

Интегралы, содержащие  $X = ax^2 + bx + c$

$$\begin{aligned}
 160.01. \quad \int \frac{dx}{X} &= \frac{2}{\sqrt{4ac-b^2}} \operatorname{arctg} \frac{2ax+b}{\sqrt{4ac-b^2}} && [4ac > b^2], \\
 &= \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| && [b^2 > 4ac], \\
 &= \frac{1}{a(p-q)} \ln \left| \frac{x-p}{x-q} \right| && [b^2 > 4ac],
 \end{aligned}$$

где  $p$  и  $q$  — корни уравнения  $ax^2 + bx + c = 0$ ,

$$\begin{aligned}
 &= -\frac{2}{\sqrt{b^2-4ac}} \operatorname{Arth} \frac{2ax+b}{\sqrt{b^2-4ac}} && [b^2 > 4ac, \quad (2ax+b)^2 < b^2-4ac], \\
 &= -\frac{2}{\sqrt{b^2-4ac}} \operatorname{Arcth} \frac{2ax+b}{\sqrt{b^2-4ac}} && [b^2 > 4ac, \quad (2ax+b)^2 > b^2-4ac], \\
 &= -\frac{2}{2ax+b} && [b^2 = 4ac].
 \end{aligned}$$

(Положить  $2ax + b = z$ .)

$$160.02. \quad \int \frac{dx}{X^2} = \frac{2ax+b}{(4ac-b^2)X} + \frac{2a}{4ac-b^2} \int \frac{dx}{X}. \quad [\text{См. } 160.01.]$$

$$\begin{aligned}
 160.03. \quad \int \frac{dx}{X^3} &= \frac{2ax+b}{2(4ac-b^2)X^2} + \frac{3a(2ax+b)}{(4ac-b^2)^2 X} + \\
 &\quad + \frac{6a^2}{(4ac-b^2)^2} \int \frac{dx}{X}. \quad [\text{См. } 160.01.]
 \end{aligned}$$

$$160.09. \quad \int \frac{dx}{X^n} = \frac{2ax+b}{(n-1)(4ac-b^2)X^{n-1}} + \frac{(2n-3)2a}{(n-1)(4ac-b^2)} \int \frac{dx}{X^{n-1}}.$$

$$160.11. \quad \int \frac{x dx}{X} = \frac{1}{2a} \ln |X| - \frac{b}{2a} \int \frac{dx}{X}. \quad [\text{См. } 160.01.]$$

$$160.12. \quad \int \frac{x dx}{X^2} = -\frac{bx+2c}{(4ac-b^2)X} - \frac{b}{4ac-b^2} \int \frac{dx}{X}. \quad [\text{См. } 160.01.]$$

$$160.19. \quad \int \frac{x dx}{X^n} = -\frac{bx+2c}{(n-1)(4ac-b^2)X^{n-1}} - \frac{b(2n-3)}{(n-1)(4ac-b^2)} \int \frac{dx}{X^{n-1}}.$$

$$160.21. \quad \int \frac{x^2 dx}{X} = \frac{x}{a} - \frac{b}{2a^2} \ln |X| + \frac{b^2-2ac}{2a^2} \int \frac{dx}{X}. \quad [\text{См. 160.01.}]$$

$$160.22. \quad \int \frac{x^2 dx}{X^2} = \frac{(b^2-2ac)x+bc}{a(4ac-b^2)X} + \frac{2c}{4ac-b^2} \int \frac{dx}{X}. \quad [\text{См. 160.01.}]$$

$$160.27. \quad \int \frac{x^m dx}{X} = \frac{x^{m-1}}{(m-1)a} - \frac{c}{a} \int \frac{x^{m-2} dx}{X} - \frac{b}{a} \int \frac{x^{m-1} dx}{X}.$$

$$160.28. \quad \int \frac{x^m dx}{X^n} = -\frac{x^{m-1}}{(2n-m-1)aX^{n-1}} + \\ + \frac{(m-1)c}{(2n-m-1)a} \int \frac{x^{m-2} dx}{X^n} - \frac{(n-m)b}{(2n-m-1)a} \int \frac{x^{m-1} dx}{X^n} \\ [m \neq 2n-1].$$

160.29. При  $m = 2n-1$

$$\int \frac{x^{2n-1} dx}{X^n} = \frac{1}{a} \int \frac{x^{2n-3} dx}{X^{n-1}} - \frac{c}{a} \int \frac{x^{2n-3} dx}{X^n} - \frac{b}{a} \int \frac{x^{2n-2} dx}{X^n}.$$

$$161.11. \quad \int \frac{dx}{xX} = \frac{1}{2c} \ln \left| \frac{x^2}{X} \right| - \frac{b}{2c} \int \frac{dx}{X}. \quad [\text{См. 160.01.}]$$

$$161.19. \quad \int \frac{dx}{xX^n} = \frac{1}{2c(n-1)X^{n-1}} - \frac{b}{2c} \int \frac{dx}{X^n} + \frac{1}{c} \int \frac{dx}{xX^{n-1}}.$$

$$161.21. \quad \int \frac{dx}{x^2 X} = \frac{b}{2c^2} \ln \left| \frac{X}{x^2} \right| - \frac{1}{cx} + \frac{b^2-2ac}{2c^2} \int \frac{dx}{X}. \quad [\text{См. 160.01.}]$$

$$161.29. \quad \int \frac{dx}{x^m X^n} = -\frac{1}{(m-1)cx^{m-1}X^{n-1}} - \\ - \frac{(2n+m-3)a}{(m-1)c} \int \frac{dx}{x^{m-2}X^n} - \frac{(n+m-2)b}{(m-1)c} \int \frac{dx}{x^{m-1}X^n} \quad [m > 1].$$