858.542. $$\int_0^{\pi/2} \frac{dx}{(1+a^2 \sin^2 x)^2} = \int_0^{\pi/2} \frac{dx}{(1+a^2 \cos^2 x)^2} = \frac{\pi}{4} \frac{2+a^2}{(1+a^2)^{3/2}}.$$

858.543. $$\int_0^{\pi/2} \frac{dx}{(1-a^2 \sin^2 x)^2} = \int_0^{\pi/2} \frac{dx}{(1-a^2 \cos^2 x)^2} = \frac{\pi}{4} \frac{2-a^2}{(1-a^2)^{3/2}} \qquad [a^2 < 1].$$

858.544. $$\int_0^{\pi} \frac{x \sin mx \, dx}{1+\cos^2 mx} = \frac{\pi^2}{4m^2} \qquad [m > 0].$$

858.545. $$\int_0^{\pi} \frac{x \, dx}{1+\cos\varphi \sin x} = \frac{\pi\varphi}{\sin\varphi}.$$

858.546. $$\int_0^{\pi} \frac{\sin^2 x \, dx}{a+b\cos x} = \frac{\pi}{a+\sqrt{a^2-b^2}} \qquad [a,\ a^2-b^2 > 0].$$

858.550. $$\int_0^{\pi/2} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{\pi}{2ab} \qquad [ab > 0].$$

858.551. $$\int_0^{\pi} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{\pi}{ab} \qquad [ab > 0].$$

858.552. $$\int_0^{\pi/2} \frac{\sin^2 x \, dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \int_0^{\pi/2} \frac{dx}{a^2 + b^2 \operatorname{ctg}^2 x} = \frac{\pi}{2a(a+b)} \qquad [a,\ b > 0].$$

858.553. $$\int_0^{\pi/2} \frac{\cos^2 x \, dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \int_0^{\pi/2} \frac{dx}{b^2 + a^2 \operatorname{tg}^2 x} = \frac{\pi}{2b(a+b)} \qquad [a,\ b > 0].$$

858.554. $$\int_0^{\pi/2} \frac{dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} = \frac{\pi}{4} \frac{a^2+b^2}{a^3 b^3} \qquad [ab > 0].$$

858.555. $$\int_0^{\pi/2} \frac{\sin^2 x \, dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} = \frac{\pi}{4a^3 b} \qquad [ab > 0].$$

858.556. $$\int_0^{\pi/2} \frac{\cos^2 x \, dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} = \frac{\pi}{4ab^3} \qquad [ab > 0].$$

858.560. $\int_0^\infty \sin(a^2x^2)\,dx = \int_0^\infty \cos(a^2x^2)\,dx = \frac{\sqrt{\pi}}{2a\sqrt{2}}$ $[a > 0]$.

858.561. $\int_0^\infty \sin\frac{\pi x^2}{2}\,dx = \int_0^\infty \cos\frac{\pi x^2}{2}\,dx = \frac{1}{2}$.

[*Интегралы Френеля.*]

858.562. $\int_0^\infty \sin(x^p)\,dx = \Gamma\left(1+\frac{1}{p}\right)\sin\frac{\pi}{2p}$ $[p > 1]$.

858.563. $\int_0^\infty \cos(x^p)\,dx = \Gamma\left(1+\frac{1}{p}\right)\cos\frac{\pi}{2p}$ $[p > 1]$.

858.564. $\int_0^\infty \sin a^2x^2 \cos mx\,dx = \frac{\sqrt{\pi}}{2a}\sin\left(\frac{\pi}{4}-\frac{m^2}{4a^2}\right)$ $[a > 0]$.

858.565. $\int_0^\infty \cos a^2x^2 \cos mx\,dx = \frac{\sqrt{\pi}}{2a}\cos\left(\frac{\pi}{4}-\frac{m^2}{4a^2}\right)$ $[a > 0]$.

858.601. $\int_0^\infty \frac{\sin mx}{x}\,dx = \frac{\pi}{2}$, 0 или $-\frac{\pi}{2}$ в зависимости от того, будет ли m положительным, нулем или отрицательным.

