

Интегралы, содержащие  $X^{1/2} = (a + bx)^{1/2}$

$$190. \quad \int \frac{x^q dx}{X^{p/2}} = \frac{1}{b^{q+1}} \int \frac{(X-a)^q dX}{X^{p/2}} \quad [q > 0].$$

При целом положительном  $q$  разложить числитель по формуле бинома Ньютона.

$$191. \quad \int \frac{dx}{X^{p/2}} = \frac{-2}{(p-2) b X^{(p-2)/2}}. \quad 191.01. \quad \int \frac{dx}{X^{1/2}} = \frac{2}{b} X^{1/2}.$$

$$191.03. \quad \int \frac{dx}{X^{3/2}} = \frac{-2}{b X^{1/2}}. \quad 191.05. \quad \int \frac{dx}{X^{5/2}} = \frac{-2}{3b X^{3/2}}.$$

$$191.1. \quad \int \frac{x dx}{X^{p/2}} = \frac{2}{b^2} \left[ \frac{-1}{(p-4) X^{(p-4)/2}} + \frac{a}{(p-2) X^{(p-2)/2}} \right].$$

$$191.11. \quad \int \frac{x dx}{X^{1/2}} = \frac{2}{b^2} \left( \frac{X^{3/2}}{3} - a X^{1/2} \right).$$

$$191.13. \quad \int \frac{x dx}{X^{3/2}} = \frac{2}{b^2} \left( X^{1/2} + \frac{a}{X^{1/2}} \right).$$

$$191.15. \quad \int \frac{x dx}{X^{5/2}} = \frac{2}{b^2} \left( \frac{-1}{X^{1/2}} + \frac{a}{3X^{3/2}} \right).$$

$$191.17. \quad \int \frac{x dx}{X^{7/2}} = \frac{2}{b^2} \left( \frac{-1}{3X^{3/2}} + \frac{a}{5X^{5/2}} \right).$$

$$191.2. \quad \int \frac{x^2 dx}{X^{p/2}} = \frac{2}{b^3} \left[ \frac{-1}{(p-6) X^{(p-6)/2}} + \frac{2a}{(p-4) X^{(p-4)/2}} - \frac{a^2}{(p-2) X^{(p-2)/2}} \right].$$

$$191.21. \quad \int \frac{x^2 dx}{X^{1/2}} = \frac{2}{b^3} \left( \frac{X^{5/2}}{5} - \frac{2aX^{3/2}}{3} + a^2 X^{1/2} \right).$$

$$191.23. \quad \int \frac{x^2 dx}{X^{3/2}} = \frac{2}{b^3} \left( \frac{X^{3/2}}{3} - 2aX^{1/2} - \frac{a^2}{X^{1/2}} \right).$$

$$191.25. \quad \int \frac{x^2 dx}{X^{5/2}} = \frac{2}{b^3} \left( X^{1/2} + \frac{2a}{X^{1/2}} - \frac{a^2}{3X^{3/2}} \right).$$

$$191.27. \quad \int \frac{x^2 dx}{X^{7/2}} = \frac{2}{b^3} \left( \frac{-1}{X^{1/2}} + \frac{2a}{3X^{3/2}} - \frac{a^2}{5X^{5/2}} \right).$$

$$192.1. \quad \int \frac{dx}{xX^{p/2}} = \frac{2}{(p-2)aX^{(p-2)/2}} + \frac{1}{a} \int \frac{dx}{xX^{(p-2)/2}} \quad [p > 1].$$

$$192.11. \quad \int \frac{dx}{xX^{1/2}} = \frac{1}{a^{1/2}} \ln \left| \frac{X^{1/2} - a^{1/2}}{X^{1/2} + a^{1/2}} \right| \quad [a > 0, X > 0],$$

$$= -\frac{2}{a^{1/2}} \operatorname{Arth} \frac{X^{1/2}}{a^{1/2}} \quad [a > X > 0],$$

$$= -\frac{2}{a^{1/2}} \operatorname{Arcth} \frac{X^{1/2}}{a^{1/2}} \quad [X > a > 0],$$

$$= \frac{2}{(-a)^{1/2}} \operatorname{arctg} \frac{X^{1/2}}{(-a)^{1/2}} \quad [a < 0, X > 0].$$

(Положить  $X^{1/2} = z$ . См. 120.1 и 140.1.)

