

№ п/п	Название формул	Системы	
		произвольная	ортогональная криволинейная
1	Закон перехода от прямоугольной к криволинейной системе координат	$x^i = x^i(\alpha^1, \alpha^2, \alpha^3)$	$x^1 = x^1(\alpha^1, \alpha^2, \alpha^3)$ $x^2 = x^2(\alpha^1, \alpha^2, \alpha^3)$ $x^3 = x^3(\alpha^1, \alpha^2, \alpha^3)$
2	Координаты локального базиса	$\vec{e}_i \left\{ \frac{\partial x^1}{\partial \alpha^i}, \frac{\partial x^2}{\partial \alpha^i}, \frac{\partial x^3}{\partial \alpha^i} \right\}$	$\vec{e}_1 \left\{ \frac{\partial x^1}{\partial \alpha^1}, \frac{\partial x^2}{\partial \alpha^1}, \frac{\partial x^3}{\partial \alpha^1} \right\}$ $\vec{e}_2 \left\{ \frac{\partial x^1}{\partial \alpha^2}, \frac{\partial x^2}{\partial \alpha^2}, \frac{\partial x^3}{\partial \alpha^2} \right\}$ $\vec{e}_3 \left\{ \frac{\partial x^1}{\partial \alpha^3}, \frac{\partial x^2}{\partial \alpha^3}, \frac{\partial x^3}{\partial \alpha^3} \right\}$
3	Элемент длины	$ds^2 = g_{ij} d\alpha^i d\alpha^j$	$ds^2 = (H_1 d\alpha^1)^2 + (H_2 d\alpha^2)^2 + (H_3 d\alpha^3)^2$
4	Компоненты фундаментальной матрицы $g_{ij}$ *	$g_{ij} = \vec{e}_i \cdot \vec{e}_j$	$g_{11} = H_1^2, g_{22} = H_2^2,$ $g_{33} = H_3^2$
5	Определитель фундаментальной матрицы $g$	$g = \det  g_{ij} $	$g = (H_1 H_2 H_3)^2$
6	Компоненты матрицы $g^{ij}$ *	$g^{ij} = \frac{1}{g} \frac{\partial g}{\partial g_{ij}}$	$g^{11} = \frac{1}{H_1^2}, g^{22} = \frac{1}{H_2^2},$ $g^{33} = \frac{1}{H_3^2}$

\* Остальные компоненты равны нулю.

системах координат

координат

прямоугольная	цилиндрическая	сферическая
$x^1 = \alpha^1$ $x^2 = \alpha^2$ $x^3 = \alpha^3$	$x^1 = \alpha^1 \cos \alpha^2$ $x^2 = \alpha^1 \sin \alpha^2$ $x^3 = \alpha^3$	$x^1 = \alpha^1 \sin \alpha^2 \cos \alpha^3$ $x^2 = \alpha^1 \sin \alpha^2 \sin \alpha^3$ $x^3 = \alpha^1 \cos \alpha^2$
$\vec{e}_1(1, 0, 0)$ $\vec{e}_2(0, 1, 0)$ $\vec{e}_3(0, 0, 1)$	$\vec{e}_1\{\cos \alpha^2, \sin \alpha^2, 0\}$ $\vec{e}_2\{-\alpha^1 \sin \alpha^2, \alpha^1 \cos \alpha^2, 0\}$ $\vec{e}_3\{0, 0, 1\}$	$\vec{e}_1\{\sin \alpha^2 \cos \alpha^3, \sin \alpha^2 \times$ $\times \sin \alpha^3, \cos \alpha^2\}$ $\vec{e}_2\{\alpha^1 \cos \alpha^2 \cos \alpha^3,$ $\alpha^1 \cos \alpha^2 \sin \alpha^3, -\alpha^1 \sin \alpha^2\}$ $\vec{e}_3\{-\alpha^1 \sin \alpha^2 \sin \alpha^3,$ $\alpha^1 \sin \alpha^2 \cos \alpha^2, 0\}$
$ds^2 = (d\alpha^1)^2 +$ $+ (d\alpha^2)^2 +$ $+ (d\alpha^3)^2$	$ds^2 = (d\alpha^1)^2 +$ $+ (\alpha^1 d\alpha^2)^2 +$ $+ (d\alpha^3)^2$	$ds^2 = (d\alpha^1)^2 + (\alpha^1 d\alpha^2)^2 +$ $+ (\alpha^1 \sin \alpha^2 d\alpha^3)^2$
$g_{11} = g_{22} =$ $= g_{33} = 1$	$g_{11} = 1, g_{22} = (\alpha^1)^2,$ $g_{33} = 1$	$g_{11} = 1, g_{22} = (\alpha^1)^2,$ $g_{33} = (\alpha^1 \sin \alpha^2)^2$
$g = 1$	$g = (\alpha^1)^2$	$g = (\alpha^1)^4 \sin^2 \alpha^2$
$g^{11} = g^{22} =$ $= g^{33} = 1$	$g^{11} = 1, g^{22} = \frac{1}{(\alpha^1)^2},$ $g^{33} = 1$	$g^{11} = 1, g^{22} = \frac{1}{(\alpha^1)^2},$ $g^{33} = \frac{1}{(\alpha^1 \sin \alpha^2)^2}$