858.611. $\int_0^\infty \frac{\cos mx - \cos nx}{x}\,dx = \ln\frac{n}{m}$ $[m,\ n > 0]$.

858.621. $\int_0^\infty \frac{\operatorname{tg} mx}{x}\,dx = \frac{\pi}{2}$, 0 или $-\frac{\pi}{2}$ в зависимости от того, будет ли m положительным, нулем или отрицательным.

858.630. $\int_0^\infty \frac{\sin^2 mx}{x}\,dx = \int_0^\infty \frac{\cos^2 mx}{x}\,dx = \infty$.

858.631. $\int_0^\infty \frac{\sin^2 mx - \sin^2 nx}{x}\,dx = \frac{1}{2}\ln\frac{m}{n}$ $[m,\ n \neq 0]$.

858.632. $$\int_0^\infty \frac{\cos^2 mx - \cos^2 nx}{x}\,dx = \frac{1}{2}\ln\frac{n}{m} \qquad [m,\ n \neq 0].$$

858.641. $$\int_0^\infty \frac{\sin^3 mx}{x}\,dx = \frac{\pi}{4} \qquad [m > 0].$$

858.649. $$\int_0^\infty \frac{\sin^{2p+1} mx}{x}\,dx = \frac{1\cdot 3\cdot 5\ \ldots\ (2p-1)}{2\cdot 4\cdot 6\ \ldots\ (2p)}\,\frac{\pi}{2}$$
$$[p = 1,\ 2,\ 3,\ \ldots;\ m > 0].$$

858.650. $$\int_0^\infty \frac{\sin mx}{x^2}\,dx = \int_0^\infty \frac{\cos mx}{x^2}\,dx = \infty.$$

858.651. $$\int_0^\infty \frac{\cos mx - \cos nx}{x^2}\,dx = (n-m)\,\frac{\pi}{2} \qquad [n > m > 0].$$

858.652. $$\int_0^\infty \frac{\sin^2 mx}{x^2}\,dx = |m|\,\frac{\pi}{2}.$$ [См. **858.711.**]

858.653. $$\int_0^\infty \frac{\sin^3 mx}{x^2}\,dx = \frac{3}{4}\,m\ln 3.$$

858.654. $$\int_0^\infty \frac{\sin^4 mx}{x^2}\,dx = |m|\,\frac{\pi}{4}.$$

858.659. $$\int_0^\infty \frac{\sin^{2p} mx}{x^2}\,dx = \frac{1\cdot 3\cdot 5\ \ldots\ (2p-3)}{2\cdot 4\cdot 6\ \ldots\ (2p-2)}\,\frac{|m|\,\pi}{2} \qquad [p = 2,\ 3,\ 4,\ \ldots].$$

858.661. $$\int_0^\infty \frac{\sin^3 mx}{x^3}\,dx = \frac{3}{8}\,m^2\pi \qquad [m > 0].$$

858.701. $$\int_0^\infty \frac{\sin mx \cos nx}{x}\,dx = \frac{\pi}{2} \qquad [m > n > 0],$$
$$= \frac{\pi}{4} \qquad [m = n > 0],$$
$$= 0 \qquad [n > m > 0].$$

858.702. $$\int_0^\infty \frac{\sin mx \sin nx}{x}\,dx = \frac{1}{2}\ln\frac{m+n}{m-n} \qquad [m > n > 0].$$

858.703. $$\int_0^\infty \frac{\cos mx \cos nx}{x}\,dx = \infty.$$

858.704. $$\int_0^\infty \frac{\sin^2 ax \sin mx}{x}\,dx = \frac{\pi}{4} \qquad [2a > m > 0],$$
$$= \frac{\pi}{8} \qquad [2a = m > 0],$$
$$= 0 \qquad [m > 2a > 0].$$

858.711. $$\int_0^\infty \frac{\sin mx \sin nx}{x^2}\,dx = \frac{m\pi}{2} \qquad [n \geqslant m > 0],$$
$$= \frac{n\pi}{2} \qquad [m \geqslant n > 0].$$

858.712. $$\int_0^\infty \frac{\sin^2 ax \sin mx}{x^2}\,dx = \frac{m+2a}{4}\ln|m+2a| + \frac{m-2a}{4}\ln|m-2a| - \frac{m}{2}\ln m \qquad [m > 0].$$

