Интегралы, содержащие $X^{1/2}=(a+bx)^{1/2}$ и $U^{1/2}=(f+gx)^{1/2}$

Здесь всюду $k=ag-bf$.

195.01. $$\int\frac{dx}{X^{1/2}U^{1/2}}=\frac{2}{\sqrt{-bg}}\operatorname{arctg}\sqrt{\frac{-gX}{bU}} \qquad \begin{bmatrix}b>0\\ g<0\end{bmatrix},$$
$$=\frac{-1}{\sqrt{-bg}}\arcsin\frac{2bgx+ag+bf}{bf-ag} \qquad \begin{bmatrix}b>0\\ g<0\end{bmatrix},$$
$$=\frac{2}{\sqrt{bg}}\ln\{\sqrt{bgX}+b\sqrt{U}\} \qquad [bg>0].$$

195.02. $$\int\frac{dx}{X^{1/2}U}=\frac{2}{\sqrt{-kg}}\operatorname{arctg}\frac{gX^{1/2}}{\sqrt{-kg}} \qquad [kg<0],$$
$$=\frac{1}{\sqrt{kg}}\ln\left|\frac{gX^{1/2}-\sqrt{kg}}{gX^{1/2}+\sqrt{kg}}\right| \qquad [kg>0].$$

195.03. $$\int\frac{dx}{X^{1/2}U^{3/2}}=-\frac{2X^{1/2}}{kU^{1/2}}.$$

195.04. $$\int\frac{U^{1/2}dx}{X^{1/2}}=\frac{X^{1/2}U^{1/2}}{b}-\frac{k}{2b}\int\frac{dx}{X^{1/2}U^{1/2}}.$$ [См. 195.01.]

195.09. $$\int\frac{U^n dx}{X^{1/2}}=\frac{2}{(2n+1)b}\left(X^{1/2}U^n-nk\int\frac{U^{n-1}dx}{X^{1/2}}\right).$$

196.01. $$\int X^{1/2}U^{1/2}dx=\frac{k+2bU}{4bg}X^{1/2}U^{1/2}-\frac{k^2}{8bg}\int\frac{dx}{X^{1/2}U^{1/2}}.$$ [См. 195.01.]

196.02. $$\int\frac{x\,dx}{X^{1/2}U^{1/2}}=\frac{X^{1/2}U^{1/2}}{bg}-\frac{ag+bf}{2bg}\int\frac{dx}{X^{1/2}U^{1/2}}.$$ [См. 195.01.]

196.03. $$\int\frac{dx}{X^{1/2}U^n}=-\frac{1}{(n-1)k}\left\{\frac{X^{1/2}}{U^{n-1}}+\left(n-\frac{3}{2}\right)b\int\frac{dx}{X^{1/2}U^{n-1}}\right\}.$$

196.04. $$\int X^{1/2}U^n dx=\frac{1}{(2n+3)g}\left(2X^{1/2}U^{n+1}+k\int\frac{U^n dx}{X^{1/2}}\right).$$ [См. 195.09.]

196.05. $$\int\frac{X^{1/2}dx}{U^n}=\frac{1}{(n-1)g}\left(-\frac{X^{1/2}}{U^{n-1}}+\frac{b}{2}\int\frac{dx}{X^{1/2}U^{n-1}}\right).$$ [См. 196.03.]

197. $$\int\frac{f(x^2)\,dx}{\sqrt{a+bx^2}}=\int f\left(\frac{au^2}{1-bu^2}\right)\frac{du}{(1-bu^2)},$$
где $u=\dfrac{x}{\sqrt{a+bx^2}}$.