Бесселевы функции от аргумента $xi\sqrt{i}$ первого рода

820.1. $\operatorname{ber} x + i \operatorname{bei} x = J_0(xi\sqrt{i}) = I_0(x\sqrt{i}) = \operatorname{ber}_0 x + i \operatorname{bei}_0 x.$

820.2. $\operatorname{ber}' x = \frac{d}{dx} \operatorname{ber} x$, и т. д.

820.3. $$\operatorname{ber} x = 1 - \frac{\left(\frac{1}{2}x\right)^4}{(2!)^2} + \frac{\left(\frac{1}{2}x\right)^8}{(4!)^2} - \ldots$$

820.4. $$\operatorname{bei} x = \frac{\left(\frac{1}{2}x\right)^2}{(1!)^2} - \frac{\left(\frac{1}{2}x\right)^6}{(3!)^2} + \frac{\left(\frac{1}{2}x\right)^{10}}{(5!)^2} - \ldots$$

820.5. $$\operatorname{ber}' x = -\frac{\left(\frac{1}{2}x\right)^3}{1!\,2!} + \frac{\left(\frac{1}{2}x\right)^7}{3!\,4!} - \frac{\left(\frac{1}{2}x\right)^{11}}{5!\,6!} + \ldots$$

820.6. $$\operatorname{bei}' x = \frac{1}{2}x - \frac{\left(\frac{1}{2}x\right)^5}{2!\,3!} + \frac{\left(\frac{1}{2}x\right)^9}{4!\,5!} - \ldots$$

821.1. Для больших значений x

$$\operatorname{ber} x \approx \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[L_0(x)\cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right) - M_0(x)\sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right)\right],$$

821.2. $$\operatorname{bei} x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[M_0(x)\cos\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}\right)+ L_0(x)\sin\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}\right)\right],$$

где

821.3. $$L_0(x)\approx 1+\frac{1^2}{1!\,8x}\cos\frac{\pi}{4}+\frac{1^2\cdot 3^2}{2!\,(8x)^2}\cos\frac{2\pi}{4}+\frac{1^2\cdot 3^2\cdot 5^2}{3!\,(8x)^3}\cos\frac{3\pi}{4}+\ldots,$$

821.4. $$M_0(x)\approx -\frac{1^2}{1!\,8x}\sin\frac{\pi}{4}-\frac{1^2\cdot 3^2}{2!\,(8x)^2}\sin\frac{2\pi}{4}-\frac{1^2\cdot 3^2\cdot 5^2}{3!\,(8x)^3}\sin\frac{3\pi}{4}-\ldots$$

821.5. $$\operatorname{ber}' x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[S_0(x)\cos\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}\right)- T_0(x)\sin\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}\right)\right].$$

821.6. $$\operatorname{bei}' x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[T_0(x)\cos\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}\right)+ S_0(x)\sin\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}\right)\right],$$

где

821.7. $$S_0(x)\approx 1-\frac{1\cdot 3}{1!\,8x}\cos\frac{\pi}{4}-\frac{1^2\cdot 3\cdot 5}{2!\,(8x)^2}\cos\frac{2\pi}{4}- -\frac{1^2\cdot 3^2\cdot 5\cdot 7}{3!\,(8x)^3}\cos\frac{3\pi}{4}-\frac{1^2\cdot 3^2\cdot 5^2\cdot 7\cdot 9}{4!\,(8x)^4}\cos\frac{4\pi}{4}-\ldots$$

821.8. $$T_0(x)\approx \frac{1\cdot 3}{1!\,8x}\sin\frac{\pi}{4}+\frac{1^2\cdot 3\cdot 5}{2!\,(8x)^2}\sin\frac{2\pi}{4}+ +\frac{1^2\cdot 3^2\cdot 5\cdot 7}{3!\,(8x)^3}\sin\frac{3\pi}{4}+\frac{1^2\cdot 3^2\cdot 5^2\cdot 7\cdot 9}{4!\,(8x)^4}\sin\frac{4\pi}{4}+\ldots$$

822.1. При n целом положительном

$$\operatorname{ber}_n x + i\operatorname{bei}_n x = J_n(xi\sqrt{i}) = i^n I_n(x\sqrt{i}).$$

822.2. $$\operatorname{ber}_n x = \sum_{p=0}^{\infty}\frac{(-1)^{n+p}\left(\frac{1}{2}x\right)^{n+2p}}{p!\,(n+p)!}\cos\frac{(n+2p)\pi}{4}.$$

822.3. $$\operatorname{bei}_n x = \sum_{p=0}^{\infty}\frac{(-1)^{n+p+1}\left(\frac{1}{2}x\right)^{n+2p}}{p!\,(n+p)!}\sin\frac{(n+2p)\pi}{4}.$$

822.4. $$\operatorname{ber}'_n x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p}\left(\frac{n}{2}+p\right)\left(\frac{1}{2}x\right)^{n+2p-1}}{p!\,(n+p)!} \cos\frac{(n+2p)\pi}{4}.$$

822.5. $$\operatorname{bei}'_n x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p+1}\left(\frac{n}{2}+p\right)\left(\frac{1}{2}x\right)^{n+2p-1}}{p!\,(n+p)!} \sin\frac{(n+2p)\pi}{4}.$$

823.1. Для больших значений x, при целом положительном n:

$$\operatorname{ber}_n x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[L_n(x)\cos\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)- M_n(x)\sin\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)\right].$$

823.2. $$\operatorname{bei}_n x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[M_n(x)\cos\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)+ L_n(x)\sin\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)\right],$$

где

823.3. $$L_n(x) \approx 1 - \frac{4n^2-1^2}{1!\,8x}\cos\frac{\pi}{4} + \frac{(4n^2-1^2)(4n^2-3^2)}{2!\,(8x)^2}\cos\frac{2\pi}{4} - \ldots$$

823.4. $$M_n(x) \approx \frac{4n^2-1^2}{1!\,8x}\sin\frac{\pi}{4} - \frac{(4n^2-1^2)(4n^2-3^2)}{2!\,(8x)^2}\sin\frac{2\pi}{4} + \ldots$$

823.5. $$\operatorname{ber}'_n x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[S_n(x)\cos\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)- T_n(x)\sin\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)\right].$$

823.6. $$\operatorname{bei}'_n x = \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}}\left[T_n(x)\cos\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)+ S_n(x)\sin\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)\right],$$

где

823.7. $$S_n(x) \approx 1 - \frac{4n^2+1\times 3}{1!\,8x}\cos\frac{\pi}{4} + \frac{(4n^2-1^2)(4n^2+3\times 5)}{2!\,(8x)^2}\cos\frac{2\pi}{4} - \frac{(4n^2-1^2)(4n^2-3^2)(4n^2+5\times 7)}{3!\,(8x)^3}\cos\frac{3\pi}{4} + \ldots$$

823.8. $$T_n(x) \approx \frac{4n^2+1\times 3}{1!\,8x}\sin\frac{\pi}{4} - \frac{(4n^2-1^2)(4n^2+3\times 5)}{2!\,(8x)^2}\sin\frac{2\pi}{4} + \frac{(4n^2-1^2)(4n^2-3^2)(4n^2+5\times 7)}{3!\,(8x)^3}\sin\frac{3\pi}{4} - \ldots$$