858.713. $$\int_0^\infty \frac{\sin^2 ax \cos mx}{x^2}\,dx = \frac{\pi}{2}\left(a - \frac{m}{2}\right) \qquad \left[a > \frac{m}{2} > 0\right],$$
$$= 0 \qquad \left[\frac{m}{2} \geqslant a \geqslant 0\right].$$

858.721. $$\int_0^\infty \frac{1-\cos mx}{x^2}\,dx = \frac{\pi|m|}{2}.$$

858.731. $$\int_0^\infty \frac{\sin^2 ax \sin mx}{x^3}\,dx = \frac{\pi am}{2} - \frac{\pi m^2}{8} \qquad \left[a \geqslant \frac{m}{2} > 0\right],$$
$$= \frac{\pi a^2}{2} \qquad \left[\frac{m}{2} \geqslant a > 0\right].$$

858.801. $$\int_0^\infty \frac{\sin mx}{\sqrt{x}}\,dx = \int_0^\infty \frac{\cos mx}{\sqrt{x}}\,dx = \frac{\sqrt{\pi}}{\sqrt{2m}} \qquad [m > 0].$$

858.802. $$\int_0^\infty \frac{\sin mx}{x\sqrt{x}}\,dx = \sqrt{2\pi m} \qquad [m > 0].$$

858.811. $$\int_0^\infty \frac{\sin mx}{x^p}\,dx = \frac{\pi m^{p-1}}{2\sin\frac{p\pi}{2}\,\Gamma(p)} \qquad [0<p<2;\ m>0].$$

К тому же численному результату приводит другая формула:

858.812. $$\int_0^\infty x^{q-1}\sin mx\,dx = \frac{\Gamma(q)}{m^q}\sin\frac{q\pi}{2}$$
$$[0<q^2<1;\ m>0].$$

Для q, близкого к нулю и равного нулю, пользоваться формулами **858.601** или **858.811.**

858.813. $$\int_0^\infty \frac{\cos mx}{x^p}\,dx = \frac{\pi m^{p-1}}{2\cos\frac{p\pi}{2}\,\Gamma(p)} \qquad [0<p<1;\ m>0].$$

К тому же численному результату приводит другая формула:

858.814. $$\int_0^\infty x^{q-1}\cos mx\,dx = \frac{\Gamma(q)}{m^q}\cos\frac{q\pi}{2} \qquad [0<q<1;\ m>0].$$

858.821. $$\int_0^\infty \frac{\sin mx\cos nx}{\sqrt{x}}\,dx =$$
$$= \left\{\frac{1}{\sqrt{m+n}}+\frac{1}{\sqrt{m-n}}\right\}\frac{\sqrt{\pi}}{2\sqrt{2}} \qquad [m>n>0],$$
$$= \left\{\frac{1}{\sqrt{m+n}}-\frac{1}{\sqrt{n-m}}\right\}\frac{\sqrt{\pi}}{2\sqrt{2}} \qquad [n>m>0].$$

858.822. $$\int_0^\infty \frac{\sin^2 mx}{x\sqrt{x}}\,dx = \sqrt{m\pi} \qquad [m>0].$$

858.823. $$\int_0^\infty \frac{\sin^3 mx}{\sqrt{x}}\,dx = \frac{(3\sqrt{3}-1)}{4}\sqrt{\frac{\pi}{6m}} \qquad [m>0].$$

858.824. $$\int_0^\infty \frac{\sin^3 mx}{x\sqrt{x}}\,dx = \frac{(3-\sqrt{3})}{4}\sqrt{2m\pi} \qquad [m>0].$$

858.831. $$\int_0^\infty \frac{\operatorname{arctg} x\,dx}{(1+x)\sqrt{x}} = \frac{\pi^2}{4}.$$

858.832. $\int_0^\infty \operatorname{arctg}\frac{a}{x}\sin mx\,dx = \frac{\pi}{2m}(1-e^{-am})$ $[a,\ m>0]$.

858.841. $\int_0^\infty \frac{\sin x\,dx}{x\sqrt{1-k^2\sin^2 x}} = K(k)$ $[0<k<1]$.

