## Интегралы, содержащие $\cos x$

**440.10.** $\int \cos x\, dx = \sin x.$

**440.101.** $\int \cos(a + bx)\, dx = \frac{1}{b} \sin(a + bx).$

**440.102.** $\int \cos \frac{x}{a}\, dx = a \sin \frac{x}{a}.$

**440.11.** $\int x \cos x\, dx = \cos x + x \sin x.$

**440.12.** $\int x^2 \cos x\, dx = 2x \cos x + (x^2 - 2) \sin x.$

**440.13.** $\int x^3 \cos x\, dx = (3x^2 - 6) \cos x + (x^3 - 6x) \sin x.$

**440.14.** $\int x^4 \cos x\, dx = (4x^3 - 24x) \cos x + (x^4 - 12x^2 + 24) \sin x.$

**440.15.** $\int x^5 \cos x\,dx =$
$= (5x^4 - 60x^2 + 120)\cos x + (x^5 - 20x^3 + 120x)\sin x.$

**440.16.** $\int x^6 \cos x\,dx = (6x^5 - 120x^3 + 720x)\cos x +$
$+ (x^6 - 30x^4 + 360x^2 - 720)\sin x.$

**440.19.** $\int x^m \cos x\,dx = x^m \sin x - m\int x^{m-1}\sin x\,dx.$ [См. **430.**]

**440.20.** $\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{x}{2} + \frac{\sin x \cos x}{2}.$

**440.21.** $\int x\cos^2 x\,dx = \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}.$

**440.22.** $\int x^2\cos^2 x\,dx = \frac{x^3}{6} + \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin 2x + \frac{x\cos 2x}{4}.$

**440.23.** $\int x^3\cos^2 x\,dx = \frac{x^4}{8} + \left(\frac{x^3}{4} - \frac{3x}{8}\right)\sin 2x + \left(\frac{3x^2}{8} - \frac{3}{16}\right)\cos 2x.$

**440.30.** $\int \cos^3 x\,dx = \sin x - \frac{\sin^3 x}{3}.$

**440.31.** $\int x\cos^3 x\,dx = \frac{x\sin 3x}{12} + \frac{\cos 3x}{36} + \frac{3}{4}x\sin x + \frac{3}{4}\cos x.$
($\cos^3 x$ выражается согласно **404.23.**)

**440.40.** $\int \cos^4 x\,dx = \frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32}.$

**440.50.** $\int \cos^5 x\,dx = \frac{5\sin x}{8} + \frac{5\sin 3x}{48} + \frac{\sin 5x}{80}.$

**440.60.** $\int \cos^6 x\,dx = \frac{5x}{16} + \frac{15\sin 2x}{64} + \frac{3\sin 4x}{64} + \frac{\sin 6x}{192}.$

**440.70.** $\int \cos^7 x\,dx = \frac{35\sin x}{64} + \frac{7\sin 3x}{64} + \frac{7\sin 5x}{320} + \frac{\sin 7x}{448}.$
(Интегрируется выражение из **404.**)

**441.11.** $\int \frac{\cos x\,dx}{x} = \ln|x| - \frac{x^2}{2\cdot 2!} + \frac{x^4}{4\cdot 4!} - \frac{x^6}{6\cdot 6!} + \ldots$
Таблицу численных значений см. [22].

**441.12.** $\int \frac{\cos x\,dx}{x^2} = -\frac{\cos x}{x} - \int \frac{\sin x\,dx}{x}.$ [См. **431.11.**]

**441.13.** $\int \frac{\cos x\,dx}{x^3} = -\frac{\cos x}{2x^2} + \frac{\sin x}{2x} - \frac{1}{2}\int \frac{\cos x\,dx}{x}.$ [См. **441.11.**]

**441.14.** $$\int \frac{\cos x\,dx}{x^4} = -\frac{\cos x}{3x^3} + \frac{\sin x}{6x^2} + \frac{\cos x}{6x} + \frac{1}{6}\int \frac{\sin x\,dx}{x}.$$

[См. **431.11.**]

**441.19.** $$\int \frac{\cos x\,dx}{x^m} = -\frac{\cos x}{(m-1)\,x^{m-1}} - \frac{1}{m-1}\int \frac{\sin x\,dx}{x^{m-1}}.$$

**441.21.** $$\int \frac{\cos^2 x\,dx}{x} = \frac{1}{2}\ln|x| + \frac{1}{2}\int \frac{\cos 2x\,d\,(2x)}{2x}.$$ [См. **441.11.**]

**441.31.** $$\int \frac{\cos^3 x\,dx}{x} = \frac{3}{4}\int \frac{\cos x\,dx}{x} + \frac{1}{4}\int \frac{\cos 3x\,d\,(3x)}{3x}.$$ [См. **441.11.**]

**441.9.** $$\int \frac{\cos^n x\,dx}{x^m}.$$

Выразить $\cos^n x$ согласно **404** и интегрировать почленно согласно **441.1.**

