ПРИНЯТЫЕ ОБОЗНАЧЕНИЯ

x, t, u, s — переменные интегрирования.

a, b, c, f, α, β, γ, p, q, A, B — произвольные вещественные числа.

k, l, m, n, r — целые числа.

ν — индекс суммирования.

$F(x, t)$ — произвольная функция от x и t.

$R(x, t)$ — рациональная функция своих аргументов.

$P_n(x)$ — многочлен n-ой степени от x.

$P_n^{(k)}(x)$ — k-ая производная многочлена $P_n(x)$.

$C_m^n = \dfrac{m!}{n!\,(m-n)!}$ — число сочетаний из m элементов по n (биномиальные коэффициенты); по определению полагаем:

$$C_m^0 = C_m^m = C_0^0 = 1.$$

$n! = n \cdot (n-1) \ldots 2 \cdot 1$ — факториал; по определению полагаем: $0! = 1$.

B_n — числа Бернулли:

n	0	1	2	3	4	5	6	7	8
B_n	1	$-\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{30}$	0	$\frac{1}{42}$	0	$-\frac{1}{30}$

E_n — числа Эйлера:

n	0	1	2	3	4	5	6	7	8
E_n	1	0	-1	0	5	0	-61	0	1385

$\Delta = \begin{vmatrix} a & b \\ c & f \end{vmatrix} = af - bc$ — определитель второго порядка.

$\delta = b^2 - 4ac$ — дискриминант квадратного трехчлена $ax^2 + bx + c$.

$\xi = \sqrt[3]{\dfrac{a}{b}}$. $\quad \eta = \sqrt[4]{\left|\dfrac{a}{b}\right|}$.

ТАБЛИЦА 1

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n\,dx}{(a+bx)^m}; \quad n=0,\ 1,\ 2,\ \ldots, \quad m=1,\ 2,\ 3,\ \ldots$$

1.1. $$\int \frac{dx}{a+bx} = \frac{1}{b}\ln|a+bx|.$$

1.2. $$\int \frac{dx}{(a+bx)^m} = \frac{-1}{(m-1)\,b\,(a+bx)^{m-1}} \qquad (m \geqslant 2).$$

1.3. $$\int \frac{x\,dx}{a+bx} = \frac{1}{b}\left(x - \frac{a}{b}\ln|a+bx|\right).$$

1.4. $$\int \frac{x\,dx}{(a+bx)^2} = \frac{1}{b^2}\left(\frac{a}{a+bx} + \ln|a+bx|\right).$$

1.5. $$\int \frac{x\,dx}{(a+bx)^m} = \frac{1}{b^2}\left[\frac{-1}{(m-2)\,(a+bx)^{m-2}} + \frac{a}{(m-1)\,(a+bx)^{m-1}}\right] \qquad (m \geqslant 3).$$

1.6. $$\int \frac{x^2\,dx}{a+bx} = \frac{1}{b}\left[\frac{x^2}{2} - \frac{a}{b}x + \left(\frac{a}{b}\right)^2 \ln|a+bx|\right].$$

1.7. $$\int \frac{x^2\,dx}{(a+bx)^2} = \frac{1}{b^2}\left[x - \frac{a}{b}\left(\frac{a}{a+bx} + 2\ln|a+bx|\right)\right].$$

1.8. $$\int \frac{x^2\,dx}{(a+bx)^3} = \frac{1}{b^3}\left[\frac{2a}{a+bx} - \frac{a^2}{2\,(a+bx)^2} + \ln|a+bx|\right].$$

1.9. $$\int \frac{x^2\,dx}{(a+bx)^m} = \frac{1}{b^3}\left[\frac{-1}{(m-3)\,(a+bx)^{m-3}} + \frac{2a}{(m-2)\,(a+bx)^{m-2}} - \frac{a^2}{(m-1)\,(a+bx)^{m-1}}\right] \qquad (m \geqslant 4)$$

1.10. $$\int \frac{x^3\,dx}{a+bx} = \frac{1}{b}\left[\frac{x^3}{3} - \frac{a}{b}\frac{x^2}{2} + \left(\frac{a}{b}\right)^2 x - \left(\frac{a}{b}\right)^3 \ln|a+bx|\right].$$

1.11. $$\int \frac{x^3\,dx}{(a+bx)^2} = \frac{1}{b^2}\left[\frac{x^2}{2} - 2\frac{a}{b}x + \left(\frac{a}{b}\right)^2\left(\frac{a}{a+bx} + 3\ln|a+bx|\right)\right].$$

1.12. $$\int \frac{x^3\,dx}{(a+bx)^3} = \frac{1}{b^3}\left\{x - \frac{a}{b}\left[\frac{3a}{a+bx} - \frac{a^2}{2\,(a+bx)^2} + 3\ln|a+bx|\right]\right\}.$$

1.13. $$\int \frac{x^3\,dx}{(a+bx)^4} = \frac{1}{b^4}\left[\frac{3a}{a+bx} - \frac{3a^2}{2\,(a+bx)^2} + \frac{a^3}{3\,(a+bx)^3} + \ln|a+bx|\right].$$

1.14. $$\int \frac{x^3\,dx}{(a+bx)^m} = \frac{1}{b^4}\left[\frac{-1}{(m-4)\,(a+bx)^{m-4}} + \frac{3a}{(m-3)\,(a+bx)^{m-3}} - \frac{3a^2}{(m-2)\,(a+bx)^{m-2}} + \frac{a^3}{(m-1)\,(a+bx)^{m-1}}\right] \qquad (m \geqslant 5).$$

1.15. $$\int \frac{x^4\,dx}{a+bx} = \frac{1}{b}\left[\frac{x^4}{4} - \frac{a}{b}\frac{x^3}{3} + \left(\frac{a}{b}\right)^2\frac{x^2}{2} - \left(\frac{a}{b}\right)^3 x + \left(\frac{a}{b}\right)^4 \ln|a+bx|\right].$$

1.16. $$\int \frac{x^4\,dx}{(a+bx)^2} = \frac{1}{b^2}\left[\frac{x^3}{3} - \frac{a}{b}x^2 + 3\left(\frac{a}{b}\right)^2 x - \left(\frac{a}{b}\right)^3\left(\frac{a}{a+bx} + 4\ln|a+bx|\right)\right].$$