858.842. $\int_0^\infty \frac{\sin x\,dx}{x\sqrt{1-k^2\cos^2 x}} = K(k)$ $[0<k<1]$.

858.843. $\int_0^\infty \frac{\sin x\cos x\,dx}{x\sqrt{1-k^2\sin^2 x}} = \frac{1}{k^2}\{E(k)-(1-k^2)K(k)\}$ $[0<k<1]$.

Для **858.841—.843** см. **773.1, 774.1.**

859.001. $\int_0^\infty \frac{\cos mx}{a^2+x^2}\,dx = \frac{\pi}{2a}e^{-ma}$ $[a>0;\ m\geqslant 0]$.

859.002. $\int_0^\infty \frac{\sin^2 mx}{a^2+x^2}\,dx = \frac{\pi}{4a}(1-e^{-2ma})$ $[a>0;\ m\geqslant 0]$.

859.003. $\int_0^\infty \frac{\cos^2 mx}{a^2+x^2}\,dx = \frac{\pi}{4a}(1+e^{-2ma})$ $[a>0;\ m\geqslant 0]$.

859.004. $\int_0^\infty \frac{x\sin mx}{a^2+x^2}\,dx = \frac{\pi}{2}e^{-ma}$ $[a\geqslant 0;\ m>0]$.

859.005. $\int_0^\infty \frac{\sin mx}{x(a^2+x^2)}\,dx = \frac{\pi}{2a^2}(1-e^{-ma})$ $[a>0;\ m\geqslant 0]$.

859.006. $\int_0^\infty \frac{\sin mx\sin nx}{a^2+x^2}\,dx = \frac{\pi}{2a}e^{-ma}\operatorname{sh} na$ $[a>0;\ m\geqslant n\geqslant 0]$.

859.007. $\int_0^\infty \frac{\cos mx\cos nx}{a^2+x^2}\,dx = \frac{\pi}{2a}e^{-ma}\operatorname{ch} na$ $[a>0;\ m\geqslant n\geqslant 0]$.

859.008. $\int_0^\infty \frac{x\sin mx\cos nx}{a^2+x^2}\,dx = \frac{\pi}{2}e^{-ma}\operatorname{ch} na$ $[a>0;\ m>n>0]$,

$= -\frac{\pi}{2}e^{-na}\operatorname{sh} ma$ $[a>0;\ n>m>0]$.

859.011. $\int\limits_0^\infty \frac{\cos mx}{(a^2+x^2)^2}\,dx = \frac{\pi}{4a^3}(1+ma)\,e^{-ma}$ $[a,\ m>0]$.

859.012. $\int\limits_0^\infty \frac{x\sin mx}{(a^2+x^2)^2}\,dx = \frac{\pi m}{4a}\,e^{-ma}$ $[a,\ m>0]$.

859.013. $\int\limits_0^\infty \frac{x^2\cos mx}{(a^2+x^2)^2}\,dx = \frac{\pi}{4a}(1-ma)\,e^{-ma}$ $[a,\ m>0]$.

859.014. $\int\limits_0^\infty \frac{\sin mx}{x(a^2+x^2)^2}\,dx = \frac{\pi}{2a^4}\left(1-\frac{2+ma}{2}\,e^{-ma}\right)$ $[a,\ m>0]$.

859.021. $\int\limits_0^\infty \frac{\cos mx\,dx}{(a^2+x^2)(b^2+x^2)} = \left(\frac{e^{-mb}}{b}-\frac{e^{-ma}}{a}\right)\frac{\pi}{2(a^2-b^2)}$

$[a,\ b,\ m>0;\ a\neq b]$.

859.022. $\int\limits_0^\infty \frac{x\sin mx\,dx}{(a^2+x^2)(b^2+x^2)} = \left(\frac{e^{-mb}-e^{-ma}}{a^2-b^2}\right)\frac{\pi}{2}$

$[a,\ b,\ m>0;\ a\neq b]$.

859.031. $\int\limits_0^\infty \frac{\cos mx}{x^4+4a^4}\,dx = \frac{\pi e^{-ma}}{8a^3}(\sin ma+\cos ma)$.

