

Интегралы, содержащие  $t = (a^2 - x^2)^{1/2}$

$$320.01. \quad \int \frac{dx}{t} = \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} \quad [x^2 < a^2].$$

Надо брать положительные значения  $t$  и  $a$ .

$$320.03. \quad \int \frac{dx}{t^3} = \frac{1}{a^2} \frac{x}{t}. \quad 320.05. \quad \int \frac{dx}{t^5} = \frac{1}{a^4} \left[ \frac{x}{t} + \frac{1}{3} \frac{x^3}{t^3} \right].$$

$$320.07. \quad \int \frac{dx}{t^7} = \frac{1}{a^6} \left[ \frac{x}{t} + \frac{2}{3} \frac{x^3}{t^3} + \frac{1}{5} \frac{x^5}{t^5} \right].$$

$$320.09. \quad \int \frac{dx}{t^9} = \frac{1}{a^8} \left[ \frac{x}{t} + \frac{3}{3} \frac{x^3}{t^3} + \frac{3}{5} \frac{x^5}{t^5} + \frac{1}{7} \frac{x^7}{t^7} \right].$$

$$320.11. \quad \int \frac{dx}{t^{11}} = \frac{1}{a^{10}} \left[ \frac{x}{t} + \frac{4}{3} \frac{x^3}{t^3} + \frac{6}{5} \frac{x^5}{t^5} + \frac{4}{7} \frac{x^7}{t^7} + \frac{1}{9} \frac{x^9}{t^9} \right].$$

$$320.13. \quad \int \frac{dx}{t^{13}} = \frac{1}{a^{12}} \left[ \frac{x}{t} + \frac{5}{3} \frac{x^3}{t^3} + \frac{10}{5} \frac{x^5}{t^5} + \frac{10}{7} \frac{x^7}{t^7} + \frac{5}{9} \frac{x^9}{t^9} + \frac{1}{11} \frac{x^{11}}{t^{11}} \right].$$

$$320.15. \quad \int \frac{dx}{t^{15}} = \frac{1}{a^{14}} \left[ \frac{x}{t} + \frac{6}{3} \frac{x^3}{t^3} + \frac{15}{5} \frac{x^5}{t^5} + \frac{20}{7} \frac{x^7}{t^7} + \frac{15}{9} \frac{x^9}{t^9} + \frac{6}{11} \frac{x^{11}}{t^{11}} + \frac{1}{13} \frac{x^{13}}{t^{13}} \right].$$

Интегралы 320.03—320.15 находят посредством подстановки:

$$z^2 = \frac{x^2}{a^2 - x^2}; \quad \text{тогда} \quad dx = \frac{a dz}{(1 + z^2)^{3/2}}.$$

$$321.01. \quad \int \frac{x dx}{t} = -t. \quad 321.03. \quad \int \frac{x dx}{t^3} = \frac{1}{t}.$$

$$321.05. \quad \int \frac{x dx}{t^5} = \frac{1}{3t^3}. \quad 321.07. \quad \int \frac{x dx}{t^7} = \frac{1}{5t^5}.$$

$$321.9. \quad \int \frac{x dx}{t^{2p+1}} = \frac{1}{(2p-1)t^{2p-1}}.$$

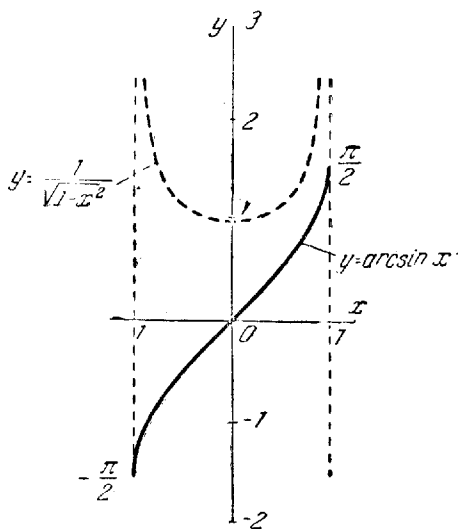


Рис. 320.01. Графики функций  $y = \frac{1}{\sqrt{1-x^2}}$  (пунктирная линия) и  $y = \arcsin x$  (сплошная линия).

