IX.

ЭЛЛИПТИЧЕСКИЕ ФУНКЦИИ

750. Обозначим $u=\int\limits_0^{\varphi}\frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}}$ $[k^2<1]$,

$$=\int\limits_0^{x}\frac{dx}{\sqrt{1-x^2}\sqrt{1-k^2x^2}} \qquad [x=\sin\varphi],$$

$=F(\varphi,\ k)$ (*эллиптический интеграл первого рода*, см. **770**).

751.1. φ называется *амплитудой*, k — *модулем*.

751.2. $\varphi=\operatorname{am}u$.

751.3. $\sin\varphi=\operatorname{sn}u=x$. **751.4.** $\cos\varphi=\operatorname{cn}u=\sqrt{1-x^2}$.

751.5. $\Delta\varphi$ или $\Delta(\varphi,\ k)=\sqrt{1-k^2\sin^2\varphi}=\operatorname{dn}u=\sqrt{1-k^2x^2}$.

751.6. $\operatorname{tg}\varphi=\operatorname{tn}u=\frac{x}{\sqrt{1-x^2}}$.

751.7. *Дополнительный модуль* $k'=\sqrt{1-k^2}$.

752.*) $u=\operatorname{am}^{-1}(\varphi,\ k)=\operatorname{sn}^{-1}(x,\ k)=\operatorname{cn}^{-1}\{\sqrt{1-x^2},\ k\}=$

$$=\operatorname{dn}^{-1}\{\sqrt{1-k^2x^2},\ k\}=\operatorname{tn}^{-1}\left[\frac{x}{\sqrt{1-x^2}},\ k\right].$$

753.1. $\operatorname{am}(-u)=-\operatorname{am}u$. **754.1.** $\operatorname{am}0=0$.

753.2. $\operatorname{sn}(-u)=-\operatorname{sn}u$. **754.2.** $\operatorname{sn}0=0$.

753.3. $\operatorname{cn}(-u)=\operatorname{cn}u$. **754.3.** $\operatorname{cn}0=1$.

753.4. $\operatorname{dn}(-u)=\operatorname{dn}u$. **754.4.** $\operatorname{dn}0=1$.

753.5. $\operatorname{tn}(-u)=-\operatorname{tn}u$. **755.1.** $\operatorname{sn}^2u+\operatorname{cn}^2u=1$.

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*) Здесь показатель степени -1 применяется в смысле обратной функции. (*Прим. ред.*)

755.2. $\operatorname{dn}^2 u + k^2 \operatorname{sn}^2 u = 1.$ **755.3.** $\operatorname{dn}^2 u - k^2 \operatorname{cn}^2 u = k'^2.$

756.1. $\operatorname{sn}(u \pm v) = \dfrac{\operatorname{sn} u \operatorname{cn} v \operatorname{dn} v \pm \operatorname{cn} u \operatorname{sn} v \operatorname{dn} u}{1 - k^2 \operatorname{sn}^2 u \operatorname{sn}^2 v}.$

756.2. $\operatorname{cn}(u \pm v) = \dfrac{\operatorname{cn} u \operatorname{cn} v \mp \operatorname{sn} u \operatorname{sn} v \operatorname{dn} u \operatorname{dn} v}{1 - k^2 \operatorname{sn}^2 u \operatorname{sn}^2 v}.$

756.3. $\operatorname{dn}(u \pm v) = \dfrac{\operatorname{dn} u \operatorname{dn} v \mp k^2 \operatorname{sn} u \operatorname{sn} v \operatorname{cn} u \operatorname{cn} v}{1 - k^2 \operatorname{sn}^2 u \operatorname{sn}^2 v}.$

756.4. $\operatorname{tn}(u \pm v) = \dfrac{\operatorname{tn} u \operatorname{dn} v \pm \operatorname{tn} v \operatorname{dn} u}{1 \mp \operatorname{tn} u \operatorname{tn} v \operatorname{dn} u \operatorname{dn} v}.$

757.1. $\operatorname{sn} 2u = \dfrac{2 \operatorname{sn} u \operatorname{cn} u \operatorname{dn} u}{1 - k^2 \operatorname{sn}^4 u}.$

757.2. $\operatorname{cn} 2u = \dfrac{\operatorname{cn}^2 u - \operatorname{sn}^2 u \operatorname{dn}^2 u}{1 - k^2 \operatorname{sn}^4 u} = \dfrac{2 \operatorname{cn}^2 u}{1 - k^2 \operatorname{sn}^4 u} - 1.$

757.3. $\operatorname{dn} 2u = \dfrac{\operatorname{dn}^2 u - k^2 \operatorname{sn}^2 u \operatorname{cn}^2 u}{1 - k^2 \operatorname{sn}^4 u} = \dfrac{2 \operatorname{dn}^2 u}{1 - k^2 \operatorname{sn}^4 u} - 1.$

757.4. $\operatorname{tn} 2u = \dfrac{2 \operatorname{tn} u \operatorname{dn} u}{1 - \operatorname{tn}^2 u \operatorname{dn}^2 u}.$

758.1. $\operatorname{sn} \dfrac{u}{2} = \sqrt{\dfrac{1 - \operatorname{cn} u}{1 + \operatorname{dn} u}}.$

758.2. $\operatorname{cn} \dfrac{u}{2} = \sqrt{\dfrac{\operatorname{cn} u + \operatorname{dn} u}{1 + \operatorname{dn} u}}.$

758.3. $\operatorname{dn} \dfrac{u}{2} = \sqrt{\dfrac{\operatorname{cn} u + \operatorname{dn} u}{1 + \operatorname{cn} u}}.$

759.1. $\operatorname{sn}(iu, k) = i \operatorname{tn}(u, k').$

759.2. $\operatorname{cn}(iu, k) = \dfrac{1}{\operatorname{cn}(u, k')}.$

759.3. $\operatorname{dn}(iu, k) = \dfrac{\operatorname{dn}(u, k')}{\operatorname{cn}(u, k')}.$

760.1. $\operatorname{sn} u = u - (1 + k^2)\dfrac{u^3}{3!} + (1 + 14k^2 + k^4)\dfrac{u^5}{5!} -$

$- (1 + 135k^2 + 135k^4 + k^6)\dfrac{u^7}{7!} + \ldots$

760.2. $\operatorname{cn} u = 1 - \dfrac{u^2}{2!} + (1 + 4k^2)\dfrac{u^4}{4!} - (1 + 44k^2 + 16k^4)\dfrac{u^6}{6!} +$

$+ (1 + 408k^2 + 912k^4 + 64k^6)\dfrac{u^8}{8!} - \ldots$

760.3. $$\operatorname{dn} u = 1 - k^2 \frac{u^2}{2!} + (4 + k^2) k^2 \frac{u^4}{4!} - (16 + 44k^2 + k^4) k^2 \frac{u^6}{6!} + (64 + 912k^2 + 408k^4 + k^6) k^2 \frac{u^8}{8!} - \ldots$$

760.4. $$\operatorname{am} u = u - k^2 \frac{u^3}{3!} + (4 + k^2) k^2 \frac{u^5}{5!} - (16 + 44k^2 + k^4) k^2 \frac{u^7}{7!} + (64 + 912k^2 + 408k^4 + k^6) k^2 \frac{u^9}{9!} - \ldots$$