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		произвольная	ортогональная криволинейная
7	Координаты взаимного базиса $\vec{e}^i$	$\vec{e}^i = g^{ij} \vec{e}_j$	$\vec{e}^1 = \frac{\vec{e}_1}{H_1^2}$ $\vec{e}^2 = \frac{\vec{e}_2}{H_2^2}$ $\vec{e}^3 = \frac{\vec{e}_3}{H_3^2}$
8	Координаты ортонормированного базиса $\vec{k}_i$	—	$\vec{k}_1 = \frac{\vec{e}_1}{H_1}$ $\vec{k}_2 = \frac{\vec{e}_2}{H_2}$ $\vec{k}_3 = \frac{\vec{e}_3}{H_3}$
9	Физические компоненты вектора $\vec{a}$	$a^i_{(\Phi)} \equiv A^i \equiv A_i = \vec{a} \cdot \vec{k}_i$	$A^\beta = \vec{a} \cdot \vec{k}_\beta = \frac{a_\beta}{H_\beta} =$ $= a^\beta H_\beta$ $\langle \beta = 1, 2, 3 \rangle$
10	Символы Кристоффеля 1-го рода*	$\Gamma_{ij,k} = \frac{1}{2} \left( \frac{\partial g_{ik}}{\partial \alpha^j} + \frac{\partial g_{jk}}{\partial \alpha^i} - \frac{\partial g_{ij}}{\partial \alpha^k} \right)$	$\Gamma_{\beta\beta,\beta} = H_\beta \frac{\partial H_\beta}{\partial \alpha^\beta},$ $\Gamma_{\beta\gamma,\beta} = -\Gamma_{\beta\beta,\gamma} =$ $= H_\beta \frac{\partial H_\beta}{\partial \alpha^\gamma}$ $\langle \beta \neq \gamma; \beta = \gamma = 1, 2, 3 \rangle$

\* Остальные компоненты равны нулю.