$$322.01. \quad \int \frac{x^2 dx}{t} = -\frac{xt}{2} + \frac{a^2}{2} \arcsin \frac{x}{a}.$$

$$322.03. \quad \int \frac{x^2 dx}{t^3} = \frac{x}{t} - \arcsin \frac{x}{a}. \quad 322.05. \quad \int \frac{x^2 dx}{t^5} = \frac{1}{3a^2} \frac{x^3}{t^3}.$$

$$322.07. \quad \int \frac{x^2 dx}{t^7} = \frac{1}{a^4} \left[ \frac{1}{3} \frac{x^3}{t^3} + \frac{1}{5} \frac{x^5}{t^5} \right].$$

$$322.09. \quad \int \frac{x^2 dx}{t^9} = \frac{1}{a^6} \left[ \frac{1}{3} \frac{x^3}{t^3} + \frac{2}{5} \frac{x^5}{t^5} + \frac{1}{7} \frac{x^7}{t^7} \right].$$

$$322.11. \quad \int \frac{x^2 dx}{t^{11}} = \frac{1}{a^8} \left[ \frac{1}{3} \frac{x^3}{t^3} + \frac{3}{5} \frac{x^5}{t^5} + \frac{3}{7} \frac{x^7}{t^7} + \frac{1}{9} \frac{x^9}{t^9} \right].$$

$$322.13. \quad \int \frac{x^2 dx}{t^{13}} = \frac{1}{a^{10}} \left[ \frac{1}{3} \frac{x^3}{t^3} + \frac{4}{5} \frac{x^5}{t^5} + \frac{6}{7} \frac{x^7}{t^7} + \frac{4}{9} \frac{x^9}{t^9} + \frac{1}{11} \frac{x^{11}}{t^{11}} \right].$$

$$322.15. \quad \int \frac{x^2 dx}{t^{15}} = \frac{1}{a^{12}} \left[ \frac{1}{3} \frac{x^3}{t^3} + \frac{5}{5} \frac{x^5}{t^5} + \frac{10}{7} \frac{x^7}{t^7} + \frac{10}{9} \frac{x^9}{t^9} + \frac{5}{11} \frac{x^{11}}{t^{11}} + \frac{1}{13} \frac{x^{13}}{t^{13}} \right].$$

$$323.01. \quad \int \frac{x^3 dx}{t} = \frac{t^3}{3} - a^2 t. \quad 323.03. \quad \int \frac{x^3 dx}{t^3} = t + \frac{a^2}{t}.$$

$$323.05. \quad \int \frac{x^3 dx}{t^5} = -\frac{1}{t} + \frac{a^2}{3t^3}.$$

$$323.9. \quad \int \frac{x^3 dx}{t^{2p+1}} = -\frac{1}{(2p-3)t^{2p-3}} + \frac{a^2}{(2p-1)t^{2p-1}}.$$

$$324.01. \quad \int \frac{x^4 dx}{t} = -\frac{x^3 t}{4} - \frac{3}{8} a^2 x t + \frac{3}{8} a^4 \arcsin \frac{x}{a}.$$

$$324.03. \quad \int \frac{x^4 dx}{t^3} = \frac{x t}{2} + \frac{a^2 x}{t} - \frac{3}{2} a^2 \arcsin \frac{x}{a}.$$

$$324.05. \quad \int \frac{x^4 dx}{t^5} = -\frac{x}{t} + \frac{1}{3} \frac{x^3}{t^3} + \arcsin \frac{x}{a}.$$

$$324.07. \quad \int \frac{x^4 dx}{t^7} = \frac{1}{5a^2} \frac{x^5}{t^5}. \quad 324.09. \quad \int \frac{x^4 dx}{t^9} = \frac{1}{a^4} \left[ \frac{1}{5} \frac{x^5}{t^5} + \frac{1}{7} \frac{x^7}{t^7} \right].$$

$$324.11. \quad \int \frac{x^4 dx}{t^{11}} = \frac{1}{a^6} \left[ \frac{1}{5} \frac{x^5}{t^5} + \frac{2}{7} \frac{x^7}{t^7} + \frac{1}{9} \frac{x^9}{t^9} \right].$$

$$324.13. \quad \int \frac{x^4 dx}{t^{13}} = \frac{1}{a^8} \left[ \frac{1}{5} \frac{x^5}{t^5} + \frac{3}{7} \frac{x^7}{t^7} + \frac{3}{9} \frac{x^9}{t^9} + \frac{1}{11} \frac{x^{11}}{t^{11}} \right].$$

