

Интегралы, содержащие $X^{1/2} = (ax^2 + bx + c)^{1/2}$

$$\begin{aligned}
 380.001. \quad \int \frac{dx}{X^{1/2}} &= \frac{1}{a^{1/2}} \ln |2(aX)^{1/2} + 2ax + b| \quad [a > 0], \\
 &= \frac{1}{a^{1/2}} \operatorname{Arsh} \frac{2ax + b}{(4ac - b^2)^{1/2}} \quad \left[\begin{array}{l} a > 0, \\ 4ac > b^2 \end{array} \right], \\
 &= \frac{1}{a^{1/2}} \ln |2ax + b| \quad [a > 0, b^2 = 4ac, 2ax + b > 0], \\
 &= -\frac{1}{a^{1/2}} \ln |2ax + b| \quad [a > 0, b^2 = 4ac, 2ax + b < 0], \\
 &= \frac{-1}{(-a)^{1/2}} \arcsin \frac{(2ax + b)}{(b^2 - 4ac)^{1/2}} \quad \left[\begin{array}{l} a < 0, \quad b^2 > 4ac, \\ |2ax + b| < (b^2 - 4ac)^{1/2} \end{array} \right].
 \end{aligned}$$

$$380.003. \quad \int \frac{dx}{X^{3/2}} = \frac{4ax + 2b}{(4ac - b^2)X^{1/2}}.$$

$$380.005. \quad \int \frac{dx}{X^{5/2}} = \frac{4ax + 2b}{3(4ac - b^2)X^{1/2}} \left(\frac{1}{X} + \frac{8a}{4ac - b^2} \right).$$

$$380.009. \int \frac{dx}{X^{(2n+1)/2}} = \frac{4ax+2b}{(2n-1)(4ac-b^2)X^{(2n-1)/2}} + \\ + \frac{8a(n-1)}{(2n-1)(4ac-b^2)} \int \frac{dx}{X^{(2n-1)/2}}.$$

$$380.011. \int \frac{x dx}{X^{1/2}} = \frac{X^{1/2}}{a} - \frac{b}{2a} \int \frac{dx}{X^{1/2}}. \quad [\text{См. } 380.001.]$$

$$380.013. \int \frac{x dx}{X^{3/2}} = -\frac{2bx+4c}{(4ac-b^2)X^{1/2}}.$$

$$380.019. \int \frac{x dx}{X^{(2n+1)/2}} = -\frac{1}{(2n-1)aX^{(2n-1)/2}} - \frac{b}{2a} \int \frac{dx}{X^{(2n+1)/2}}.$$

$$380.021. \int \frac{x^2 dx}{X^{1/2}} = \left(\frac{x}{2a} - \frac{3b}{4a^2}\right)X^{1/2} + \frac{3b^2-4ac}{8a^2} \int \frac{dx}{X^{1/2}}. \quad [\text{См. } 380.001.]$$

$$380.111. \int \frac{dx}{xX^{1/2}} = -\frac{1}{c^{1/2}} \ln \left| \frac{2(cX)^{1/2}}{x} + \frac{2c}{x} + b \right| \quad [c > 0], \\ = -\frac{1}{c^{1/2}} \operatorname{Arsh} \frac{bx+2c}{|x|(4ac-b^2)^{1/2}} \quad \left[\begin{array}{l} c > 0, \\ 4ac > b^2 \end{array} \right], \\ = -\frac{1}{c^{1/2}} \ln \left| \frac{bx+2c}{x} \right| \quad [c > 0, b^2 = 4ac, bx+2c > 0], \\ = \frac{1}{c^{1/2}} \ln \left| \frac{bx+2c}{x} \right| \quad [c > 0, b^2 = 4ac, bx+2c < 0], \\ = \frac{1}{(-c)^{1/2}} \arcsin \frac{bx+2c}{|x|(b^2-4ac)^{1/2}} \quad \left[\begin{array}{l} c < 0, \\ b^2 > 4ac \end{array} \right].$$

$$380.119. \int \frac{dx}{xX^{(2n+1)/2}} = \frac{1}{(2n-1)cX^{(2n-1)/2}} + \\ + \frac{1}{c} \int \frac{dx}{xX^{(2n-1)/2}} - \frac{b}{2c} \int \frac{dx}{X^{(2n+1)/2}}.$$