$$192.13. \quad \int \frac{dx}{xX^{3/2}} = \frac{2}{aX^{1/2}} + \frac{1}{a} \int \frac{dx}{xX^{1/2}}. \quad [\text{См. } 192.11.]$$

$$192.15. \quad \int \frac{dx}{xX^{5/2}} = \frac{2}{3aX^{3/2}} + \frac{2}{a^2 X^{1/2}} + \frac{1}{a^2} \int \frac{dx}{xX^{1/2}}. \quad [\text{См. } 192.11.]$$

$$192.17. \quad \int \frac{dx}{xX^{7/2}} = \frac{2}{5aX^{5/2}} + \frac{2}{3a^2 X^{3/2}} + \frac{2}{a^3 X^{1/2}} + \frac{1}{a^3} \int \frac{dx}{xX^{1/2}}. \quad [\text{См. } 192.11.]$$

$$192.2. \quad \int \frac{dx}{x^2 X^{p/2}} = \frac{-1}{axX^{(p-2)/2}} - \frac{pb}{2a} \int \frac{dx}{xX^{p/2}}.$$

$$192.21. \quad \int \frac{dx}{x^2 X^{1/2}} = \frac{-X^{1/2}}{ax} - \frac{b}{2a} \int \frac{dx}{xX^{1/2}}. \quad [\text{См. } 192.11.]$$

$$192.23. \quad \int \frac{dx}{x^2 X^{3/2}} = \frac{-1}{axX^{1/2}} - \frac{3b}{a^2 X^{1/2}} - \frac{3b}{2a^2} \int \frac{dx}{xX^{1/2}}. \quad [\text{См. } 192.11.]$$

- 192.25.  $\int \frac{dx}{x^2 X^{5/2}} = \frac{-1}{ax X^{3/2}} - \frac{5b}{3a^2 X^{3/2}} - \frac{5b}{a^3 X^{1/2}} - \frac{5b}{2a^3} \int \frac{dx}{x X^{1/2}}$ . [См. 192.11.]
- 192.9.  $\int \frac{dx}{x^p X^{1/2}} = \frac{-X^{1/2}}{(p-1)ax^{p-1}} - \frac{(2p-3)b}{(2p-2)a} \int \frac{dx}{x^{p-1} X^{1/2}}$ .
193.  $\int X^{\rho/2} dx = \frac{2X^{(\rho+2)/2}}{(\rho+2)b}$ .
- 193.01.  $\int X^{1/2} dx = \frac{2X^{3/2}}{3b}$ .      193.03.  $\int X^{3/2} dx = \frac{2X^{5/2}}{5b}$ .
- 193.1.  $\int x X^{\rho/2} dx = \frac{2}{b^2} \left( \frac{X^{(\rho+4)/2}}{\rho+4} - \frac{aX^{(\rho+2)/2}}{\rho+2} \right)$ .
- 193.11.  $\int x X^{1/2} dx = \frac{2}{b^2} \left( \frac{X^{5/2}}{5} - \frac{aX^{3/2}}{3} \right)$ .
- 193.13.  $\int x X^{3/2} dx = \frac{2}{b^2} \left( \frac{X^{7/2}}{7} - \frac{aX^{5/2}}{5} \right)$ .
- 193.2.  $\int x^2 X^{\rho/2} dx = \frac{2}{b^3} \left( \frac{X^{(\rho+6)/2}}{\rho+6} - \frac{2aX^{(\rho+4)/2}}{\rho+4} + \frac{a^2 X^{(\rho+2)/2}}{\rho+2} \right)$ .
- 193.21.  $\int x^2 X^{1/2} dx = \frac{2}{b^3} \left( \frac{X^{7/2}}{7} - \frac{2aX^{5/2}}{5} + \frac{a^2 X^{3/2}}{3} \right)$ .
- 194.1.  $\int \frac{X^{\rho/2} dx}{x} = \frac{2X^{\rho/2}}{\rho} + a \int \frac{X^{(\rho-2)/2} dx}{x}$ .
- 194.11.  $\int \frac{X^{1/2} dx}{x} = 2X^{1/2} + a \int \frac{dx}{x X^{1/2}}$ . [См. 192.11.]
- 194.13.  $\int \frac{X^{3/2} dx}{x} = \frac{2X^{3/2}}{3} + 2aX^{1/2} + a^2 \int \frac{dx}{x X^{1/2}}$ . [См. 192.11.]
- 194.15.  $\int \frac{X^{5/2} dx}{x} = \frac{2X^{5/2}}{5} + \frac{2aX^{3/2}}{3} + 2a^2 X^{1/2} + a^3 \int \frac{dx}{x X^{1/2}}$ . [См. 192.11.]
- 194.2.  $\int \frac{X^{\rho/2} dx}{x^2} = -\frac{X^{(\rho+2)/2}}{ax} + \frac{\rho b}{2a} \int \frac{X^{\rho/2} dx}{x}$ .
- 194.21.  $\int \frac{X^{1/2} dx}{x^2} = -\frac{X^{1/2}}{x} + \frac{b}{2} \int \frac{dx}{x X^{1/2}}$ . [См. 192.11.]
- 194.31.  $\int \frac{X^{1/2} dx}{x^3} = -\frac{(2a+bx) X^{1/2}}{4ax^2} - \frac{b^2}{8a} \int \frac{dx}{x X^{1/2}}$ . [См. 192.11.]