$$324.15. \quad \int \frac{x^4 dx}{t^{15}} = \frac{1}{a^{10}} \left[ \frac{1}{5} \frac{x^5}{t^5} + \frac{4}{7} \frac{x^7}{t^7} + \frac{6}{9} \frac{x^9}{t^9} + \frac{4}{11} \frac{x^{11}}{t^{11}} + \frac{1}{13} \frac{x^{13}}{t^{13}} \right].$$

$$325.01. \quad \int \frac{x^5 dx}{t} = -\frac{t^5}{5} + \frac{2a^2 t^3}{3} - a^4 t.$$

$$325.03. \quad \int \frac{x^5 dx}{t^3} = -\frac{t^3}{3} + 2a^2 t + \frac{a^4}{t}.$$

$$325.05. \quad \int \frac{x^5 dx}{t^5} = -t - \frac{2a^2}{t} + \frac{a^4}{3t^3}. \quad 325.07. \quad \int \frac{x^5 dx}{t^7} = \frac{1}{t} - \frac{2a^2}{3t^3} + \frac{a^4}{5t^5}.$$

$$325.9. \quad \int \frac{x^5 dx}{t^{2p+1}} = \frac{1}{(2p-5)t^{2p-5}} - \frac{2a^2}{(2p-3)t^{2p-3}} + \frac{a^4}{(2p-1)t^{2p-1}}.$$

$$326.01. \quad \int \frac{x^6 dx}{t} = -\frac{x^5 t}{6} - \frac{5}{24} a^2 x^3 t - \frac{5}{16} a^4 x t + \frac{5}{16} a^6 \arcsin \frac{x}{a}.$$

$$326.03. \quad \int \frac{x^6 dx}{t^3} = -\frac{x^5}{4t} - \frac{5}{8} \frac{a^2 x^3}{t} + \frac{15}{8} \frac{a^4 x}{t} - \frac{15}{8} a^4 \arcsin \frac{x}{a}.$$

$$326.05. \quad \int \frac{x^6 dx}{t^5} = -\frac{x^5}{2t^3} + \frac{10}{3} \frac{a^2 x^3}{t^3} - \frac{5}{2} \frac{a^4 x}{t^3} + \frac{5}{2} a^2 \arcsin \frac{x}{a}.$$

$$326.07. \int \frac{x^5 dx}{t^7} = \frac{23}{15} \frac{x^5}{t^5} - \frac{7}{3} \frac{a^2 x^3}{t^5} + \frac{a^4 x}{t^5} - \arcsin \frac{x}{a}.$$

$$326.09. \int \frac{x^5 dx}{t^9} = \frac{1}{7a^2} \frac{x^7}{t^7}. \quad 326.11. \int \frac{x^6 dx}{t^{11}} = \frac{1}{a^4} \left[ \frac{1}{7} \frac{x^7}{t^7} + \frac{1}{9} \frac{x^9}{t^9} \right].$$

$$326.13. \int \frac{x^6 dx}{t^{13}} = \frac{1}{a^6} \left[ \frac{1}{7} \frac{x^7}{t^7} + \frac{2}{9} \frac{x^9}{t^9} + \frac{1}{11} \frac{x^{11}}{t^{11}} \right].$$

$$326.15. \int \frac{x^6 dx}{t^{15}} = \\ = \frac{1}{a^8} \left[ \frac{1}{7} \frac{x^7}{t^7} + \frac{3}{9} \frac{x^9}{t^9} + \frac{3}{11} \frac{x^{11}}{t^{11}} + \frac{1}{13} \frac{x^{13}}{t^{13}} \right].$$

$$327.01. \int \frac{x^7 dx}{t} = \frac{1}{7} t^7 - \frac{3}{5} a^2 t^5 + \frac{3}{3} a^4 t^3 - a^6 t.$$

$$327.03. \int \frac{x^7 dx}{t^3} = \frac{1}{5} t^5 - \frac{3}{3} a^2 t^3 + 3a^4 t + \frac{a^6}{t}.$$

$$327.05. \int \frac{x^7 dx}{t^5} = \frac{1}{3} t^3 - 3a^2 t - \frac{3a^4}{t} + \frac{a^6}{3t^3}.$$

$$327.07. \int \frac{x^7 dx}{t^7} = t + \frac{3a^2}{t} - \frac{3a^4}{3t^3} + \frac{a^6}{5t^5}.$$

$$327.9. \int \frac{x^7 dx}{t^{2p+1}} = -\frac{1}{(2p-7)t^{2p-7}} + \\ + \frac{3a^2}{(2p-5)t^{2p-5}} - \frac{3a^4}{(2p-3)t^{2p-3}} + \\ + \frac{a^6}{(2p-1)t^{2p-1}}.$$

$$341.01. \int \frac{dx}{xt} = \int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a+t}{x} \right| \\ [x^2 < a^2].$$