координат

прямоугольная	цилиндрическая	сферическая
$\vec{e}^1 \{1, 0, 0\}$ $\vec{e}^2 \{0, 1, 0\}$ $\vec{e}^3 \{0, 0, 1\}$	$\vec{e}^1 \{\cos \alpha^2, \sin \alpha^2, 0\}$ $\vec{e}^2 \left\{ -\frac{\sin \alpha^2}{\alpha^1}, \frac{\cos \alpha^2}{\alpha^1}, 0 \right\}$ $\vec{e}^3 \{0, 0, 1\}$	$\vec{e}^1 \{ \sin \alpha^2 \cos \alpha^3, \sin \alpha^2 \times$ $\times \sin \alpha^3, \cos \alpha^2 \}$ $\vec{e}^2 \left\{ \frac{\cos \alpha^2 \cos \alpha^3}{\alpha^1}, \right.$ $\left. \frac{\cos \alpha^2 \sin \alpha^3}{\alpha^1}, -\frac{\sin \alpha^2}{\alpha^1} \right\}$ $\vec{e}^3 \left\{ -\frac{\sin \alpha^3}{\alpha^1 \sin \alpha^2}, \right.$ $\left. \frac{\cos \alpha^3}{\alpha^1 \sin \alpha^2}, 0 \right\}$
$\vec{k}_1 \{1, 0, 0\}$ $\vec{k}_2 \{0, 1, 0\}$ $\vec{k}_3 \{0, 0, 1\}$	$\vec{k}_1 \{\cos \alpha^2, \sin \alpha^2, 0\}$ $\vec{k}_2 \{-\sin \alpha^2, \cos \alpha^2, 0\}$ $\vec{k}_3 \{0, 0, 1\}$	$\vec{k}_1 \{ \sin \alpha^2 \cos \alpha^3, \sin \alpha^2 \times$ $\times \sin \alpha^3, \cos \alpha^2 \}$ $\vec{k}_2 \{ \cos \alpha^2, \cos \alpha^3, \cos \alpha^2 \times$ $\times \sin \alpha^3, -\sin \alpha^2 \}$ $\vec{k}_3 \{ -\sin \alpha^3, \cos \alpha^3, 0 \}$
$A^1 = a^1$ $A^2 = a^2$ $A^3 = a^3$	$A^1 = \vec{a} \cdot \vec{k}_1 = a^1$ $A^2 = \vec{a} \cdot \vec{k}_2 = a^2 \alpha^1$ $A^3 = \vec{a} \cdot \vec{k}_3 = a^3$	$A^1 = \vec{a} \cdot \vec{k}_1 = a^1$ $A^2 = \vec{a} \cdot \vec{k}_2 = a^2 \alpha^1$ $A^3 = \vec{a} \cdot \vec{k}_3 = a^3 \alpha^1 \sin \alpha^2$
	$\Gamma_{12,2} = -\Gamma_{22,1} = \alpha^1$	$\Gamma_{21,2} = -\Gamma_{22,1} = \alpha^1$ $\Gamma_{31,3} = -\Gamma_{33,1} =$ $= \alpha^1 \sin^2 \alpha^2$ $\Gamma_{33,3} = -\Gamma_{33,2} =$ $= (\alpha^1)^2 \sin \alpha^2 \cos \alpha^3$

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		Произвольная	ортогональная криволинейная
11	Символы Кристоффеля 2-го рода*	$\Gamma_{ij}^k = g^{kl}\Gamma_{ij,l}$	$\Gamma_{\beta\beta}^\beta = \frac{1}{H_\beta} = \frac{\partial H_\beta}{\partial \alpha^\beta}$ $\Gamma_{\beta\gamma}^\gamma = -\frac{H_\beta}{H_\gamma^2} \frac{\partial H_\beta}{\partial \alpha^\gamma}$ $\Gamma_{\beta\gamma}^\beta = \frac{1}{H_\beta} \frac{\partial H_\beta}{\partial \alpha^\gamma}$ $\langle \beta = \gamma; \gamma = 1, 2, 3 \rangle$
12	Элемент объема	$dV = \sqrt{g} d\alpha^1 d\alpha^2 d\alpha^3$	$dV = H_1 H_2 H_3 d\alpha^1 d\alpha^2 d\alpha^3$
13	Градиент скаляра $\varphi$	$\text{grad } \varphi = \nabla_i \varphi \vec{e}^i$	$\frac{\partial \varphi}{\partial \alpha^1} \frac{\vec{k}_1}{H_1} +$ $+ \frac{\partial \varphi}{\partial \alpha^2} \frac{\vec{k}_2}{H_2} +$ $+ \frac{\partial \varphi}{\partial \alpha^3} \frac{\vec{k}_3}{H_3}$
14	Дивергенция вектора $\vec{a}$	$\text{div } \vec{a} = \nabla_i a^i$	$\left\{ \frac{\partial}{\partial \alpha^1} (H_2 H_3 A^1) + \right.$ $+ \frac{\partial}{\partial \alpha^2} (H_1 H_3 A^2) +$ $\left. + \frac{\partial}{\partial \alpha^3} (H_1 H_2 A^3) \right\} / H_1 H_2 H_3$