1.17. $$\int \frac{x^4\,dx}{(a+bx)^3} = \frac{1}{b^3}\left\{\frac{x^2}{2} - 3\frac{a}{b}x + \left(\frac{a}{b}\right)^2\left[\frac{4a}{a+bx} - \frac{a^2}{2\,(a+bx)^2} + 6\ln|a+bx|\right]\right\}.$$

1.18. $$\int \frac{x^4\,dx}{(a+bx)^4} = \frac{1}{b^4}\left\{x - \frac{a}{b}\left[\frac{6a}{a+bx} - \frac{2a^2}{(a+bx)^2} + \frac{a^3}{3\,(a+bx)^3} + 4\ln|a+bx|\right]\right\}.$$

1.19. $$\int \frac{x^4\,dx}{(a+bx)^5} = \frac{1}{b^5}\left[\frac{4a}{a+bx} - \frac{3a^2}{(a+bx)^2} + \frac{4a^3}{3\,(a+bx)^3} - \frac{a^4}{4\,(a+bx)^4} + \ln|a+bx|\right].$$

1.20. $$\int \frac{x^n\,dx}{a+bx} = \sum_{\nu=0}^{n-1} \frac{(-1)^\nu a^\nu x^{n-\nu}}{(n-\nu)\,b^{\nu+1}} + \frac{(-a)^n}{b^{n+1}}\ln|a+bx| \qquad (n \geqslant 1).$$

1.21. $$\int \frac{x^n\,dx}{(a+bx)^m} = -\frac{x^n}{(m-1)\,b\,(a+bx)^{m-1}} + \frac{n}{(m-1)\,b}\int \frac{x^{n-1}\,dx}{(a+bx)^{m-1}} \qquad (m \geqslant 2).$$

ТАБЛИЦА 2

ИНТЕГРАЛЫ ВИДА

$$\int \frac{dx}{x^n (a+bx)^m} \qquad \begin{matrix} n=1,\ 2,\ 3,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

2.1. $$\int \frac{dx}{x(a+bx)} = -\frac{1}{a} \ln \left| \frac{a}{x} + b \right|.$$

2.2. $$\int \frac{dx}{x(a+bx)^2} = \frac{1}{a} \left(\frac{1}{a+bx} - \frac{1}{a} \ln \left| \frac{a}{x} + b \right| \right).$$

2.3. $$\int \frac{dx}{x(a+bx)^3} = \frac{1}{a} \left[\frac{1}{2(a+bx)^2} + \frac{1}{a(a+bx)} - \frac{1}{a^2} \ln \left| \frac{a}{x} + b \right| \right].$$

2.4. $$\int \frac{dx}{x(a+bx)^4} = \frac{1}{a} \left[\frac{1}{3(a+bx)^3} + \frac{1}{2a(a+bx)^2} + \frac{1}{a^2(a+bx)} - \frac{1}{a^3} \ln \left| \frac{a}{x} + b \right| \right].$$

2.5. $$\int \frac{dx}{x(a+bx)^5} = \frac{1}{a} \left[\frac{1}{4(a+bx)^4} + \frac{1}{3a(a+bx)^3} + \frac{1}{2a^2(a+bx)^2} + \frac{1}{a^3(a+bx)} - \frac{1}{a^4} \ln \left| \frac{a}{x} + b \right| \right].$$

2.6. $$\int \frac{dx}{x(a+bx)^m} = \sum_{\nu=1}^{m-1} \frac{1}{\nu a^{m-\nu}(a+bx)^\nu} - \frac{1}{a^m} \ln \left| \frac{a+bx}{x} \right| \quad (m \geqslant 2).$$

2.7. $$\int \frac{dx}{x^2(a+bx)} = -\frac{1}{a} \left(\frac{1}{x} - \frac{b}{a} \ln \left| \frac{a}{x} + b \right| \right).$$

2.8. $$\int \frac{dx}{x^2(a+bx)^2} = -\frac{1}{a^2} \left(\frac{b}{a+bx} + \frac{1}{x} - \frac{2b}{a} \ln \left| \frac{a}{x} + b \right| \right).$$

2.9. $$\int \frac{dx}{x^2(a+bx)^3} = -\frac{1}{a^2} \left[\frac{b}{2(a+bx)^2} + \frac{2b}{a(a+bx)} + \frac{1}{ax} - \frac{3b}{a^2} \ln \left| \frac{a}{x} + b \right| \right].$$

2.10. $$\int \frac{dx}{x^2(a+bx)^4} = -\frac{1}{a^2} \left[\frac{b}{3(a+bx)^3} + \frac{b}{a(a+bx)^2} + \frac{3ab}{a^2(a+bx)} + \frac{1}{a^2 x} - \frac{4b}{a^3} \ln \left| \frac{a}{x} + b \right| \right].$$

2.11. $$\int \frac{dx}{x^3(a+bx)^m} = \frac{-1}{ax(a+bx)^{m-1}} - \frac{mb}{a} \int \frac{dx}{x(a+bx)^m} \quad \text{(см. 2.6)}.$$

2.12. $$\int \frac{dx}{x^3(a+bx)} = \frac{1}{a} \left(\frac{b}{ax} - \frac{1}{2x^2} - \frac{b^2}{a^2} \ln \left| \frac{a}{x} + b \right| \right).$$

2.13. $$\int \frac{dx}{x^3(a+bx)^2} = \frac{1}{a^2} \left[\frac{b^2}{a(a+bx)} + \frac{2b}{ax} - \frac{1}{2x^2} - \frac{3b^2}{a^3} \ln \left| \frac{a}{x} + b \right| \right].$$

2.14. $$\int \frac{dx}{x^3(a+bx)^3} = \frac{1}{a^3} \left[\frac{3b^2}{a(a+bx)} + \frac{b^2}{2(a+bx)^2} + \frac{3b}{ax} - \frac{1}{2x^2} - \frac{6b^2}{a^3} \ln \left| \frac{a}{x} + b \right| \right].$$