Заметим, что

$$-\frac{1}{a} \ln \left| \frac{a+t}{x} \right| = -\frac{1}{a} \operatorname{Arsch} \left| \frac{x}{a} \right| = \\ = -\frac{1}{a} \operatorname{Arch} \left| \frac{a}{x} \right| = -\frac{1}{2a} \ln \left( \frac{a+t}{a-t} \right).$$

Надо брать положительные значения  $\operatorname{Arsch}$ ,  $\operatorname{Arch}$ ,  $a$  и  $t$ .

$$341.03. \int \frac{dx}{xt^3} = \frac{1}{a^2 t} - \frac{1}{a^3} \ln \left| \frac{a+t}{x} \right|.$$

$$341.05. \int \frac{dx}{xt^5} = \frac{1}{3a^2 t^3} + \frac{1}{a^4 t} - \frac{1}{a^5} \ln \left| \frac{a+t}{x} \right|.$$

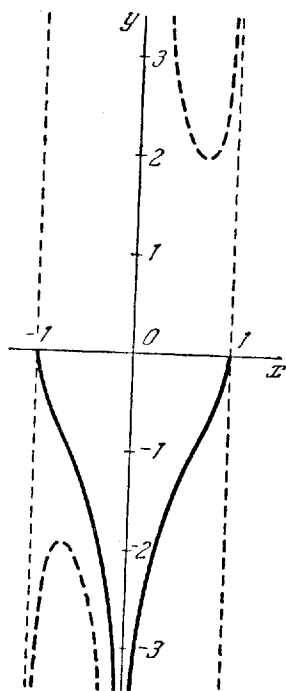


Рис. 341.01. Графики функций  $y = \frac{1}{x \sqrt{1-x^2}}$  (пунктирная линия) и  $y = -\ln \left| \frac{1+\sqrt{1-x^2}}{x} \right|$  (сплошная линия).

$$341.07. \int \frac{dx}{xt^7} = \frac{1}{5a^2t^5} + \frac{1}{3a^4t^3} + \frac{1}{a^6t} - \frac{1}{a^7} \ln \left| \frac{a+t}{x} \right|.$$

$$341.09. \int \frac{dx}{xt^9} = \frac{1}{7a^2t^7} + \frac{1}{5a^4t^5} + \frac{1}{3a^6t^3} + \frac{1}{a^8t} - \frac{1}{a^9} \ln \left| \frac{a+t}{x} \right|.$$

$$342.01. \int \frac{dx}{x^2t} = -\frac{t}{a^2x}.$$

$$342.03. \int \frac{dx}{x^2t^3} = \frac{1}{a^4} \left( -\frac{t}{x} + \frac{x}{t} \right).$$

$$342.05. \int \frac{dx}{x^2t^5} = \frac{1}{a^6} \left( -\frac{t}{x} + \frac{2x}{t} + \frac{x^3}{3t^3} \right).$$

$$342.07. \int \frac{dx}{x^2t^7} = \frac{1}{a^8} \left( -\frac{t}{x} + \frac{3x}{t} + \frac{3x^3}{3t^3} + \frac{x^5}{5t^5} \right).$$

$$342.09. \int \frac{dx}{x^2t^9} = \frac{1}{a^{10}} \left( -\frac{t}{x} + \frac{4x}{t} + \frac{6x^3}{3t^3} + \frac{4x^5}{5t^5} + \frac{x^7}{7t^7} \right).$$

$$343.01. \int \frac{dx}{x^3t} = -\frac{t}{2a^2x^2} - \frac{1}{2a^3} \ln \left| \frac{a+t}{x} \right|. \quad [\text{См. } 341.01.]$$

$$343.03. \int \frac{dx}{x^3t^3} = -\frac{1}{2a^2x^2t} + \frac{3}{2a^4t} - \frac{3}{2a^5} \ln \left| \frac{a+t}{x} \right|.$$

$$343.05. \int \frac{dx}{x^3t^5} = -\frac{1}{2a^2x^2t^3} + \frac{5}{6a^4t^3} + \frac{5}{2a^6t} - \frac{5}{2a^7} \ln \left| \frac{a+t}{x} \right|.$$

$$344.01. \int \frac{dx}{x^4t} = -\frac{1}{a^4} \left( \frac{t}{x} + \frac{t^3}{3x^3} \right).$$

$$344.03. \int \frac{dx}{x^4t^3} = -\frac{1}{a^6} \left( -\frac{x}{t} + \frac{2t}{x} + \frac{t^3}{3x^3} \right).$$

$$344.05. \int \frac{dx}{x^4t^5} = -\frac{1}{a^8} \left( -\frac{x^3}{3t^3} - \frac{3x}{t} + \frac{3t}{x} + \frac{t^3}{3x^3} \right).$$