\* Остальные компоненты равны нулю

координат		
прямоугольная	цилиндрическая	сферическая
	$\Gamma_{22}^1 = -\alpha^1$ $\Gamma_{21}^2 = \frac{1}{\alpha^1}$	$\Gamma_{22}^1 = -\alpha^1$ $\Gamma_{33}^1 = -\alpha^1 \sin^2 \alpha^2$ $\Gamma_{33}^2 = -\sin \alpha^2 \cos \alpha^2$ $\Gamma_{21}^2 = \frac{1}{\alpha^1}$ $\Gamma_{31}^3 = \frac{1}{\alpha^1}$ $\Gamma_{32}^3 = \operatorname{ctg} \alpha^2$
$dV = dx^1 dx^2 dx^3$	$dV = \alpha^1 da^1 da^2 da^3$	$dV = (\alpha^1)^2 \sin^2 \alpha^2 da^1 da^2 da^3$
$\frac{\partial \varphi}{\partial x^1} \vec{k}_1 +$ $+ \frac{\partial \varphi}{\partial x^2} \vec{k}_2 +$ $+ \frac{\partial \varphi}{\partial x^3} \vec{k}_3$	$\frac{\partial \varphi}{\partial \alpha^1} \vec{k}_1 + \frac{1}{\alpha^1} \frac{\partial \varphi}{\partial \alpha^2} \vec{k}_2 +$ $+ \frac{\partial \varphi}{\partial \alpha^3} \vec{k}_3$	$\frac{\partial \varphi}{\partial \alpha^1} \vec{k}_1 + \frac{1}{\alpha^1} \frac{\partial \varphi}{\partial \alpha^2} \vec{k}_2 -$ $- \frac{1}{\alpha^1 \sin \alpha^2} \frac{\partial \varphi}{\partial \alpha^3} \vec{k}_3$
$\frac{\partial A^1}{\partial x^1} +$ $+ \frac{\partial A^2}{\partial x^2} +$ $+ \frac{\partial A^3}{\partial x^3}$	$\frac{1}{\alpha^1} \left\{ \frac{\partial}{\partial \alpha^1} (\alpha^1 A^1) + \right.$ $+ \frac{\partial A^2}{\partial \alpha^2} +$ $\left. + \frac{\partial}{\partial \alpha^3} \left( \alpha^1 \frac{\partial A^3}{\partial \alpha^3} \right) \right\}$	$\frac{1}{(\alpha^1)^2 \sin^2 \alpha^2} \left\{ \frac{\partial}{\partial \alpha^1} [(\alpha^1)^2 \times \right.$ $\times \sin \alpha^2 A^1] +$ $+ \frac{\partial}{\partial \alpha^2} (\alpha^1 \sin \alpha^2 A^2) +$ $\left. + \frac{\partial}{\partial \alpha^3} (\alpha^1 A^3) \right\}$

№ п/п	Название формул	Системы	
		произвольная	ортогональная криволинейная
15	Ротор вектора $\vec{a}$	$\operatorname{rot} \vec{a} =$ $= -\frac{1}{\sqrt{g}} \epsilon^{ijk} \nabla_i a_j \vec{e}_k$	$\frac{1}{H_1 H_2 H_3} \begin{vmatrix} H_1 \vec{k}_1 H_2 \vec{k}_2 H_3 \vec{k}_3 \\ \frac{\partial}{\partial \alpha^1} & \frac{\partial}{\partial \alpha^2} & \frac{\partial}{\partial \alpha^3} \\ H_1 A^1 & H_2 A^2 & H_3 A^3 \end{vmatrix}$
16	Лапласиан скаляра $\varphi$	$\Delta \varphi = g^{ij} \Delta_i \Delta_j \varphi$	$\frac{1}{H_1 H_2 H_3} \times$ $\left\langle \left\{ \frac{\partial}{\partial \alpha^1} \left( \frac{H_2 H_3}{H_1} \frac{\partial \varphi}{\partial \alpha^1} \right) + \right. \right.$ $\left. + \frac{\partial}{\partial \alpha^2} \left( \frac{H_1 H_3}{H_2} \frac{\partial \varphi}{\partial \alpha^2} \right) + \right.$ $\left. + \frac{\partial}{\partial \alpha^3} \left( \frac{H_1 H_2}{H_3} \frac{\partial \varphi}{\partial \alpha^3} \right) \right\rangle$
17	Лапласиан вектора $\vec{a}$	$\Delta^* \vec{a} = g^{ij} \nabla_i \nabla_j a_k \vec{e}^k$	