2.15. $$\int \frac{dx}{x^3(a+bx)^m} = \frac{(m+1)\,bx - a}{2a^2x^2(a+bx)^{m-1}} + \frac{m(m+1)\,b^2}{2a^2} \int \frac{dx}{x(a+bx)^m} \quad \text{(см. 2.6)}.$$

2.16. $$\int \frac{dx}{x^4(a+bx)} = -\frac{1}{a} \left(\frac{b^2}{a^2x} - \frac{b}{2ax^2} + \frac{1}{3x^3} - \frac{b^3}{a^3} \ln \left| \frac{a}{x} + b \right| \right).$$

2.17. $$\int \frac{dx}{x^4(a+bx)^2} = -\frac{1}{a^3} \left[\frac{b^2}{a^2(a+bx)} + \frac{3b^2}{a^2x} - \frac{b}{ax^2} + \frac{1}{3x^3} - \frac{4b^3}{a^3} \ln \left| \frac{a}{x} + b \right| \right].$$

2.18. $$\int \frac{dx}{x^4(a+bx)^m} = \frac{(m+2)\,abx - [m(m+3)+2]\,b^2x^2 - 2a^2}{6a^3x^3(a+bx)^{m-1}} - \frac{m(m+1)(m+2)\,b^3}{6a^3} \int \frac{dx}{x(a+bx)^m} \quad \text{(см. 2.6)}.$$

2.19. $$\int \frac{dx}{x^n(a+bx)} = \sum_{\nu=1}^{n-1} \frac{(-1)^\nu b^{\nu-1}}{(n-\nu)\,a^\nu x^{n-\nu}} + \frac{(-1)^{n-1} b^{n-1}}{a^n} \ln \left| \frac{x}{a+bx} \right| \quad (n \geqslant 2).$$

2.20. $$\int \frac{dx}{x^n(a+bx)^m} = \frac{-1}{(n-1)\,ax^{n-1}(a+bx)^{m-1}} - \frac{(n+m-2)\,b}{(n-1)\,a} \int \frac{dx}{x^{n-1}(a+bx)^m} \quad (n \geqslant 2);$$

$$= \frac{1}{(m-1)\,ax^{n-1}(a+bx)^{m-1}} + \frac{n+m-2}{(m-1)\,a} \int \frac{dx}{x^n(a+bx)^{m-1}} \quad (m \geqslant 2).$$

ТАБЛИЦА 3

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\frac{(a+bx)^m}{(c+fx)^k}\,dx, \quad af\neq bc;$$

$n=0, 1, 2, \ldots, m=1, 2, 3, \ldots, k=1, 2, 3, \ldots$

3.1. $\int \frac{a+bx}{c+fx}\,dx=\frac{1}{f}\left(bx+\frac{\Delta}{f}\ln|c+fx|\right).$

3.2. $\int \frac{(a+bx)^2}{c+fx}\,dx=\frac{1}{f}\left[\frac{b^2}{2}x^2+b\left(a+\frac{\Delta}{f}\right)x+\frac{\Delta^2}{f^2}\ln|c+fx|\right].$

3.3. $\int \frac{(a+bx)^m}{c+fx}\,dx=$

$$=\sum_{\nu=1}^{m} C_m^\nu a^{m-\nu}b^\nu\int\frac{x^\nu\,dx}{c+fx}+\frac{a^m}{f}\ln|c+fx| \qquad \text{(см. 1.20).}$$

3.4. $\int \frac{a+bx}{(c+fx)^2}\,dx=-\frac{1}{f^2}\left(\frac{\Delta}{c+fx}-b\ln|c+fx|\right).$

3.5. $\int\left(\frac{a+bx}{c+fx}\right)^2 dx=\frac{1}{f^3}\left[b^2x-\frac{\Delta}{f}\left(\frac{\Delta}{c+fx}-2b\ln|c+fx|\right)\right].$

3.6. $\int\left(\frac{a+bx}{c+fx}\right)^m dx=\frac{1}{f^m}\sum_{\nu=0}^{m} C_m^\nu\Delta^{m-\nu}b^\nu\int\frac{dx}{(c+fx)^{m-\nu}}$ (см. 1.2).

3.7. $\int \frac{(a+bx)^m}{(c+fx)^k}\,dx=-\frac{(a+bx)^m}{(k-1)f(c+fx)^{k-1}}+$

$$+\frac{mb}{(k-1)f}\int\frac{(a+bx)^{m-1}}{(c+fx)^{k-1}}\,dx \quad (k\geqslant 2).$$

3.8. $\int x\frac{a+bx}{c+fx}\,dx=\frac{1}{f}\left[\frac{b}{2}x^2+\frac{\Delta}{f}\left(x-\frac{c}{f}\ln|c+fx|\right)\right].$

3.9. $\int x\frac{(a+bx)^2}{c+fx}\,dx=$

$$=\frac{1}{f}\left[\frac{b^2}{3}x^3+\frac{b}{2}\left(a+\frac{\Delta}{f}\right)x^2+\frac{\Delta^2}{f^2}\left(x-\frac{c}{f}\ln|c+fx|\right)\right].$$

3.10. $\int x\frac{a+bx}{(c+fx)^2}\,dx=\frac{1}{f^2}\left(bx+\frac{c\Delta}{f(c+fx)}+\left(\frac{2\Delta}{f}-a\right)\ln|c+fx|\right].$

3.11. $\int x\left(\frac{a+bx}{c+fx}\right)^2 dx=\frac{1}{f^2}\left\{\frac{b^2}{2}x^2+\frac{2b\Delta}{f}x+\right.$

$$\left.+\frac{\Delta}{f^2}\left[\frac{c\Delta}{c+fx}+(\Delta-2bc)\ln|c+fx|\right]\right\}.$$

3.12. $\int x\left(\frac{a+bx}{c+fx}\right)^m dx=\frac{(a+bx)^m}{2f^2(c+fx)^{m-2}}+\frac{c(a+bx)^m}{(m-1)f^2(c+fx)^{m-1}}-$

$$-\frac{m[(m-1)\Delta+2bc]}{2(m-1)f^2}\int\left(\frac{a+bx}{c+fx}\right)^{m-1}dx \quad (m\geqslant 2) \quad \text{(см. 3.6).}$$

