

Интегралы, содержащие $X = a^2 + x^2$

$$120. \quad \int \frac{dx}{1+x^2} = \operatorname{arctg} x \quad (\text{см. рисунок на стр. 30}).$$

$$120.01. \quad \int \frac{dx}{a^2 + b^2 x^2} = \frac{1}{ab} \operatorname{arctg} \frac{bx}{a}.$$

$$120.1. \quad \int \frac{dx}{X} = \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a}.$$

$$120.2. \quad \int \frac{dx}{X^2} = \frac{x}{2a^2 X} + \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a}.$$

$$120.3. \quad \int \frac{dx}{X^3} = \frac{x}{4a^2 X^2} + \frac{3x}{8a^4 X} + \frac{3}{8a^5} \operatorname{arctg} \frac{x}{a}.$$

$$120.4. \quad \int \frac{dx}{X^4} = \frac{x}{6a^2 X^3} + \frac{5x}{24a^4 X^2} + \frac{5x}{16a^6 X} + \frac{5}{16a^7} \operatorname{arctg} \frac{x}{a}.$$

$$120.9. \quad \int \frac{dx}{(a^2 + b^2 x^2)^{n+1}} = \frac{x}{2na^2 (a^2 + b^2 x^2)^n} + \frac{2n-1}{2na^2} \int \frac{dx}{(a^2 + b^2 x^2)^n}.$$

121. Интегралы вида

$$\int \frac{x^{2m+1} dx}{(a^2 \pm x^2)^n}$$

подстановкой $x^2 = z$ приводятся к

$$\frac{1}{2} \int \frac{z^m dz}{(a^2 \pm z)^n},$$

о которых см. 89—105 (для m положительного, отрицательного или равного 0).

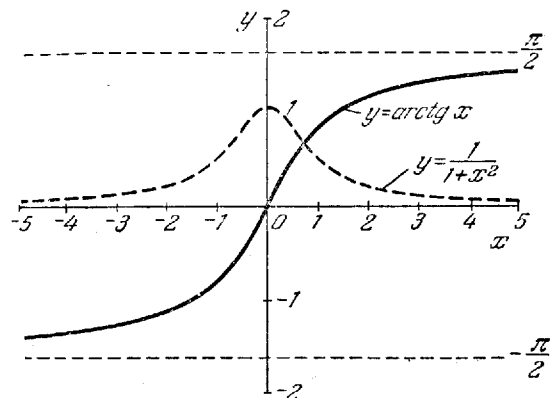


Рис. 120. Графики функций $y = \frac{1}{1+x^2}$ (пунктирная линия) и $y = \arctg x$ (сплошная линия).

$$121.1. \int \frac{x dx}{X} = \int \frac{x dx}{a^2 + x^2} = \frac{1}{2} \ln(a^2 + x^2).$$

$$121.2. \int \frac{x dx}{X^2} = -\frac{1}{2X}. \quad 121.3. \int \frac{x dx}{X^3} = -\frac{1}{4X^2}.$$

$$121.4. \int \frac{x dx}{X^4} = -\frac{1}{6X^3}.$$

$$121.9. \int \frac{x dx}{X^{n+1}} = -\frac{1}{2nX^n} \quad [n \neq 0].$$

$$122.1. \int \frac{x^2 dx}{X} = x - a \arctg \frac{x}{a}.$$

$$122.2. \int \frac{x^2 dx}{X^2} = -\frac{x}{2X} + \frac{1}{2a} \arctg \frac{x}{a}.$$

$$122.3. \int \frac{x^2 dx}{X^3} = -\frac{x}{4X^2} + \frac{x}{8a^2X} + \frac{1}{8a^3} \arctg \frac{x}{a}.$$

$$122.4. \int \frac{x^2 dx}{X^4} = -\frac{x}{6X^3} + \frac{x}{24a^2 X^2} + \frac{x}{16a^4 X} + \frac{1}{16a^5} \operatorname{arctg} \frac{x}{a}.$$

$$122.9. \int \frac{x^2 dx}{X^{n+1}} = \frac{-x}{2nX^n} + \frac{1}{2n} \int \frac{dx}{X^n}.$$

$$123.1. \int \frac{x^2 dx}{X} = \frac{x^2}{2} - \frac{a^2}{2} \ln X.$$

$$123.2. \int \frac{x^2 dx}{X^2} = \frac{a^2}{2X} + \frac{1}{2} \ln X.$$

$$123.3. \int \frac{x^2 dx}{X^3} = -\frac{1}{2X} + \frac{a^2}{4X^2}.$$

$$123.4. \int \frac{x^2 dx}{X^4} = -\frac{1}{4X^2} + \frac{a^2}{6X^3}.$$

$$123.9. \int \frac{x^2 dx}{X^{n+1}} = \frac{-1}{2(n-1)X^{n-1}} + \frac{a^2}{2nX^n}. \quad [n > 1].$$

$$124.1. \int \frac{x^4 dx}{X} = \frac{x^3}{3} - a^2 x + a^3 \operatorname{arctg} \frac{x}{a}.$$

$$124.2. \int \frac{x^4 dx}{X^2} = x + \frac{a^2 x}{2X} - \frac{3a}{2} \operatorname{arctg} \frac{x}{a}.$$

