

Интегралы, содержащие $X=a^2-x^2$

$$140. \quad \int \frac{dx}{1-x^2} = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|. \quad (\text{См. примечание к 140.1.})$$

Функция $1/(1-x^2)$ и интеграл от нее могут быть определены и для отрицательных значений x . См. рис. 140.

$$140.01. \quad \int \frac{dx}{x^2-1} = - \int \frac{dx}{1-x^2}. \quad [\text{См. 140}].$$

$$140.02. \quad \int \frac{dx}{a^2-b^2x^2} = \frac{1}{2ab} \ln \left| \frac{a+bx}{a-bx} \right|.$$

Заметим, что

$$\frac{1}{2ab} \ln \frac{a+bx}{a-bx} = \frac{1}{ab} \operatorname{Arth} \frac{bx}{a} \quad [b^2x^2 < a^2],$$

$$\frac{1}{2ab} \ln \frac{bx+a}{bx-a} = \frac{1}{ab} \operatorname{Arcth} \frac{bx}{a} \quad [b^2x^2 > a^2].$$

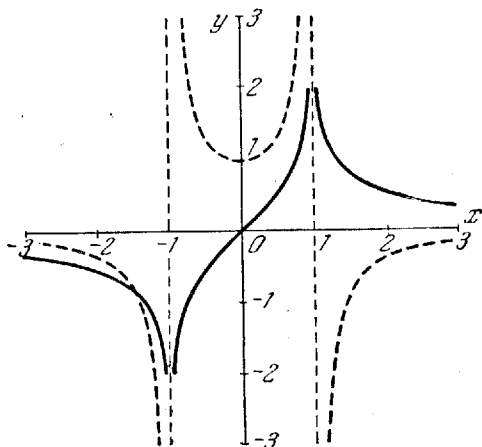


Рис. 140. Графики функций $y = \frac{1}{1-x^2}$ (пунктирная линия) и $y = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right|$ (сплошная линия).

$$140.1. \quad \int \frac{dx}{X} = \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|. \quad [\text{См. замечание к 140.02}].$$

$$140.2. \quad \int \frac{dx}{X^2} = \frac{x}{2a^2X} + \frac{1}{4a^3} \ln \left| \frac{a+x}{a-x} \right|.$$

$$140.3. \quad \int \frac{dx}{X^3} = \frac{x}{4a^2X^2} + \frac{3x}{8a^4X} + \frac{3}{16a^5} \ln \left| \frac{a+x}{a-x} \right|.$$

$$140.4. \int \frac{dx}{X^4} = \frac{x}{6a^2X^3} + \frac{5x}{24a^4X^2} + \frac{5x}{16a^6X} + \frac{5}{32a^7} \ln \left| \frac{a+x}{a-x} \right|.$$

$$140.9. \int \frac{dx}{(a^2 - b^2x^2)^{n+1}} = \frac{x}{2na^2(a^2 - b^2x^2)^n} + \frac{2n-1}{2na^2} \int \frac{dx}{(a^2 - b^2x^2)^n}.$$

$$141.1. \int \frac{x dx}{X} = \int \frac{x dx}{a^2 - x^2} = -\frac{1}{2} \ln |a^2 - x^2|.$$

$$141.2. \int \frac{x dx}{X^2} = \frac{1}{2X}. \quad 141.3. \int \frac{x dx}{X^3} = \frac{1}{4X^2}.$$

$$141.4. \int \frac{x dx}{X^3} = \frac{1}{6X^3}. \quad 141.9. \int \frac{x dx}{X^{n+1}} = \frac{1}{2nX^n} \quad [n \neq 0].$$

$$142.1. \int \frac{x^2 dx}{X} = -x + \frac{a}{2} \ln \left| \frac{a+x}{a-x} \right|.$$

$$142.2. \int \frac{x^2 dx}{X^2} = \frac{x}{2X} - \frac{1}{4a} \ln \left| \frac{a+x}{a-x} \right|.$$