3.13. $\int x^2\frac{a+bx}{c+fx}\,dx=\frac{1}{f}\left[\frac{b}{3}x^3+\frac{\Delta}{f}\left(\frac{x^2}{2}-\frac{c}{f}x+\frac{c^2}{f^2}\ln|c+fx|\right)\right].$

3.14. $\int x^2\frac{(a+bx)^2}{c+fx}\,dx=\frac{1}{f}\left[\frac{b^2}{4}x^4+\frac{b}{3}\left(a+\frac{\Delta}{f}\right)x^3+\right.$

$$\left.+\frac{\Delta^2}{f^2}\left(\frac{x^2}{2}-\frac{c}{f}x+\frac{c^2}{f^2}\ln|c+fx|\right)\right].$$

3.15. $\int x^2\frac{a+bx}{(c+fx)^2}\,dx=\frac{1}{f^2}\left[\frac{b}{2}x^2+\left(\frac{2\Delta}{f}-a\right)x-\right.$

$$\left.-\frac{c^2\Delta}{f^2(c+fx)}-\frac{c}{f}\left(\frac{3\Delta}{f}-a\right)\ln|c+fx|\right].$$

3.16. $\int x^2\left(\frac{a+bx}{c+fx}\right)^2 dx=\frac{1}{f^2}\left\{\frac{b^2}{3}x^3+\frac{b\Delta}{f}x^2+\frac{c\Delta^2x}{f^3(c+fx)}+\right.$

$$\left.+\frac{\Delta}{f^3}[(\Delta-2bc)(c+fx)-2c(\Delta-bc)\ln|c+fx|]\right\}.$$

3.17. $\int\frac{a+bx}{x(c+fx)}\,dx=\frac{a}{c}\ln|x|-\frac{\Delta}{cf}\ln|c+fx|.$

3.18. $\int\frac{(a+bx)^2}{x(c+fx)}\,dx=\frac{b^2}{f}x+\frac{1}{c}\left(a^2\ln|x|-\frac{\Delta^2}{f^2}\ln|c+fx|\right).$

3.19. $\int\frac{a+bx}{x(c+fx)^2}\,dx=\frac{\Delta}{cf(c+fx)}-\frac{a}{c^2}\ln\left|\frac{c}{x}+f\right|.$

3.20. $\int\frac{1}{x}\left(\frac{a+bx}{c+fx}\right)^2 dx=$

$$=\frac{\Delta^2}{cf^2(c+fx)}+\frac{a^2}{c^2}\ln|x|-\left[\left(\frac{a}{c}\right)^2-\left(\frac{b}{f}\right)^2\right]\ln|c+fx|.$$

ТАБЛИЦА 4

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\frac{dx}{(a+bx)^m(c+fx)^k},\quad af\neq bc;$$

$n=0,\ 1,\ 2,\ \ldots,\ m=1,\ 2,\ 3,\ \ldots,\ k=1,\ 2,\ 3,\ \ldots$

4.1. $$\int\frac{dx}{(a+bx)(c+fx)}=-\frac{1}{\Delta}\ln\left|\frac{a+bx}{c+fx}\right|.$$

4.2. $$\int\frac{dx}{(a+bx)(c+fx)^2}=\frac{-1}{\Delta(c+fx)}+\frac{b}{\Delta^2}\ln\left|\frac{a+bx}{c+fx}\right|.$$

4.3. $$\int\frac{dx}{(a+bx)^2(c+fx)^2}=-\frac{1}{\Delta^2}\left(\frac{b}{a+bx}+\frac{f}{c+fx}\right)+\frac{2bf}{\Delta^3}\ln\left|\frac{a+bx}{c+fx}\right|.$$

4.4. $$\int\frac{dx}{(a+bx)(c+fx)^3}=\frac{2b(c+fx)-\Delta}{2\Delta^2(c+fx)^2}-\frac{b^2}{\Delta^3}\ln\left|\frac{a+bx}{c+fx}\right|.$$

4.5. $$\int\frac{dx}{(a+bx)^2(c+fx)^3}=\frac{b^2}{\Delta^3(a+bx)}-\frac{f}{2\Delta^2(c+fx)^2}+$$
$$+\frac{2bf}{\Delta^3(c+fx)}-\frac{3b^2f}{\Delta^4}\ln\left|\frac{a+bx}{c+fx}\right|.$$

4.6. $$\int\frac{dx}{(a+bx)^3(c+fx)^3}=\frac{b^3}{2\Delta^3(a+bx)^2}+\frac{3b^2f}{\Delta^4(a+bx)}-$$
$$-\frac{f^2}{2\Delta^3(c+fx)^2}+\frac{3bf^2}{\Delta^4(c+fx)}-\frac{6b^2f^2}{\Delta^5}\ln\left|\frac{a+bx}{c+fx}\right|.$$

4.7. $$\int\frac{dx}{(a+bx)^m(c+fx)^k}=-\frac{1}{(k-1)\Delta(a+bx)^{m-1}(c+fx)^{k-1}}-$$
$$-\frac{(m+k-2)b}{(k-1)\Delta}\int\frac{dx}{(a+bx)^m(c+fx)^{k-1}}\qquad(k\geqslant 2);$$
$$=\frac{1}{(m-1)\Delta(a+bx)^{m-1}(c+fx)^{k-1}}+$$
$$+\frac{(m+k-2)f}{(m-1)\Delta}\int\frac{dx}{(a+bx)^{m-1}(c+fx)^k}\qquad(m\geqslant 2).$$

4.8. $$\int\frac{x\,dx}{(a+bx)(c+fx)}=\frac{1}{\Delta}\left[\frac{a}{b}\ln|a+bx|-\frac{c}{f}\ln|c+fx|\right].$$

4.9. $$\int\frac{x\,dx}{(a+bx)(c+fx)^2}=\frac{c}{f\Delta(c+fx)}-\frac{af}{b\Delta^2}\ln\left|\frac{a+bx}{c+fx}\right|$$