<sup>1</sup> Символ  $\Delta^*$  обозначает оператор Лапласа вектора и тензора,

координат

прямоугольная	цилиндрическая	сферическая
$\begin{array}{c} \vec{k}_1 \vec{k}_2 \vec{k}_3 \\ \frac{\partial}{\partial x^1} \frac{\partial}{\partial x^2} \frac{\partial}{\partial x^3} \\ A^1 \quad A^2 \quad A^3 \end{array}$	$\frac{1}{\alpha^1} \begin{array}{c} \vec{k}_1 \alpha^1 \vec{k}_2 \vec{k}_3 \\ \frac{\partial}{\partial \alpha^1} \frac{\partial}{\partial \alpha^2} \frac{\partial}{\partial \alpha^3} \\ A^1 \alpha^1 A^2 A^3 \end{array}$	$\frac{1}{(\alpha^1)^2 \sin \alpha^2} \times \begin{array}{c} \vec{k}_1 \alpha^1 \vec{k}_2 \alpha^1 \sin \alpha^2 \vec{k}_3 \\ \frac{\partial}{\partial \alpha^1} \frac{\partial}{\partial \alpha^2} \frac{\partial}{\partial \alpha^3} \\ A^1 \alpha^1 A^2 \alpha^1 \sin \alpha^2 A^3 \end{array}$
$\begin{array}{l} \frac{\partial^2 \varphi}{\partial (x^1)^2} + \\ + \frac{\partial^2 \varphi}{\partial (x^2)^2} + \\ + \frac{\partial^2 \varphi}{\partial (x^3)^2} \end{array}$	$\frac{1}{\alpha^1} \left\{ \frac{1}{\partial \alpha^1} \left( \alpha^1 \frac{\partial \varphi}{\partial \alpha^1} \right) + \frac{\partial}{\partial \alpha^2} \left( \frac{1}{\alpha^1} \frac{\partial \varphi}{\partial \alpha^2} \right) + \frac{\partial}{\partial \alpha^3} \left( \alpha^1 \frac{\partial \varphi}{\partial \alpha^2} \right) \right\}$	$\begin{array}{l} \frac{1}{(\alpha^1)^2 \sin \alpha^2} \times \\ \times \left\{ \frac{\partial}{\partial \alpha^1} \left[ (\alpha^1)^2 \times \right. \right. \\ \times \sin \alpha^2 \frac{\partial \varphi}{\partial \alpha^1} \left. \right] + \\ + \frac{\partial}{\partial \alpha^2} \left( \sin \alpha^2 \frac{\partial \varphi}{\partial \alpha^2} \right) + \\ + \frac{\partial}{\partial \alpha^3} \left( \frac{1}{\sin \alpha^2} \frac{\partial \varphi}{\partial \alpha^2} \right) \left. \right\} \end{array}$
$\Delta^* \vec{a} = \Delta \vec{a}$	$\begin{array}{l} \Delta a_1 - \frac{2}{(\alpha^1)^3} \frac{\partial a_2}{\partial \alpha^2} + \\ + \frac{a_2}{(\alpha^1)^4} - \frac{a_1}{(\alpha^1)^3} \\ \Delta a_2 - \frac{2}{\alpha^1} \frac{\partial a_2}{\partial \alpha^1} + \\ + \frac{2}{\alpha^1} \frac{\partial a_1}{\partial \alpha^2} + a_1 \\ \Delta a_3 \end{array}$	$\begin{array}{l} \Delta a_1 - \frac{2}{(\alpha^1)^3 \sin^2 \alpha^2} \frac{\partial a_2}{\partial \alpha^2} - \\ - \frac{2}{(\alpha^1)^3} \frac{\partial a_2}{\partial \alpha^2} - \frac{2a_1}{(\alpha^1)^2} - \\ - \frac{2a_2}{(\alpha^1)^2} \operatorname{ctg} \alpha^2 \\ \Delta a_2 + \frac{2}{(\alpha^1)^2 \sin^2 \alpha^2} \frac{\partial a_2}{\partial \alpha^2} - \\ - \frac{a_2}{(\alpha^1)^2 \sin^2 \alpha^2} \Delta a_3 + \frac{2}{\alpha^1} \times \\ \times \left( \frac{\partial a_1}{\partial \alpha^2} - \frac{\partial a_2}{\partial \alpha^1} \right) + \frac{2}{(\alpha^1)^2} \times \\ \times \operatorname{ctg} \alpha^2 \left( \frac{\partial a_2}{\partial \alpha^2} - \frac{\partial a_2}{\partial \alpha^2} \right) \end{array}$