$$142.3. \int \frac{x^2 dx}{X^3} = \frac{x}{4X^2} - \frac{x}{8a^2X} - \frac{1}{16a^3} \ln \left| \frac{a+x}{a-x} \right|.$$

$$142.4. \int \frac{x^2 dx}{X^4} = \frac{x}{6X^3} - \frac{x}{24a^2X^2} - \frac{x}{16a^4X} - \frac{1}{32a^5} \ln \left| \frac{a+x}{a-x} \right|.$$

$$142.9. \int \frac{x^2 dx}{X^{n+1}} = \frac{x}{2nX^n} - \frac{1}{2n} \int \frac{dx}{X^n}.$$

$$143.1. \int \frac{x^3 dx}{X} = -\frac{x^2}{2} - \frac{a^2}{2} \ln |X|.$$

$$143.2. \int \frac{x^3 dx}{X^2} = \frac{a^2}{2X} + \frac{1}{2} \ln |X|.$$

$$143.3. \int \frac{x^3 dx}{X^3} = -\frac{1}{2X} + \frac{a^2}{4X^2}. \quad 143.4. \int \frac{x^3 dx}{X^4} = -\frac{1}{4X^2} + \frac{a^2}{6X^3}.$$

$$143.9. \int \frac{x^3 dx}{X^{n+1}} = \frac{-1}{2(n-1)X^{n-1}} + \frac{a^2}{2nX^n} \quad [n > 1].$$

$$144.1. \int \frac{x^4 dx}{X} = -\frac{x^3}{3} - a^2x + \frac{a^3}{2} \ln \left| \frac{a+x}{a-x} \right|.$$

$$144.2. \int \frac{x^4 dx}{X^2} = x + \frac{a^2x}{2X} - \frac{3a}{4} \ln \left| \frac{a+x}{a-x} \right|.$$

$$144.3. \int \frac{x^4 dx}{X^3} = \frac{a^2x}{4X^2} - \frac{5x}{8X} + \frac{3}{16a} \ln \left| \frac{a+x}{a-x} \right|.$$

$$144.4. \int \frac{x^4 dx}{X^4} = \frac{a^2x}{6X^3} - \frac{7x}{24X^2} + \frac{x}{16a^2X} + \frac{1}{32a^3} \ln \left| \frac{a+x}{a-x} \right|.$$

$$145.1. \int \frac{x^5 dx}{X} = -\frac{x^4}{4} - \frac{a^2x^2}{2} - \frac{a^4}{2} \ln |X|.$$

$$145.2. \int \frac{x^5 dx}{X^2} = \frac{x^2}{2} + \frac{a^4}{2X} + a^2 \ln |X|.$$

$$145.3. \int \frac{x^5 dx}{X^3} = -\frac{a^2}{X} + \frac{a^4}{4X^2} - \frac{1}{2} \ln |X|.$$

$$145.4. \int \frac{x^5 dx}{X^4} = \frac{1}{2X} - \frac{a^2}{2X^2} + \frac{a^4}{6X^3}.$$

$$145.9. \int \frac{x^5 dx}{X^{n+1}} = \frac{1}{2(n-2)X^{n-2}} - \frac{a^2}{(n-1)X^{n-1}} + \frac{a^4}{2nX^n} \quad [n > 2].$$

$$146.1. \int \frac{x^6 dx}{X} = -\frac{x^5}{5} - \frac{a^2 x^3}{3} - a^4 x + \frac{a^5}{2} \ln \left| \frac{a+x}{a-x} \right|.$$

$$147.1. \int \frac{x^7 dx}{X} = -\frac{x^6}{6} - \frac{a^2 x^4}{4} - \frac{a^4 x^2}{2} - \frac{a^6}{2} \ln |X|.$$