4.10. $$\int\frac{x\,dx}{(a+bx)^2(c+fx)^2}=$$
$$=\frac{1}{\Delta^2}\left[\frac{a}{a+bx}+\frac{c}{c+fx}\right]-\frac{af+bc}{\Delta^3}\ln\left|\frac{a+bx}{c+fx}\right|.$$

4.11. $$\int\frac{x^2\,dx}{(a+bx)(c+fx)}=\frac{x}{bf}+\frac{1}{\Delta}\left[\frac{c^2}{f^2}\ln|c+fx|-\frac{a^2}{b^2}\ln|a+bx|\right].$$

4.12. $$\int\frac{x^2\,dx}{(a+bx)(c+fx)^2}=$$
$$=\frac{-c^2}{f^2\Delta(c+fx)}+\frac{a^2}{b\Delta^2}\ln|a+bx|+\frac{bc^2-2acf}{f^2\Delta^2}\ln|c+fx|.$$

4.13. $$\int\frac{x^2\,dx}{(a+bx)^2(c+fx)^2}=$$
$$=-\frac{1}{\Delta^2}\left[\frac{a^2}{b(a+bx)}+\frac{c^2}{f(c+fx)}\right]+\frac{2ab}{\Delta^3}\ln\left|\frac{a+bx}{c+fx}\right|.$$

4.14. $$\int\frac{x^3\,dx}{(a+bx)(c+fx)}=$$
$$=\frac{x^2}{2bf}-\frac{af+bc}{b^2f^2}x+\frac{1}{\Delta}\left[\frac{a^3}{b^3}\ln|a+bx|-\frac{c^3}{f^3}\ln|c+fx|\right].$$

4.15. $$\int\frac{dx}{x(a+bx)(c+fx)}=\frac{1}{ac}\ln|x|+\frac{b}{a\Delta}\ln|a+bx|-\frac{f}{c\Delta}\ln|c+fx|.$$

4.16. $$\int\frac{dx}{x(a+bx)(c+fx)^2}=\frac{af(af+\Delta)+(af+bc)\Delta}{a^2cf(\Delta-bc)(c+fx)}+\frac{1}{ac^2}\ln|x|+$$
$$+\frac{b^2c^2-2a^2f^2}{a^2c^2f(\Delta-bc)}\ln|a+bx|+\frac{(af+bc)^2}{a^2c^2f(\Delta-bc)}\ln|c+fx|.$$

4.17. $$\int\frac{dx}{x^2(a+bx)(c+fx)}=$$
$$=-\frac{1}{acx}-\frac{af+bc}{a^2c^2}\ln|x|-\frac{b^2}{a^2\Delta}\ln|a+bx|+\frac{f^2}{c^2\Delta}\ln|c+fx|.$$

ТАБЛИЦА 5

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{(a^2+b^2x^2)^m}; \qquad \begin{matrix} a>0, & n=0,\ 1,\ 2,\ \ldots, \\ b>0, & m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

5.1. $$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\operatorname{arctg}\frac{bx}{a}.$$

5.2. $$\int \frac{dx}{(a^2+b^2x^2)^2} = \frac{x}{2a^2(a^2+b^2x^2)} + \frac{1}{2a^3b}\operatorname{arctg}\frac{bx}{a}.$$

5.3. $$\int \frac{dx}{(a^2+b^2x^2)^m} = \frac{x}{2(m-1)a^2(a^2+b^2x^2)^{m-1}} + \frac{2m-3}{2(m-1)a^2}\int \frac{dx}{(a^2+b^2x^2)^{m-1}} \qquad (m\geqslant 2).$$

5.4. $$\int \frac{x\,dx}{a^2+b^2x^2} = \frac{1}{2b^2}\ln(a^2+b^2x^2).$$

5.5. $$\int \frac{x\,dx}{(a^2+b^2x^2)^2} = -\frac{1}{2b^2(a^2+b^2x^2)}.$$

5.6. $$\int \frac{x\,dx}{(a^2+b^2x^2)^m} = -\frac{1}{2(m-1)b^2(a^2+b^2x^2)^{m-1}} \qquad (m\geqslant 2).$$

5.7. $$\int \frac{x^2\,dx}{a^2+b^2x^2} = \frac{x}{b^2} - \frac{a}{b^3}\operatorname{arctg}\frac{bx}{a}.$$

5.8. $$\int \frac{x^2\,dx}{(a^2+b^2x^2)^2} = -\frac{x}{2b^2(a^2+b^2x^2)} + \frac{1}{2ab^3}\operatorname{arctg}\frac{bx}{a}.$$

5.9. $$\int \frac{x^2\,dx}{(a^2+b^2x^2)^m} = -\frac{x}{2(m-1)b^2(a^2+b^2x^2)^{m-1}} + \frac{1}{2(m-1)b^2}\int \frac{dx}{(a^2+b^2x^2)^{m-1}} \qquad (m\geqslant 2) \quad (\text{см. } 5.3).$$

5.10. $$\int \frac{x^3\,dx}{a^2+b^2x^2} = \frac{x^2}{2b^2} - \frac{a^2}{2b^4}\ln(a^2+b^2x^2).$$

5.11. $$\int \frac{x^3\,dx}{(a^2+b^2x^2)^2} = \frac{a^2}{2b^4(a^2+b^2x^2)} + \frac{1}{2b^4}\ln(a^2+b^2x^2).$$

5.12. $$\int \frac{x^3\,dx}{(a^2+b^2x^2)^m} = -\frac{1}{2(m-2)b^4(a^2+b^2x^2)^{m-2}} + \frac{a^2}{2(m-1)b^4(a^2+b^2x^2)^{m-1}} \qquad (m\geqslant 3).$$

5.13. $$\int \frac{x^n\,dx}{(a^2+b^2x^2)^m} = -\frac{x^{n-1}}{2(m-1)b^2(a^2+b^2x^2)^{m-1}} + \frac{n-1}{2(m-1)b^2}\int \frac{x^{n-2}dx}{(a^2+b^2x^2)^{m-1}} \qquad (m\geqslant 2).$$

5.14. $$\int \frac{dx}{x(a^2+b^2x^2)} = \frac{1}{2a^2}\ln\frac{x^2}{a^2+b^2x^2}.$$

