Интегралы, содержащие $a^3 \pm x^3$

165.01. $$\int \frac{dx}{a^3+x^3} = \frac{1}{6a^2} \ln \frac{(a+x)^2}{a^2-ax+x^2} + \frac{1}{a^2\sqrt{3}} \operatorname{arctg} \frac{2x-a}{a\sqrt{3}}.$$

165.02. $$\int \frac{dx}{(a^3+x^3)^2} = \frac{x}{3a^3(a^3+x^3)} + \frac{2}{3a^3} \int \frac{dx}{a^3+x^3}.$$

165.11. $$\int \frac{x\,dx}{a^3+x^3} = \frac{1}{6a} \ln \frac{a^2-ax+x^2}{(a+x)^2} + \frac{1}{a\sqrt{3}} \operatorname{arctg} \frac{2x-a}{a\sqrt{3}}.$$

165.12. $$\int \frac{x\,dx}{(a^3+x^3)^2} = \frac{x^2}{3a^3(a^3+x^3)} + \frac{1}{3a^3} \int \frac{x\,dx}{a^3+x^3}.$$

165.21. $$\int \frac{x^2 dx}{a^3+x^3} = \frac{1}{3} \ln |a^3+x^3|.$$

165.22. $\int \frac{x^2dx}{(a^3+x^3)^2} = -\frac{1}{3(a^3+x^3)}.$

165.31. $\int \frac{x^3dx}{a^3+x^3} = x - a^3 \int \frac{dx}{a^3+x^3}.$ [См. 165.01.]

165.32. $\int \frac{x^3dx}{(a^3+x^3)^2} = \frac{-x}{3(a^3+x^3)} + \frac{1}{3}\int \frac{dx}{a^3+x^3}.$ [См. 165.01.]

165.41. $\int \frac{x^4dx}{a^3+x^3} = \frac{x^2}{2} - a^3 \int \frac{x\,dx}{a^3+x^3}.$ [См. 165.11.]

165.42. $\int \frac{x^4dx}{(a^3+x^3)^2} = -\frac{x^2}{3(a^3+x^3)} + \frac{2}{3}\int \frac{x\,dx}{a^3+x^3}.$ [См. 165.11.]

165.51. $\int \frac{x^5dx}{a^3+x^3} = \frac{x^3}{3} - \frac{a^3}{3}\ln|a^3+x^3|.$

165.52. $\int \frac{x^5dx}{(a^3+x^3)^2} = \frac{a^3}{3(a^3+x^3)} + \frac{1}{3}\ln|a^3+x^3|.$

166.11. $\int \frac{dx}{x(a^3+x^3)} = \frac{1}{3a^3}\ln\left|\frac{x^3}{a^3+x^3}\right|.$

166.12. $\int \frac{dx}{x(a^3+x^3)^2} = \frac{1}{3a^3(a^3+x^3)} + \frac{1}{3a^6}\ln\left|\frac{x^3}{a^3+x^3}\right|.$

166.21. $\int \frac{dx}{x^2(a^3+x^3)} = -\frac{1}{a^3x} - \frac{1}{a^3}\int \frac{x\,dx}{a^3+x^3}.$ [См. 165.11.]

166.22. $\int \frac{dx}{x^2(a^3+x^3)^2} = -\frac{1}{a^6x} - \frac{x^2}{3a^6(a^3+x^3)} - \frac{4}{3a^6}\int \frac{x\,dx}{a^3+x^3}.$ [См. 165.11.]

166.31. $\int \frac{dx}{x^3(a^3+x^3)} = -\frac{1}{2a^3x^2} - \frac{1}{a^3}\int \frac{dx}{a^3+x^3}.$ [См. 165.01.]

166.32. $\int \frac{dx}{x^3(a^3+x^3)^2} = -\frac{1}{2a^6x^2} - \frac{x}{3a^6(a^3+x^3)} - \frac{5}{3a^6}\int \frac{dx}{a^3+x^3}.$ [См. 165.01.]

166.41. $\int \frac{dx}{x^4(a^3+x^3)} = -\frac{1}{3a^3x^3} + \frac{1}{3a^6}\ln\left|\frac{a^3+x^3}{x^3}\right|.$

166.42. $\int \frac{dx}{x^4(a^3+x^3)^2} = -\frac{1}{3a^6x^3} - \frac{1}{3a^6(a^3+x^3)} + \frac{2}{3a^9}\ln\left|\frac{a^3+x^3}{x^3}\right|.$

168.01. $\int \frac{dx}{a^3-x^3} = \frac{1}{6a^2}\ln\frac{a^2+ax+x^2}{(a-x)^2} + \frac{1}{a^2\sqrt{3}}\operatorname{arctg}\frac{2x+a}{a\sqrt{3}}.$

168.02. $\int \frac{dx}{(a^3-x^3)^2} = \frac{x}{3a^3(a^3-x^3)} + \frac{2}{3a^3}\int \frac{dx}{a^3-x^3}.$ [См. 168.01.]