а символ  $\Delta$  — оператор Лапласа скаляра.



№ п/п	Название формул	Системы	
		произвольная	цилиндрическая
18	Лапласиан тензора $\tilde{a}$ (симметричного)	$\Delta^* a = g^{il} \nabla_i \nabla_j a_{kl} e^k \otimes e^l$	$\Delta a_{11} - \frac{4}{(\alpha^1)^3} \frac{\partial a_{11}}{\partial \alpha^2} -$ $- \frac{2}{(\alpha^1)^2} a_{11} + \frac{2}{(\alpha^1)^4} a_{22}$ $\Delta a_{22} - \frac{4}{\alpha^1} \frac{\partial a_{22}}{\partial \alpha^1} +$ $+ \frac{4}{\alpha^1} \frac{\partial a_{12}}{\partial \alpha^2} +$ $+ \frac{2}{(\alpha^1)^2} a_{22} + 2a_{11}$
		прямоугольная	$\Delta a_{33}$ $\Delta a_{12} - \frac{2}{\alpha^1} \frac{\partial a_{12}}{\partial \alpha^1} -$ $- \frac{2}{(\alpha^1)^3} \frac{\partial a_{22}}{\partial \alpha^2} +$ $+ \frac{2}{\alpha^1} \frac{\partial a_{11}}{\partial \alpha^2} - \frac{5}{(\alpha^1)^2} a_{12}$
		$\Delta^* \tilde{a} = \Delta \tilde{a}$	$\Delta a_{13} - \frac{2}{(\alpha^1)^3} \frac{\partial a_{23}}{\partial \alpha^2} -$ $- \frac{1}{(\alpha^1)^2} a_{13}$ $\Delta a_{23} - \frac{2}{\alpha^1} \frac{\partial a_{23}}{\partial \alpha^1} +$ $+ \frac{2}{\alpha^1} \frac{\partial a_{13}}{\partial \alpha^2}$