$$380.121. \int \frac{dx}{x^2 X^{1/2}} = -\frac{X^{1/2}}{cx} - \frac{b}{2c} \int \frac{dx}{xX^{1/2}}. \quad [\text{См. } 380.111.]$$

$$380.201. \int X^{1/2} dx = \frac{2ax+b}{4a} X^{1/2} + \frac{4ac-b^2}{8a} \int \frac{dx}{X^{1/2}}. \quad [\text{См. } 380.001.]$$

$$380.209. \int X^{(2n+1)/2} dx = \frac{(2ax+b)X^{(2n+1)/2}}{4a(n+1)} + \\ + \frac{(4ac-b^2)(2n+1)}{8a(n+1)} \int X^{(2n-1)/2} dx.$$

$$380.211. \int xX^{1/2} dx = \frac{X^{3/2}}{3a} - \frac{b(2ax+b)}{8a^2} X^{1/2} - \\ - \frac{b(4ac-b^2)}{16a^2} \int \frac{dx}{X^{1/2}}. \quad [\text{См. } 380.001.]$$

$$380.219. \int xX^{(2n+1)/2} dx = \frac{X^{(2n+3)/2}}{(2n+3)a} - \frac{b}{2a} \int X^{(2n+1)/2} dx.$$

$$380.311. \int \frac{X^{1/2} dx}{x} = X^{1/2} + \frac{b}{2} \int \frac{dx}{X^{1/2}} + c \int \frac{dx}{xX^{1/2}}.$$

[См. 380.001 и 380.111.]

$$380.319. \int \frac{X^{(2n+1)/2} dx}{x} = \frac{X^{(2n+1)/2}}{2n+1} + \frac{b}{2} \int X^{(2n-1)/2} dx + c \int \frac{X^{(2n-1)/2} dx}{x}.$$

$$380.321. \int \frac{X^{1/2} dx}{x^2} = -\frac{X^{1/2}}{x} + a \int \frac{dx}{X^{1/2}} + \frac{b}{2} \int \frac{dx}{xX^{1/2}}.$$

[См. 380.001 и 380.111.]

$$383.1. \int \frac{dx}{x(ax^2 + bx)^{1/2}} = -\frac{2}{bx} (ax^2 + bx)^{1/2}.$$

$$383.2. \int \frac{dx}{(2ax - x^2)^{1/2}} = \arcsin \frac{x-a}{a}.$$

$$383.3. \int \frac{x dx}{(2ax - x^2)^{1/2}} = -(2ax - x^2)^{1/2} + a \arcsin \left(\frac{x-a}{a} \right).$$

$$383.4. \int (2ax - x^2)^{1/2} dx = \frac{x-a}{2} (2ax - x^2)^{1/2} + \frac{a^2}{2} \arcsin \frac{x-a}{a}.$$

$$384.1. \int \frac{dx}{x(x^n + a^2)^{1/2}} = -\frac{2}{na} \ln \left| \frac{a + (x^n + a^2)^{1/2}}{x^{n/2}} \right|.$$

$$384.2. \int \frac{dx}{x(x^n - a^2)^{1/2}} = \frac{2}{na} \arccos \left| \frac{a}{x^{n/2}} \right|.$$

$$384.3. \int \frac{x^{1/2} dx}{(a^3 - x^3)^{1/2}} = \frac{2}{3} \arcsin \left(\frac{x}{a} \right)^{3/2}.$$

$$387. \int \frac{dx}{(ax^2 + b) \sqrt{fx^2 + g}} =$$

$$= \frac{1}{\sqrt{b} \sqrt{ag - bf}} \operatorname{arctg} \frac{x \sqrt{ag - bf}}{\sqrt{b} \sqrt{fx^2 + g}} \quad [ag > bf],$$

$$= \frac{1}{2 \sqrt{b} \sqrt{bf - ag}} \ln \left| \frac{\sqrt{b} \sqrt{fx^2 + g} + x \sqrt{bf - ag}}{\sqrt{b} \sqrt{fx^2 + g} - x \sqrt{bf - ag}} \right| \quad [bf > ag].$$