168.11. $$\int \frac{x\,dx}{a^3-x^3} = \frac{1}{6a} \ln \frac{a^2+ax+x^2}{(a-x)^2} - \frac{1}{a\sqrt{3}} \operatorname{arctg} \frac{2x+a}{a\sqrt{3}}.$$

168.12. $$\int \frac{x\,dx}{(a^3-x^3)^2} = \frac{x^2}{3a^3(a^3-x^3)} + \frac{1}{3a^3} \int \frac{x\,dx}{a^3-x^3}.$$ [См. **168.11.**]

168.21. $$\int \frac{x^2 dx}{a^3-x^3} = -\frac{1}{3} \ln |a^3-x^3|.$$

168.22. $$\int \frac{x^2 dx}{(a^3-x^3)^2} = \frac{1}{3(a^3-x^3)}.$$

168.31. $$\int \frac{x^3 dx}{a^3-x^3} = -x + a^3 \int \frac{dx}{a^3-x^3}.$$ [См. **168.01.**]

168.32. $$\int \frac{x^3 dx}{(a^3-x^3)^2} = \frac{x}{3(a^3-x^3)} - \frac{1}{3} \int \frac{dx}{a^3-x^3}.$$ [См. **168.01.**]

168.41. $$\int \frac{x^4 dx}{a^3-x^3} = -\frac{x^2}{2} + a^3 \int \frac{x\,dx}{a^3-x^3}.$$ [См. **168.11.**]

168.42. $$\int \frac{x^4 dx}{(a^3-x^3)^2} = \frac{x^2}{3(a^3-x^3)} - \frac{2}{3} \int \frac{x\,dx}{a^3-x^3}.$$ [См. **168.11.**]

168.51. $$\int \frac{x^5 dx}{a^3-x^3} = -\frac{x^3}{3} - \frac{a^3}{3} \ln |a^3-x^3|.$$

168.52. $$\int \frac{x^5 dx}{(a^3-x^3)^2} = \frac{a^3}{3(a^3-x^3)} + \frac{1}{3} \ln |a^3-x^3|.$$

169.11. $$\int \frac{dx}{x(a^3-x^3)} = \frac{1}{3a^3} \ln \left| \frac{x^3}{a^3-x^3} \right|.$$

169.12. $$\int \frac{dx}{x(a^3-x^3)^2} = \frac{1}{3a^3(a^3-x^3)} + \frac{1}{3a^6} \ln \left| \frac{x^3}{a^3-x^3} \right|.$$

169.21. $$\int \frac{dx}{x^2(a^3-x^3)} = -\frac{1}{a^3 x} + \frac{1}{a^3} \int \frac{x\,dx}{a^3-x^3}.$$ [См. **168.11.**]

169.22. $$\int \frac{dx}{x^2(a^3-x^3)^2} = -\frac{1}{a^6 x} + \frac{x^2}{3a^6(a^3-x^3)} + \frac{4}{3a^6} \int \frac{x\,dx}{a^3-x^3}.$$ [См. **168.11.**]

169.31. $$\int \frac{dx}{x^3(a^3-x^3)} = -\frac{1}{2a^3x^2} + \frac{1}{a^3} \int \frac{dx}{a^3-x^3}.$$ [См. **168.01.**]

169.32. $$\int \frac{dx}{x^3(a^3-x^3)^2} = -\frac{1}{2a^6x^2} + \frac{x}{3a^6(a^3-x^3)} + \frac{5}{3a^6} \int \frac{dx}{a^3-x^3}.$$ [См. **168.01.**]

169.41. $$\int \frac{dx}{x^4(a^3-x^3)} = -\frac{1}{3a^3x^3} + \frac{1}{3a^6} \ln \left| \frac{x^3}{a^3-x^3} \right|.$$

169.42. $$\int \frac{dx}{x^4(a^3-x^3)^2} = -\frac{1}{3a^6x^3} + \frac{1}{3a^6(a^3-x^3)} + \frac{2}{3a^9} \ln \left| \frac{x^3}{a^3-x^3} \right|.$$