859.032. $\int\limits_0^\infty \frac{\sin mx}{x(x^4+4a^4)}\,dx = \frac{\pi}{8a^4}(1-e^{-ma}\cos ma)$.

859.033. $\int\limits_0^\infty \frac{x\sin mx}{x^4+4a^4}\,dx = \frac{\pi}{4a^2}\,e^{-ma}\sin ma$.

859.034. $\int\limits_0^\infty \frac{x^2\cos mx}{x^4+4a^4}\,dx = \frac{\pi}{4a}\,e^{-ma}(\cos ma-\sin ma)$.

859.035. $\int\limits_0^\infty \frac{x^3\sin mx}{x^4+4a^4}\,dx = \frac{\pi}{2}\,e^{-ma}\cos ma$.

Для **859.031 —.035** $a,\ m>0$.

859.041. $\int_0^\infty \frac{\cos mx}{\sqrt{a^2+x^2}}\,dx = K_0(ma)$. [См. **815.1.**] $[ma>0]$.

859.042. $\int_0^1 \frac{\cos mx}{\sqrt{1-x^2}}\,dx = \frac{\pi}{2} J_0(m)$.

859.043. $\int_0^\pi \frac{dx}{\sqrt{a\pm b\cos x}} = \frac{2}{\sqrt{a+b}} K\left(\sqrt{\frac{2b}{a+b}}\right)$ $[0<b<a]$.

$\left[\frac{2b}{a+b} = k^2 = \sin^2\theta.\right.$ См. $\left.\mathbf{773.1.}\right]$

859.100. $\int_0^{\pi/2} \frac{dx}{1+2a\sin x+a^2} = \int_0^{\pi/2} \frac{dx}{1+2a\cos x+a^2} =$

$$= \frac{\frac{\pi}{2} - \arcsin\frac{2a}{1+a^2}}{|1-a^2|} = \frac{\arccos\frac{2a}{1+a^2}}{|1-a^2|}.$$

[См. примечание к **859.112.**]

859.101. $\int_0^{\pi/2} \frac{dx}{1-2a\sin x+a^2} = \int_0^{\pi/2} \frac{dx}{1-2a\cos x+a^2} =$

$$= \frac{\frac{\pi}{2} + \arcsin\frac{2a}{1+a^2}}{|1-a^2|}.$$

[См. примечание к **859.112.**]

859.111. $\int_0^\pi \frac{dx}{1+2a\sin x+a^2} = \frac{\pi - 2\arcsin\frac{2a}{1+a^2}}{|1-a^2|} = \frac{2\arccos\frac{2a}{1+a^2}}{|1-a^2|}$.

[См. примечание к **859.112.**]

859.112. $\int_0^\pi \frac{dx}{1-2a\sin x+a^2} = \frac{\pi + 2\arcsin\frac{2a}{1+a^2}}{|1-a^2|}$.

В **859.100—.112** $a>0$; $a\neq 1$; arcsin и arccos везде в первом квадранте.

859.113. $\int_0^\pi \frac{dx}{a^2\pm 2ab\cos x+b^2} = \frac{\pi}{|a^2-b^2|}$ $[a^2\neq b^2]$.

859.121. $$\int_0^\pi \frac{\sin x\,dx}{1-2a\cos x+a^2}=\frac{2}{a}\operatorname{Arth} a=\frac{1}{a}\ln\frac{1+a}{1-a} \qquad [a^2<1],$$
$$=\frac{2}{a}\operatorname{Arcth} a=\frac{1}{a}\ln\frac{a+1}{a-1} \qquad [a^2>1].$$

859.122. $$\int_0^\pi \frac{\cos mx\,dx}{1-2a\cos x+a^2}=\frac{\pi a^m}{1-a^2} \qquad [a^2<1;\ m=0,\ 1,\ 2,\ \ldots],$$
$$=\frac{\pi}{a^m(a^2-1)} \qquad [a^2>1;\ m=0,\ 1,\ 2,\ \ldots].$$