координат

Г сферическая

$\Delta a_{11} - \frac{4}{(\alpha^1)^2} \frac{\partial a_{11}}{\partial \alpha^2} -$ $\frac{4}{(\alpha^1)^2 \sin^2 \alpha^2} \frac{\partial a_{13}}{\partial \alpha^3} -$ $\frac{4}{(\alpha^1)^2} a_{11} - \frac{4}{(\alpha^1)^3} \times$ $\text{ctg } \alpha^2 a_{12} + \frac{2}{(\alpha^1)^4} a_{22} +$ $\frac{2}{(\alpha^1)^4 \sin^2 \alpha^2} a_{33}$	$\Delta a_{12} - \frac{2}{\alpha^1} \frac{\partial a_{12}}{\partial \alpha^1} -$ $\frac{2}{(\alpha^1)^3} \frac{\partial a_{22}}{\partial \alpha^2} -$ $\frac{2}{(\alpha^1)^2 \sin^2 \alpha^2} \frac{\partial a_{23}}{\partial \alpha^3} +$ $+ \frac{2}{\alpha_2} \frac{\partial a_{11}}{\partial \alpha^2} -$ $\frac{2 \text{ctg } \alpha^2}{(\alpha^1)^2 \sin^2 \alpha^2} \frac{\partial a_{13}}{\partial \alpha^3} -$ $- \frac{5}{(\alpha^1)^2} a_{12} -$ $- \frac{2}{(\alpha^1)^2} \text{ctg } \alpha^2 a_{22} +$ $+ \frac{2 \text{ctg } \alpha^2}{(\alpha^1)^3 \sin^2 \alpha^2} a_{33} -$ $- \frac{\text{ctg}^2 \alpha^2}{(\alpha^1)^2} a_{12}$	$- \frac{2 \text{ctg } \alpha^2}{(\alpha^1)^3} a_{23} -$ $- \frac{\text{ctg}^2 \alpha^2}{(\alpha^1)^2} a_{13}$ $\Delta a_{23} - \frac{4}{\alpha^1} \frac{\partial a_{23}}{\partial \alpha^1} -$ $- \frac{2 \text{ctg } \alpha^2}{(\alpha^1)^2} \frac{\partial a_{23}}{\partial \alpha^2} +$ $+ \frac{2}{\alpha^1} \frac{\partial a_{13}}{\partial \alpha^2} +$ $+ \frac{2}{\alpha^1} \frac{\partial a_{12}}{\partial \alpha^3} -$ $- \frac{2 \text{ctg } \alpha^2}{(\alpha^1)^2 \sin^2 \alpha^2} \frac{\partial a_{33}}{\partial \alpha^3} +$ $+ \frac{2 \text{ctg } \alpha^2}{(\alpha^1)^2} \frac{\partial a_{22}}{\partial \alpha^2} +$ $+ \frac{1}{(\alpha^1)^2 \sin^2 \alpha^2} a_{23} -$ $- \frac{3 \text{ctg } \alpha^2}{\alpha^1} a_{13} -$ $- \frac{3 \text{ctg}^2 \alpha^2}{(\alpha^1)^2} a_{33}$
$\Delta a_{22} - \frac{4}{\alpha^1} \frac{\partial a_{22}}{\partial \alpha^1} +$ $+ \frac{4}{\alpha^1} \frac{\partial a_{12}}{\partial \alpha^2} -$ $\frac{4 \text{ctg } \alpha^2}{(\alpha^1)^2 \sin^2 \alpha^2} \frac{\partial a_{23}}{\partial \alpha^3} -$ $- \text{ctg}^2 \alpha^2 a_{22} +$ $\frac{2 \text{ctg}^2 \alpha^2}{(\alpha^1)^2 \sin^2 \alpha^2} a_{33} + 2a_{11}$ $\Delta a_{33} - \frac{4}{\alpha^1} \frac{\partial a_{33}}{\partial \alpha^1} -$ $\frac{4}{(\alpha^1)^2} \text{ctg } \alpha^2 \frac{\partial a_{33}}{\partial \alpha^2} +$ $+ \frac{4}{(\alpha^1)^2} \frac{\partial a_{12}}{\partial \alpha^2} +$ $+ \frac{4 \text{ctg } \alpha^2}{(\alpha^1)^2} \frac{\partial a_{22}}{\partial \alpha^2} +$ $+ \frac{2}{(\alpha^1)^2 \sin^2 \alpha^2} a_{33} +$ $+ 2 \sin^2 \alpha^2 a_{11} +$ $+ \frac{2 \cos^2 \alpha^2}{(\alpha^1)^2} a_{22} +$ $+ \frac{4}{\alpha^1} \sin^2 \alpha^2 \cos^2 \alpha^2 a_{12}$	$\Delta a_{12} - \frac{2}{\alpha^1} \frac{\partial a_{12}}{\partial \alpha^1} -$ $\frac{2 \text{ctg } \alpha^2}{(\alpha^1)^2} \frac{\partial a_{13}}{\partial \alpha^2} -$ $\frac{2}{(\alpha^1)^3} \frac{\partial a_{22}}{\partial \alpha^2} -$ $\frac{2}{(\alpha^1)^2 \sin^2 \alpha^2} \frac{\partial a_{32}}{\partial \alpha^3} +$ $+ \frac{2 \text{ctg } \alpha^2}{(\alpha^1)^2} \frac{\partial a_{12}}{\partial \alpha^2} -$ $- \frac{5}{(\alpha^1)^2} a_{13} +$ $+ \frac{2}{\alpha^1} \frac{\partial a_{11}}{\partial \alpha^2} +$ $+ \frac{1}{(\alpha^1)^2 \sin^2 \alpha^2} a_{13} -$	