**442.10.** $$\int \frac{dx}{\cos x} = \int \sec x\,dx = \ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|,$$
$$= \ln|\sec x + \operatorname{tg} x| = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x},$$
$$= \lambda(x) \text{ (лямбда-функция).}$$ [См. **640.**]

**442.11.** $$\int \frac{x\,dx}{\cos x} = \frac{x^2}{2} + \frac{x^4}{4\cdot 2!} + \frac{5x^6}{6\cdot 4!} + \frac{61x^8}{8\cdot 6!} + \frac{1385x^{10}}{10\cdot 8!} + \ldots$$
$$\ldots + \frac{E_n x^{2n+2}}{(2n+2)\,(2n)!} + \ldots$$ [См. **45.**]

**442.12.** $$\int \frac{x^2 dx}{\cos x} = \frac{x^3}{3} + \frac{x^5}{5\cdot 2!} + \frac{5x^7}{7\cdot 4!} + \frac{61x^9}{9\cdot 6!} + \frac{1385x^{11}}{11\cdot 8!} + \ldots$$
$$\ldots + \frac{E_{n-1}x^{2n+1}}{(2n+1)\,(2n-2)!} + \ldots$$ [См. **45.**]

**442.19.** $\int \frac{x^m dx}{\cos x}$. Разложить $\frac{1}{\cos x}$ согласно **415.05,** умножить на $x^m$ и интегрировать $[m \neq 0]$.

**442.20.** $$\int \frac{dx}{\cos^2 x} = \int \sec^2 x\,dx = \operatorname{tg} x.$$

**442.21.** $$\int \frac{x\,dx}{\cos^2 x} = x\operatorname{tg} x + \ln|\cos x|.$$

**442.30.** $$\int \frac{dx}{\cos^3 x} = \frac{\sin x}{2\cos^2 x} + \frac{1}{2}\ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|.$$

**442.31.** $$\int \frac{x\,dx}{\cos^3 x} = \frac{x\sin x}{2\cos^2 x} - \frac{1}{2\cos x} + \frac{1}{2}\int \frac{x\,dx}{\cos x}.$$ [См. **442.11.**]

**442.40.** $\int \frac{dx}{\cos^4 x} = \frac{\sin x}{3\cos^3 x} + \frac{2}{3}\operatorname{tg} x = \operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3}$.

**442.41.** $\int \frac{x\,dx}{\cos^4 x} = \frac{x\sin x}{3\cos^3 x} - \frac{1}{6\cos^2 x} + \frac{2}{3} x \operatorname{tg} x + \frac{2}{3}\ln|\cos x|$.

**442.50.** $\int \frac{dx}{\cos^5 x} = \frac{\sin x}{4\cos^4 x} + \frac{3}{8}\frac{\sin x}{\cos^2 x} + \frac{3}{8}\ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|$.

**442.60.** $\int \frac{dx}{\cos^6 x} = \frac{\sin x}{5\cos^5 x} + \frac{4}{15}\frac{\sin x}{\cos^3 x} + \frac{8}{15}\operatorname{tg} x$.

**442.90.** $\int \frac{dx}{\cos^n x} = \int \sec^n x\,dx =$

$$= \frac{\sin x}{(n-1)\cos^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} x} \qquad [n>1].$$

**442.91.** $\int \frac{x\,dx}{\cos^n x} = \frac{x\sin x}{(n-1)\cos^{n-1} x} - \frac{1}{(n-1)(n-2)\cos^{n-2} x} +$

$$+ \frac{n-2}{n-1}\int \frac{x\,dx}{\cos^{n-2} x} \qquad [n>2].$$

**443.01.** $\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2}$.

**443.02.** $\int \frac{dx}{1-\cos x} = -\operatorname{ctg}\frac{x}{2}$.

**443.03.** $\int \frac{x\,dx}{1+\cos x} = x\operatorname{tg}\frac{x}{2} + 2\ln\left|\cos\frac{x}{2}\right|$.

**443.04.** $\int \frac{x\,dx}{1-\cos x} = -x\operatorname{ctg}\frac{x}{2} + 2\ln\left|\sin\frac{x}{2}\right|$.

**443.05.** $\int \frac{\cos x\,dx}{1+\cos x} = x - \operatorname{tg}\frac{x}{2}$.

**443.06.** $\int \frac{\cos x\,dx}{1-\cos x} = -x - \operatorname{ctg}\frac{x}{2}$.

**443.07.** $\int \frac{dx}{\cos x\,(1+\cos x)} = \ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| - \operatorname{tg}\frac{x}{2}$.

**443.08.** $\int \frac{dx}{\cos x\,(1-\cos x)} = \ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| - \operatorname{ctg}\frac{x}{2}$.

**444.01.** $\int \frac{dx}{(1+\cos x)^2} = \frac{1}{2}\operatorname{tg}\frac{x}{2} + \frac{1}{6}\operatorname{tg}^3\frac{x}{2}$.

**444.02.** $\int \frac{dx}{(1-\cos x)^2} = -\frac{1}{2}\operatorname{ctg}\frac{x}{2} - \frac{1}{6}\operatorname{ctg}^3\frac{x}{2}$.