$$124.3. \int \frac{x^4 dx}{X^3} = \frac{a^2 x}{4X^2} - \frac{5x}{8X} + \frac{3}{8a} \operatorname{arctg} \frac{x}{a}.$$

$$124.4. \int \frac{x^4 dx}{X^4} = \frac{a^2 x}{6X^3} - \frac{7x}{24X^2} + \frac{x}{16a^2 X} + \frac{1}{16a^3} \operatorname{arctg} \frac{x}{a}.$$

$$125.1. \int \frac{x^5 dx}{X} = \frac{x^4}{4} - \frac{a^2 x^2}{2} + \frac{a^4}{2} \ln X.$$

$$125.2. \int \frac{x^5 dx}{X^2} = \frac{x^2}{2} - \frac{a^4}{2X} - a^2 \ln X.$$

$$125.3. \int \frac{x^5 dx}{X^3} = \frac{a^2}{X} - \frac{a^4}{4X^2} + \frac{1}{2} \ln X.$$

$$125.4. \int \frac{x^5 dx}{X^4} = -\frac{1}{2X} + \frac{a^2}{2X^2} - \frac{a^4}{6X^3}.$$

$$125.9. \int \frac{x^5 dx}{X^{n+1}} = \frac{-1}{2(n-2)X^{n-2}} + \frac{a^2}{(n-1)X^{n-1}} - \frac{a^4}{2nX^n}. \quad [n > 2].$$

$$126.1. \int \frac{x^6 dx}{X} = \frac{x^5}{5} - \frac{a^2 x^3}{3} + a^4 x - a^5 \operatorname{arctg} \frac{x}{a}.$$

$$127.1. \int \frac{x^7 dx}{X} = \frac{x^6}{6} - \frac{a^2 x^4}{4} + \frac{a^4 x^2}{2} - \frac{a^6}{2} \ln X.$$

$$128.1. \int \frac{x^8 dx}{X} = \frac{x^7}{7} - \frac{a^2 x^5}{5} + \frac{a^4 x^3}{3} - a^6 x + a^7 \operatorname{arctg} \frac{x}{a}.$$

На этой странице $X = a^2 + x^2$.

$$131.1. \int \frac{dx}{xX} = \int \frac{dx}{x(a^2 + x^2)} = \frac{1}{2a^2} \ln \frac{x^2}{a^2 + x^2}.$$

$$131.2. \int \frac{dx}{xX^2} = \frac{1}{2a^2X} + \frac{1}{2a^4} \ln \frac{x^2}{X}.$$

$$131.3. \int \frac{dx}{xX^3} = \frac{1}{4a^2X^2} + \frac{1}{2a^4X} + \frac{1}{2a^6} \ln \frac{x^2}{X}.$$

$$131.4. \int \frac{dx}{xX^4} = \frac{1}{6a^2X^3} + \frac{1}{4a^4X^2} + \frac{1}{2a^6X} + \frac{1}{2a^8} \ln \frac{x^2}{X}.$$

$$132.1. \int \frac{dx}{x^2X} = -\frac{1}{a^2x} - \frac{1}{a^3} \operatorname{arctg} \frac{x}{a}.$$

$$132.2. \int \frac{dx}{x^2X^2} = -\frac{1}{a^4x} - \frac{x}{2a^4X} - \frac{3}{2a^5} \operatorname{arctg} \frac{x}{a}.$$

$$132.3. \int \frac{dx}{x^2X^3} = -\frac{1}{a^6x} - \frac{x}{4a^4X^2} - \frac{7x}{8a^6X} - \frac{15}{8a^7} \operatorname{arctg} \frac{x}{a}.$$

$$133.1. \int \frac{dx}{x^3X} = -\frac{1}{2a^2x^2} - \frac{1}{2a^4} \ln \frac{x^2}{X}.$$

$$133.2. \int \frac{dx}{x^3X^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4X} - \frac{1}{a^6} \ln \frac{x^2}{X}.$$

$$133.3. \int \frac{dx}{x^3X^3} = -\frac{1}{2a^6x^2} - \frac{1}{a^6X} - \frac{1}{4a^4X^2} - \frac{3}{2a^8} \ln \frac{x^2}{X}.$$

$$134.1. \int \frac{dx}{x^4X} = -\frac{1}{3a^2x^3} + \frac{1}{a^4x} + \frac{1}{a^5} \operatorname{arctg} \frac{x}{a}.$$

$$134.2. \int \frac{dx}{x^4X^2} = -\frac{1}{3a^4x^3} + \frac{2}{a^6x} + \frac{x}{2a^6X} + \frac{5}{2a^7} \operatorname{arctg} \frac{x}{a}.$$

$$135.1. \int \frac{dx}{x^5X} = -\frac{1}{4a^2x^4} + \frac{1}{2a^4x^2} + \frac{1}{2a^6} \ln \frac{x^2}{X}.$$

$$135.2. \int \frac{dx}{x^5X^2} = -\frac{1}{4a^4x^4} + \frac{1}{a^6x^2} + \frac{1}{2a^6X} + \frac{3}{2a^8} \ln \frac{x^2}{X}.$$

$$136. \int \frac{dx}{(f+gx)(a^2+x^2)} = \frac{1}{(f^2+a^2g^2)} \left[g \ln |f+gx| - \frac{g}{2} \ln (a^2+x^2) + \frac{f}{a} \operatorname{arctg} \frac{x}{a} \right].$$