Интегралы 342 и 344 находят посредством подстановки:

$$z^2 = \frac{x^2}{t^2}, \quad \text{тогда} \quad dx = \frac{a dz}{(1+z^2)^{3,2}}.$$

$$345.01. \int \frac{dx}{x^3t} = -\left[ \frac{t}{4a^2x^4} + \frac{3}{8} \frac{t}{a^4x^2} + \frac{3}{8a^5} \ln \left| \frac{a+t}{x} \right| \right].$$

$$345.03. \int \frac{dx}{x^3t^3} = -\left[ \frac{1}{4a^2x^4t} + \frac{5}{8a^4x^2t} - \frac{15}{8a^6t} + \frac{15}{8a^7} \ln \left| \frac{a+t}{x} \right| \right].$$

$$346.01. \quad \int \frac{dx}{x^6 t} = -\frac{1}{a^6} \left( \frac{t}{x} + \frac{2t^3}{3x^3} + \frac{t^5}{5x^5} \right).$$

$$346.03. \quad \int \frac{dx}{x^6 t^3} = -\frac{1}{a^6} \left( -\frac{x}{t} + \frac{3t}{x} + \frac{3t^3}{3x^3} + \frac{t^5}{5x^5} \right).$$

$$350.01. \quad \int t dx = \frac{xt}{2} + \frac{a^2}{2} \arcsin \frac{x}{a}.$$

$$350.03. \quad \int t^3 dx = \frac{xt^3}{4} + \frac{3}{8} a^2 xt + \frac{3}{8} a^4 \arcsin \frac{x}{a}.$$

$$350.05. \quad \int t^5 dx = \frac{xt^5}{6} + \frac{5}{24} a^2 xt^3 + \frac{5}{16} a^4 xt + \frac{5}{16} a^6 \arcsin \frac{x}{a}.$$

$$351.01. \quad \int xt dx = -\frac{t^3}{3}. \quad 351.03. \quad \int xt^3 dx = -\frac{t^5}{5}.$$

$$351.9. \quad \int xt^{2p+1} dx = -\frac{t^{2p+3}}{2p+3}.$$

$$352.01. \quad \int x^2 t dx = -\frac{xt^3}{4} + \frac{a^2 xt}{8} + \frac{a^4}{8} \arcsin \frac{x}{a}.$$

$$352.03. \quad \int x^2 t^3 dx = -\frac{xt^5}{6} + \frac{a^2 xt^3}{24} + \frac{a^4 xt}{16} + \frac{a^6}{16} \arcsin \frac{x}{a}.$$

$$353.01. \quad \int x^3 t dx = \frac{t^5}{5} - \frac{a^2 t^3}{3}. \quad 353.03. \quad \int x^3 t^3 dx = \frac{t^7}{7} - \frac{a^2 t^5}{5}.$$

$$353.9. \quad \int x^3 t^{2p+1} dx = \frac{t^{2p+5}}{2p+5} - \frac{a^2 t^{2p+3}}{2p+3}.$$

$$354.01. \quad \int x^4 t dx = -\frac{x^3 t^3}{6} - \frac{a^2 x t^3}{8} + \frac{a^4 x t}{16} + \frac{a^6}{16} \arcsin \frac{x}{a}.$$

$$354.03. \quad \int x^4 t^3 dx = -\frac{x^3 t^5}{8} - \frac{a^2 x t^5}{16} + \frac{a^4 x t^3}{64} + \frac{3}{128} a^6 x t + \frac{3}{128} a^8 \arcsin \frac{x}{a}.$$

$$355.01. \quad \int x^5 t dx = -\frac{t^7}{7} + \frac{2a^2 t^5}{5} - \frac{a^4 t^3}{3}.$$

$$355.03. \quad \int x^5 t^3 dx = -\frac{t^9}{9} + \frac{2a^2 t^7}{7} - \frac{a^4 t^5}{5}.$$

$$355.9. \quad \int x^5 t^{2p+1} dx = -\frac{t^{2p+7}}{2p+7} + \frac{2a^2 t^{2p+5}}{2p+5} - \frac{a^4 t^{2p+3}}{2p+3}.$$

$$361.01. \quad \int \frac{t dx}{x} = t - a \ln \left| \frac{a+t}{x} \right|. \quad (\text{См. замечание к } 341.01.)$$

$$361.03. \quad \int \frac{t^3 dx}{x} = \frac{t^3}{3} + a^2 t - a^3 \ln \left| \frac{a+t}{x} \right|.$$