5.15. $$\int \frac{dx}{x(a^2+b^2x^2)^2} = \frac{1}{2a^2(a^2+b^2x^2)} + \frac{1}{2a^4}\ln\frac{x^2}{a^2+b^2x^2}.$$

5.16. $$\int \frac{dx}{x(a^2+b^2x^2)^m} = \frac{1}{2(m-1)a^2(a^2+b^2x^2)^{m-1}} + \frac{1}{a^2}\int \frac{dx}{x(a^2+b^2x^2)^{m-1}} \qquad (m\geqslant 2).$$

5.17. $$\int \frac{dx}{x^2(a^2+b^2x^2)} = -\frac{1}{a^2x} - \frac{b}{a^3}\operatorname{arctg}\frac{bx}{a}.$$

5.18. $$\int \frac{dx}{x^2(a^2+b^2x^2)^2} = \frac{1}{a^4x} - \frac{b^2x}{2a^4(a^2+b^2x^2)} - \frac{3b}{2a^5}\operatorname{arctg}\frac{bx}{a}.$$

5.19. $$\int \frac{dx}{x^2(a^2+b^2x^2)^m} = \frac{-1}{a^2x(a^2+b^2x^2)^{m-1}} - \frac{2(m-1)b^2}{a^2}\int \frac{dx}{(a^2+b^2x^2)^m} \quad (\text{см. } 5.3).$$

5.20. $$\int \frac{dx}{x^3(a^2+b^2x^2)} = -\frac{1}{2a^2x^2} - \frac{b^2}{2a^4}\ln\frac{x^2}{a^2+b^2x^2}.$$

5.21. $$\int \frac{dx}{x^3(a^2+b^2x^2)^2} = -\frac{1}{2a^2x^2} - \frac{b^2}{2a^4(a^2+b^2x^2)} - \frac{b^2}{a^6}\ln\frac{x^2}{a^2+b^2x^2}.$$

5.22. $$\int \frac{dx}{x^3(a^2+b^2x^2)^m} = -\frac{1}{2a^2x^2(a^2+b^2x^2)^{m-1}} - \frac{mb^2}{a^2}\int \frac{dx}{x(a^2+b^2x^2)^m} \quad (\text{см. } 5.16).$$

5.23. $$\int \frac{dx}{x^n(a^2+b^2x^2)^m} = -\frac{1}{(n-1)a^2x^{n-1}(a^2+b^2x^2)^{m-1}} - \frac{(2m+n-3)b^2}{(n-1)a^2}\int \frac{dx}{x^{n-2}(a^2+b^2x^2)^m} \qquad (n\geqslant 2).$$

ТАБЛИЦА 6

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{(a^2-b^2x^2)^m}; \qquad a>0,\ n=0,\ 1,\ 2,\ \ldots,\quad b>0,\ m=1,\ 2,\ 3,\ \ldots$$

6.1. $$\int \frac{dx}{a^2-b^2x^2} = \frac{1}{2ab}\ln\left|\frac{a+bx}{a-bx}\right|.$$

6.2. $$\int \frac{dx}{(a^2-b^2x^2)^2} = \frac{x}{2a^2(a^2-b^2x^2)} + \frac{1}{4a^3b}\ln\left|\frac{a+bx}{a-bx}\right|.$$

6.3. $$\int \frac{dx}{(a^2-b^2x^2)^m} = \frac{x}{2(m-1)a^2(a^2-b^2x^2)^{m-1}} + \frac{2m-3}{2(m-1)a^2}\int \frac{dx}{(a^2-b^2x^2)^{m-1}} \qquad (m\geqslant 2).$$

6.4. $$\int \frac{x\,dx}{a^2-b^2x^2} = -\frac{1}{2b^2}\ln|a^2-b^2x^2|.$$

6.5. $$\int \frac{x\,dx}{(a^2-b^2x^2)^2} = \frac{1}{2b^2(a^2-b^2x^2)}.$$

6.6. $$\int \frac{x\,dx}{(a^2-b^2x^2)^m} = \frac{1}{2(m-1)b^2(a^2-b^2x^2)^{m-1}} \qquad (m\geqslant 2).$$

6.7. $$\int \frac{x^2\,dx}{a^2-b^2x^2} = -\frac{x}{b^2} + \frac{a}{2b^3}\ln\left|\frac{a+bx}{a-bx}\right|.$$

6.8. $$\int \frac{x^2\,dx}{(a^2-b^2x^2)^2} = \frac{x}{2b^2(a^2-b^2x^2)} - \frac{1}{4ab^3}\ln\left|\frac{a+bx}{a-bx}\right|.$$

6.9. $$\int \frac{x^2\,dx}{(a^2-b^2x^2)^m} = \frac{x}{2(m-1)b^2(a^2-b^2x^2)^{m-1}} - \frac{1}{2(m-1)b^2}\int \frac{dx}{(a^2-b^2x^2)^{m-1}} \qquad (m\geqslant 2)\ (\text{см. } 6.3).$$

6.10. $$\int \frac{x^3\,dx}{a^2-b^2x^2} = -\frac{x^2}{2b^2} - \frac{a^2}{2b^4}\ln|a^2-b^2x^2|.$$

6.11. $$\int \frac{x^3\,dx}{(a^2-b^2x^2)^2} = \frac{a^2}{2b^4(a^2-b^2x^2)} + \frac{1}{2b^4}\ln|a^2-b^2x^2|.$$

6.12. $$\int \frac{x^3\,dx}{(a^2-b^2x^2)^m} = -\frac{1}{2(m-2)b^4(a^2-b^2x^2)^{m-2}} + \frac{a^2}{2(m-1)b^4(a^2-b^2x^2)^{m-1}} \qquad (m\geqslant 3)$$

6.13. $$\int \frac{x^n\,dx}{(a^2-b^2x^2)^m} = \frac{x^{n-1}}{2(m-1)b^2(a^2-b^2x^2)^{m-1}} - \frac{n-1}{2(m-1)b^2}\int \frac{x^{n-2}}{(a^2-b^2x^2)^{m-1}}\,dx \qquad (m\geqslant 2).$$