**444.03.** $\int \frac{\cos x\,dx}{(1+\cos x)^2} = \frac{1}{2}\operatorname{tg}\frac{x}{2} - \frac{1}{6}\operatorname{tg}^3\frac{x}{2}$.

**444.04.** $\int \frac{\cos x\,dx}{(1-\cos x)^2} = \frac{1}{2}\operatorname{ctg}\frac{x}{2} - \frac{1}{6}\operatorname{ctg}^3\frac{x}{2}$.

**444.05.** $\int \frac{dx}{1+\cos^2 x} = \frac{1}{2\sqrt{2}}\arcsin\left(\frac{1-3\cos^2 x}{1+\cos^2 x}\right)$. [См. **446.6.**]

**444.06.** $\int \frac{dx}{1-\cos^2 x} = \int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x$. [См. **432.20.**]

**445.** $\int \cos mx \cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)}$

$[m^2 \neq n^2$. Если $m^2 = n^2$, то см. **440.20.**$]$

**446.00.** $\int \frac{dx}{a+b\cos x} =$

$$= \frac{2}{\sqrt{a^2-b^2}}\operatorname{arctg}\frac{(a-b)\operatorname{tg}\frac{x}{2}}{\sqrt{a^2-b^2}} \qquad [a^2 > b^2],$$

$$= \frac{1}{\sqrt{b^2-a^2}}\ln\left|\frac{(b-a)\operatorname{tg}\frac{x}{2} + \sqrt{b^2-a^2}}{(b-a)\operatorname{tg}\frac{x}{2} - \sqrt{b^2-a^2}}\right| \qquad [b^2 > a^2],$$

$$= \frac{2}{\sqrt{b^2-a^2}}\operatorname{Arth}\frac{(b-a)\operatorname{tg}\frac{x}{2}}{\sqrt{b^2-a^2}} \qquad \left[b^2 > a^2,\ \left|(b-a)\operatorname{tg}\frac{x}{2}\right| < \sqrt{b^2-a^2}\right],$$

$$= \frac{2}{\sqrt{b^2-a^2}}\operatorname{Arcth}\frac{(b-a)\operatorname{tg}\frac{x}{2}}{\sqrt{b^2-a^2}} \qquad \left[b^2 > a^2,\ \left|(b-a)\operatorname{tg}\frac{x}{2}\right| > \sqrt{b^2-a^2}\right].$$

Подынтегральная функция обращается в бесконечность (если $b^2 > a^2$) при $x = 2n\pi \pm \arccos\left(-\frac{a}{b}\right)$.

**446.01.** $\int \frac{\cos x\,dx}{a+b\cos x} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+b\cos x}$.

**446.02.** $\int \frac{dx}{(a+b\cos x)\cos x} = \frac{1}{a}\ln\left|\operatorname{tg}\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| - \frac{b}{a}\int \frac{dx}{a+b\cos x}$.

**446.03.** $\int \frac{dx}{(a+b\cos x)^2} = \frac{b\sin x}{(b^2-a^2)(a+b\cos x)} - \frac{a}{b^2-a^2}\int \frac{dx}{a+b\cos x}$.

**446.04.** $\int \frac{\cos x\,dx}{(a+b\cos x)^2} = \frac{a\sin x}{(a^2-b^2)(a+b\cos x)} - \frac{b}{a^2-b^2}\int \frac{dx}{a+b\cos x}$.

[К **446.01—446.04** см. **446.00.**]

**446.2.** $$\int \frac{dx}{a^2+b^2-2ab\cos x} = \frac{2}{|a^2-b^2|}\operatorname{arctg}\left[\left|\frac{a+b}{a-b}\right|\operatorname{tg}\frac{x}{2}\right] \qquad [a \neq b].$$

[См. **446.00.**]

**446.5.** $$\int \frac{dx}{a^2+b^2\cos^2 x} = \frac{1}{a\sqrt{a^2+b^2}}\operatorname{arctg}\frac{a\operatorname{tg} x}{\sqrt{a^2+b^2}} \qquad [a>0].$$

**446.6.** При $a=b=1$

$$\int \frac{dx}{1+\cos^2 x} = \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{\operatorname{tg} x}{\sqrt{2}}\right).$$

Другое выражение, отличающееся на константу, дается в **444.05.**

**446.7.** $$\int \frac{dx}{a^2-b^2\cos^2 x} = \frac{1}{a\sqrt{a^2-b^2}}\operatorname{arctg}\frac{a\operatorname{tg} x}{\sqrt{a^2-b^2}}$$

$$[a^2>b^2,\ a>0],$$

$$= \frac{1}{2a\sqrt{b^2-a^2}}\ln\left|\frac{a\operatorname{tg} x-\sqrt{b^2-a^2}}{a\operatorname{tg} x+\sqrt{b^2-a^2}}\right|$$

$$[b^2>a^2,\ a>0].$$

Если $b^2=a^2$, см. **444.06.**