$$148.1. \int \frac{x^8 dx}{X} = -\frac{x^7}{7} - \frac{a^2 x^5}{5} - \frac{a^4 x^3}{3} - a^6 x + \frac{a}{2} \ln \left| \frac{a+x}{a-x} \right|.$$

$$151.1. \int \frac{dx}{xX} = \int \frac{dx}{x(a^2 - x^2)} = \frac{1}{2a^2} \ln \left| \frac{x^2}{a^2 - x^2} \right|.$$

$$151.2. \int \frac{dx}{xX^2} = \frac{1}{2a^2 X} + \frac{1}{2a^4} \ln \left| \frac{x^2}{X} \right|.$$

$$151.3. \int \frac{dx}{xX^3} = \frac{1}{4a^2 X^2} + \frac{1}{2a^4 X} + \frac{1}{2a^6} \ln \left| \frac{x^2}{X} \right|.$$

$$151.4. \int \frac{dx}{xX^4} = \frac{1}{6a^2 X^3} + \frac{1}{4a^4 X^2} + \frac{1}{2a^6 X} + \frac{1}{2a^8} \ln \left| \frac{x^2}{X} \right|.$$

$$152.1. \int \frac{dx}{x^2 X} = -\frac{1}{a^2 x} + \frac{1}{2a^3} \ln \left| \frac{a+x}{a-x} \right|.$$

$$152.2. \int \frac{dx}{x^2 X^2} = -\frac{1}{a^4 x} + \frac{x}{2a^4 X} + \frac{3}{4a^5} \ln \left| \frac{a+x}{a-x} \right|.$$

$$152.3. \int \frac{dx}{x^2 X^3} = -\frac{1}{a^6 x} + \frac{x}{4a^4 X^2} + \frac{7x}{8a^6 X} + \frac{15}{16a^7} \ln \left| \frac{a+x}{a-x} \right|.$$

$$153.1. \int \frac{dx}{x^3 X} = -\frac{1}{2a^2 x^2} + \frac{1}{2a^4} \ln \left| \frac{x^2}{X} \right|.$$

$$153.2. \int \frac{dx}{x^3 X^2} = -\frac{1}{2a^4 x^2} + \frac{1}{2a^4 X} + \frac{1}{a^6} \ln \left| \frac{x^2}{X} \right|.$$

$$153.3. \int \frac{dx}{x^3 X^3} = -\frac{1}{2a^6 x^2} + \frac{1}{a^6 X} + \frac{1}{4a^4 X^2} + \frac{3}{2a^8} \ln \left| \frac{x^2}{X} \right|.$$

$$154.1. \int \frac{dx}{x^4 X} = -\frac{1}{3a^2 x^3} - \frac{1}{a^4 x} + \frac{1}{2a^5} \ln \left| \frac{a+x}{a-x} \right|.$$

$$154.2. \int \frac{dx}{x^4 X^2} = -\frac{1}{3a^4 x^3} - \frac{2}{a^6 x} + \frac{x}{2a^6 X} + \frac{5}{4a^7} \ln \left| \frac{a+x}{a-x} \right|.$$

$$155.1. \quad \int \frac{dx}{x^5 X} = -\frac{1}{4a^2 x^4} - \frac{1}{2a^4 x^2} + \frac{1}{2a^6} \ln \left| \frac{x^2}{X} \right|.$$

$$155.2. \quad \int \frac{dx}{x^5 X^3} = -\frac{1}{4a^4 x^4} - \frac{1}{a^6 x^2} + \frac{1}{2a^6 X} + \frac{3}{2a^8} \ln \left| \frac{x^2}{X} \right|.$$

В 155.1 и 155.2 $X = a^2 - x^2$.

$$156. \quad \int \frac{dx}{(f + gx)(a^2 - x^2)} = \frac{1}{a^2 g^2 - f^2} \left[g \ln |f + gx| - \right. \\ \left. - \frac{g}{2} \ln |a^2 - x^2| - \frac{f}{2a} \ln \left| \frac{a + x}{a - x} \right| \right].$$