6.14. $$\int \frac{dx}{x(a^2-b^2x^2)} = \frac{1}{2a^2}\ln\left|\frac{x^2}{a^2-b^2x^2}\right|.$$

6.15. $$\int \frac{dx}{x(a^2-b^2x^2)^2} = \frac{1}{2a^2(a^2-b^2x^2)} + \frac{1}{2a^4}\ln\left|\frac{x^2}{a^2-b^2x^2}\right|.$$

6.16. $$\int \frac{dx}{x(a^2-b^2x^2)^m} = \frac{1}{2(m-1)a^2(a^2-b^2x^2)^{m-1}} + \frac{1}{a^2}\int \frac{dx}{x(a^2-b^2x^2)^{m-1}} \qquad (m\geqslant 2).$$

6.17. $$\int \frac{dx}{x^2(a^2-b^2x^2)} = -\frac{1}{a^2x} + \frac{b}{2a^3}\ln\left|\frac{a+bx}{a-bx}\right|.$$

6.18. $$\int \frac{dx}{x^2(a^2-b^2x^2)^2} = -\frac{1}{a^4x} + \frac{b^2x}{2a^4(a^2-b^2x^2)} + \frac{3b}{4a^5}\ln\left|\frac{a+bx}{a-bx}\right|.$$

6.19. $$\int \frac{dx}{x^2(a^2-b^2x^2)^m} = -\frac{1}{a^2x(a^2-b^2x^2)^{m-1}} + \frac{(2m-1)b^2}{a^2}\int \frac{dx}{(a^2-b^2x^2)^m} \qquad (\text{см. } 6.3).$$

6.20. $$\int \frac{dx}{x^3(a^2-b^2x^2)} = -\frac{1}{2a^2x^2} + \frac{b^2}{2a^4}\ln\left|\frac{x^2}{a^2-b^2x^2}\right|.$$

6.21. $$\int \frac{dx}{x^3(a^2-b^2x^2)^2} = -\frac{1}{2a^4x^2} + \frac{b^2}{2a^4(a^2-b^2x^2)} + \frac{b^2}{a^6}\ln\left|\frac{x^2}{a^2-b^2x^2}\right|.$$

6.22. $$\int \frac{dx}{x^3(a^2-b^2x^2)^m} = -\frac{1}{2a^2x^2(a^2-b^2x^2)^{m-1}} + \frac{mb^2}{a^2}\int \frac{dx}{x(a^2-b^2x^2)^m} \qquad (\text{см. } 6.16).$$

6.23. $$\int \frac{dx}{x^n(a^2-b^2x^2)^m} = -\frac{1}{(n-1)a^2x^{n-1}(a^2-b^2x^2)^{m-1}} + \frac{(2m+n-3)b^2}{(n-1)a^2}\int \frac{dx}{x^{n-2}(a^2-b^2x^2)^m} \qquad (n\geqslant 2).$$

ТАБЛИЦА 7

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n\,dx}{(a+bx^3)^m}; \quad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

7.1. $$\int \frac{dx}{a+bx^3} = \frac{1}{6b\xi^2}\ln\left|\frac{(\xi+x)^2}{\xi^2-\xi x+x^2}\right| + \frac{1}{\sqrt{3}\,b\xi^2}\operatorname{arctg}\frac{2x-\xi}{\sqrt{3}\,\xi}.$$

7.2. $$\int \frac{dx}{(a+bx^3)^2} = \frac{x}{3a\,(a+bx^3)} + \frac{2}{3a}\int \frac{dx}{a+bx^3} \quad \text{(см. 7.1).}$$

7.3. $$\int \frac{dx}{(a+bx^3)^m} = \frac{x}{3\,(m-1)\,a\,(a+bx^3)^{m-1}} + \frac{3m-4}{3\,(m-1)\,a}\int \frac{dx}{(a+bx^3)^{m-1}} \quad (m \geqslant 2).$$

7.4. $$\int \frac{x\,dx}{a+bx^3} = \frac{-1}{6b\xi}\ln\left|\frac{(\xi+x)^2}{\xi^2-\xi x+x^2}\right| + \frac{1}{\sqrt{3}\,b\xi}\operatorname{arctg}\frac{2x-\xi}{\sqrt{3}\,\xi}.$$

7.5. $$\int \frac{x\,dx}{(a+bx^3)^2} = \frac{x^2}{3a\,(a+bx^3)} + \frac{1}{3a}\int \frac{x\,dx}{a+bx^3} \quad \text{(см. 7.4).}$$

7.6. $$\int \frac{x\,dx}{(a+bx^3)^3} = -\frac{7ax^3+4bx^5}{18a^2\,(a+bx^3)^2} + \frac{2}{9a^2}\int \frac{x\,dx}{a+bx^3} \quad \text{(см. 7.4).}$$

7.7. $$\int \frac{x\,dx}{(a+bx^3)^m} = \frac{x^2}{3\,(m-1)\,a\,(a+bx^3)^{m-1}} + \frac{3m-5}{3\,(m-1)\,a}\int \frac{x\,dx}{(a+bx^3)^{m-1}} \quad (m \geqslant 2).$$

7.8. $$\int \frac{x^2\,dx}{a+bx^3} = \frac{1}{3b}\ln|a+bx^3|.$$

7.9. $$\int \frac{x^2\,dx}{(a+bx^3)^m} = -\frac{1}{3\,(m-1)\,b\,(a+bx^3)^{m-1}} \quad (m \geqslant 2).$$

7.10. $$\int \frac{x^3\,dx}{a+bx^3} = \frac{x}{b} - \frac{\xi}{6b}\ln\left|\frac{(\xi+x)^2}{\xi^2-\xi x+x^2}\right| - \frac{\xi}{\sqrt{3}\,b}\operatorname{arctg}\frac{2x-\xi}{\sqrt{3}\,\xi}.$$

7.11. $$\int \frac{x^3\,dx}{(a+bx^3)^2} = -\frac{x}{3b\,(a+bx^3)} + \frac{1}{3b}\int \frac{dx}{a+bx^3} \quad \text{(см. 7.1).}$$