На этой странице  $t = (a^2 - x^2)^{1/2}$

- 361.05. 
$$\int \frac{t^5 dx}{x} = \frac{t^5}{5} + \frac{a^2 t^3}{3} + a^4 t - a^5 \ln \left| \frac{a+t}{x} \right|.$$
- 361.07. 
$$\int \frac{t^7 dx}{x} = \frac{t^7}{7} + \frac{a^2 t^5}{5} + \frac{a^4 t^3}{3} + a^6 t - a^7 \ln \left| \frac{a+t}{x} \right|.$$
- 362.01. 
$$\int \frac{t dx}{x^2} = -\frac{t}{x} - \arcsin \frac{x}{a}.$$
- 362.03. 
$$\int \frac{t^3 dx}{x^2} = -\frac{t^3}{x} - \frac{3}{2} xt - \frac{3}{2} a^2 \arcsin \frac{x}{a}.$$
- 362.05. 
$$\int \frac{t^5 dx}{x^2} = -\frac{t^5}{x} - \frac{5}{4} xt^3 - \frac{15}{8} a^2 xt - \frac{15}{8} a^4 \arcsin \frac{x}{a}.$$
- 363.01. 
$$\int \frac{t dx}{x^3} = -\frac{t}{2x^2} + \frac{1}{2a} \ln \left| \frac{a+t}{x} \right|. \quad (\text{См. замечание к 341.01.})$$
- 363.03. 
$$\int \frac{t^3 dx}{x^3} = -\frac{t^3}{2x^2} - \frac{3t}{2} + \frac{3a}{2} \ln \left| \frac{a+t}{x} \right|.$$
- 363.05. 
$$\int \frac{t^5 dx}{x^3} = -\frac{t^5}{2x^2} - \frac{5}{6} t^3 - \frac{5}{2} a^2 t + \frac{5}{2} a^3 \ln \left| \frac{a+t}{x} \right|.$$
- 364.01. 
$$\int \frac{t dx}{x^4} = -\frac{t^3}{3a^2 x^3}.$$
- 364.03. 
$$\int \frac{t^3 dx}{x^4} = -\frac{t^3}{3x^3} + \frac{t}{x} + \arcsin \frac{x}{a}.$$
- 364.05. 
$$\int \frac{t^5 dx}{x^4} = -\frac{a^2 t^3}{3x^3} + \frac{2a^2 t}{x} + \frac{xt}{2} + \frac{5}{2} a^2 \arcsin \frac{x}{a}.$$
- 365.01. 
$$\int \frac{t dx}{x^5} = -\frac{t}{4x^4} + \frac{t}{8a^2 x^2} + \frac{1}{8a^3} \ln \left| \frac{a+t}{x} \right|.$$
- 365.03. 
$$\int \frac{t^3 dx}{x^5} = -\frac{t^3}{4x^4} + \frac{3}{8} \frac{t^3}{a^2 x^2} + \frac{3}{8} \frac{t}{a^2} - \frac{3}{8a} \ln \left| \frac{a+t}{x} \right|.$$
- 366.01. 
$$\int \frac{t dx}{x^6} = -\frac{t^3}{5a^2 x^3} \left( \frac{1}{x^2} + \frac{2}{3a^2} \right).$$
- 366.03. 
$$\int \frac{t^5 dx}{x^6} = -\frac{t^5}{5a^2 x^5}.$$
- 367.01. 
$$\int \frac{t dx}{x^7} = -\frac{t}{6x^6} + \frac{t}{24a^2 x^4} + \frac{t}{16a^4 x^2} + \frac{1}{16a^5} \ln \left| \frac{a+t}{x} \right|.$$
- 368.01. 
$$\int \frac{t dx}{x^8} = -\frac{t^3}{7a^2 x^3} \left( \frac{1}{x^4} + \frac{4}{5a^2 x^2} + \frac{8}{15a^4} \right).$$