} dx = \frac{x^{p+1}}{p+1} \operatorname{Arcsch} \frac{x}{a} + \frac{a}{p+1} \int \frac{x^p dx}{\sqrt{x^2+a^2}} \\ [p \neq -1]. \quad [\text{См. } 200.01-207.01.]$$

$$741.1. \quad \int \frac{1}{x} \operatorname{Arcsch} \frac{x}{a} dx = -\frac{a}{x} + \frac{1}{2 \cdot 3 \cdot 3} \frac{a^3}{x^3} - \frac{1 \cdot 3}{2 \cdot 4 \cdot 5 \cdot 5} \frac{a^5}{x^5} + \\ + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} \frac{a^7}{x^7} - \dots \quad [x^2 > a^2], \\ = -\frac{1}{2} \left(\ln \frac{a}{x} \right) \ln \frac{4a}{x} + \frac{1}{2^3} \frac{x^2}{a^2} - \frac{1 \cdot 3}{2 \cdot 4^3} \frac{x^4}{a^4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6^3} \frac{x^6}{a^6} - \dots \\ [0 < x < a], \\ = \frac{1}{2} \ln \left| \frac{a}{x} \right| \ln \left| \frac{4a}{x} \right| - \frac{1}{2^3} \frac{x^2}{a^2} + \frac{1 \cdot 3}{2 \cdot 4^3} \frac{x^4}{a^4} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6^3} \frac{x^6}{a^6} + \dots \\ [-a < x < 0].$$

$$741.9. \quad \int \frac{1}{x^p} \operatorname{Arcsch} \frac{x}{a} dx = \\ = -\frac{1}{(p-1)x^{p-1}} \operatorname{Arcsch} \frac{x}{a} - \frac{a}{p-1} \int \frac{dx}{x^p \sqrt{x^2+a^2}} \\ [p \neq 1]. \quad [\text{См. } 222.01-226.01.]$$

В 740—741.9 всюду $a > 0, x > 0$ (кроме 741.1).