859.123. $$\int_0^\pi \frac{x\sin x\,dx}{1-2a\cos x+a^2}=\frac{\pi}{a}\ln(1+a) \qquad [a^2\leqslant 1],$$
$$=\frac{\pi}{a}\ln\left(1+\frac{1}{a}\right) \qquad [a^2\geqslant 1].$$

859.124. $$\int_0^\pi \frac{(a-b\cos x)\,dx}{a^2-2ab\cos x+b^2}=\frac{\pi}{a} \qquad [a>b>0],$$
$$=0 \qquad [b>a>0].$$

859.131. $$\int_0^\pi \frac{\sin^2 x\,dx}{a^2-2ab\cos x+b^2}=\frac{\pi}{2a^2} \qquad [a>b>0],$$
$$=\frac{\pi}{2b^2} \qquad [b>a>0].$$

859.132. $$\int_0^\pi \frac{\cos^2 x\,dx}{1-2a\cos x+a^2}=\frac{\pi}{2}\left(\frac{1+a^2}{1-a^2}\right) \qquad [a^2<1],$$
$$=\frac{\pi}{2a^2}\left(\frac{a^2+1}{a^2-1}\right) \qquad [a^2>1].$$

859.141. $$\int_0^\pi \frac{\sin x\sin mx\,dx}{1-2a\cos x+a^2}=\frac{\pi a^{m-1}}{2} \qquad [a^2<1;\ m=1,\ 2,\ 3,\ \ldots],$$
$$=\frac{\pi}{2a^{m+1}} \qquad [a^2>1;\ m=1,\ 2,\ 3,\ \ldots].$$

859.142. $$\int_0^\pi \frac{\cos x\cos mx\,dx}{1-2a\cos x+a^2}=\frac{\pi a^{m-1}}{2}\left(\frac{1+a^2}{1-a^2}\right) \qquad [a^2<1;\ m=1,\ 2,\ 3,\ \ldots],$$
$$=\frac{\pi}{2a^{m+1}}\left(\frac{a^2+1}{a^2-1}\right) \qquad [a^2>1;\ m=1,\ 2,\ 3,\ \ldots].$$

859.151. $$\int_0^\pi \frac{\sin x\,dx}{\sqrt{a^2-2ab\cos x+b^2}}=\frac{2}{a}\quad [a>b>0],$$
$$=\frac{2}{b}\quad [b>a>0].$$

859.161. $$\int_0^1 \frac{dx}{1+2x\cos\varphi+x^2}=\frac{\varphi}{2\sin\varphi}\quad [-\pi<\varphi<\pi].$$

При $\varphi=0$:

859.162. $$\int_0^1 \frac{dx}{1+2x+x^2}=\frac{1}{2}.$$ [См. **90.2.**]

859.163. $$\int_0^\infty \frac{dx}{1+2x\cos\varphi+x^2}=\frac{\varphi}{\sin\varphi}\quad [-\pi<\varphi<\pi].$$

При $\varphi=0$:

859.164. $$\int_0^\infty \frac{dx}{1+2x+x^2}=1.$$ [См. **90.2.**]

859.165. $$\int_0^\infty \frac{dx}{1-2x^2\cos\varphi+x^4}=\frac{\pi}{4\sin\frac{\varphi}{2}}\quad [0<\varphi<\pi].$$

859.166. $$\int_0^\infty \frac{x^p\,dx}{1+2x\cos\varphi+x^2}=\frac{\pi\sin p\varphi}{\sin p\pi\sin\varphi}\quad [0<p<1;\ 0<\varphi<\pi].$$

860.01. $$\int_0^\infty e^{-ax}\,dx=\frac{1}{a}\quad [a>0].$$ **860.02.** $$\int_0^\infty xe^{-ax}\,dx=\frac{1}{a^2}\quad [a>0].$$

860.03. $$\int_0^\infty x^2e^{-ax}\,dx=\frac{2}{a^3}\quad [a>0].$$

860.04. $$\int_0^\infty x^{1/2}e^{-ax}\,dx=\frac{\sqrt{\pi}}{2a\sqrt{a}}\quad [a>0].$$

860.05. $$\int_0^\infty x^{-1/2}e^{-ax}\,dx=\frac{\sqrt{\pi}}{\sqrt{a}}\quad [a>0].$$