7.12. $$\int \frac{x^3\,dx}{(a+bx^3)^3} = \frac{3ax-5ax^2+6bx^4-5bx^5}{18ab\,(a+bx^3)^2} + \frac{2}{3ab}\int \frac{dx}{a+bx^3} - \frac{5}{18ab}\int \frac{x\,dx}{a+bx^3} \quad \text{(см. 7.1 и 7.4).}$$

7.13. $$\int \frac{x^3\,dx}{(a+bx^3)^m} = \frac{x^4}{3\,(m-1)\,a\,(a+bx^3)^{m-1}} + \frac{3m-7}{3\,(m-1)\,a}\int \frac{x^3\,dx}{(a+bx^3)^{m-1}} \quad (m \geqslant 2);$$

$$= \frac{x}{(4-3m)\,b\,(a+bx^3)^{m-1}} - \frac{a}{(4-3m)\,b}\int \frac{dx}{(a+bx^3)^m} \quad (m \geqslant 2) \text{ (см. 7.3).}$$

7.14. $$\int \frac{x^4\,dx}{a+bx^3} = \frac{x^2}{2b} + \frac{\xi^2}{6b}\ln\left|\frac{(\xi+x)^2}{\xi^2-\xi x+x^2}\right| - \frac{\xi^2}{\sqrt{3}\,b}\operatorname{arctg}\frac{2x-\xi}{\sqrt{3}\,\xi}.$$

7.15. $$\int \frac{x^4\,dx}{(a+bx^3)^2} = \frac{-x^2}{3b\,(a+bx^3)} + \frac{2}{3b}\int \frac{x\,dx}{a+bx^3} \quad \text{(см. 7.4).}$$

7.16. $$\int \frac{x^4\,dx}{(a+bx^3)^3} = -\frac{x^2}{4b\,(a+bx^3)^2} + \frac{a}{2b}\int \frac{x\,dx}{(a+bx^3)^3} \quad \text{(см. 7.6).}$$

7.17. $$\int \frac{x^4\,dx}{(a+bx^3)^m} = \frac{x^2}{(5-3m)\,b\,(a+bx^3)^{m-1}} - \frac{2a}{(5-3m)\,b}\int \frac{x\,dx}{(a+bx^3)^m} \quad \text{(см. 7.7).}$$

7.18. $$\int \frac{x^n\,dx}{a+bx^3} = \frac{x^{n-2}}{(n-2)\,b} - \frac{a}{b}\int \frac{x^{n-3}\,dx}{a+bx^3} \quad (n \geqslant 3).$$

7.19. $$\int \frac{x^n\,dx}{(a+bx^3)^m} = \frac{x^{n+1}}{3\,(m-1)\,a\,(a+bx^3)^{m-1}} - \frac{n+4-3m}{3\,(m-1)\,a}\int \frac{x^n\,dx}{(a+bx^3)^{m-1}} \quad (m \geqslant 2);$$

$$= \frac{x^{n-2}}{(n+1-3m)\,b\,(a+bx^3)^{m-1}} - \frac{(n-2)\,a}{(n+1-3m)\,b}\int \frac{x^{n-3}}{(a+bx^3)^m}\,dx \quad (n \neq 3m-1).$$

ТАБЛИЦА 8

ИНТЕГРАЛЫ ВИДА

$$\int \frac{dx}{x^n (a+bx^3)^m}; \quad \begin{matrix} n=1,\ 2,\ 3,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

8.1. $$\int \frac{dx}{x(a+bx^3)} = \frac{1}{3a} \ln \left| \frac{bx^3}{a+bx^3} \right|.$$

8.2. $$\int \frac{dx}{x(a+bx^3)^2} = \frac{1}{3a(a+bx^3)} + \frac{1}{3a^2} \ln \left| \frac{bx^3}{a+bx^3} \right|.$$

8.3. $$\int \frac{dx}{x(a+bx^3)^3} = \frac{3a+2bx^3}{6a^2(a+bx^3)^2} + \frac{1}{3a^3} \ln \left| \frac{bx^3}{a+bx^3} \right|.$$

8.4. $$\int \frac{dx}{x(a+bx^3)^4} = \frac{11a^2+15abx^3+6b^2x^6}{18a^3(a+bx^3)^3} + \frac{1}{3a^4} \ln \left| \frac{bx^3}{a+bx^3} \right|.$$

8.5. $$\int \frac{dx}{x(a+bx^3)^m} =$$

$$= \frac{1}{3(m-1)a(a+bx^3)^{m-1}} + \frac{1}{a} \int \frac{dx}{x(a+bx^3)^{m-1}} \quad (m \geqslant 2).$$

8.6. $$\int \frac{dx}{x^2(a+bx^3)} =$$

$$= -\frac{1}{ax} + \frac{1}{6a\xi} \ln \left| \frac{(\xi+x)^2}{\xi^2-\xi x+x^2} \right| - \frac{1}{\sqrt{3}a\xi} \operatorname{arctg} \frac{2x-\xi}{\sqrt{3}\xi}.$$

8.7. $$\int \frac{dx}{x^2(a+bx^3)^2} = \frac{-3a+4bx^3}{3a^2x(a+bx^3)} + \frac{4}{3a} \int \frac{dx}{x^2(a+bx^3)} \quad \text{(см. 8.6)}.$$

8.8. $$\int \frac{dx}{x^2(a+bx^3)^3} = \frac{1}{6a(a+bx^3)^2} + \frac{7}{18a^2(a+bx^3)} - \frac{14}{9a^3x} +$$

$$+ \frac{14}{9a^3} \int \frac{dx}{x^2(a+bx^3)} \quad \text{(см. 8.6)}.$$

8.9. $$\int \frac{dx}{x^2(a+bx^3)^m} = \frac{1}{3(m-1)a(a+bx^3)^{m-1}} +$$

$$+ \frac{3m-2}{3(m-1)a} \int \frac{dx}{x^2(a+bx^3)^{m-1}} \quad (m \geqslant 2)$$

8.10. $$\frac{dx}{x^3(a+bx^3)} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{dx}{a+bx^3} \quad \text{(см. 7.1)}.$$

8.11. $$\int \frac{dx}{x^3(a+bx^3)^2} = -\frac{3a+5bx^3}{6a^2x^2(a+bx^3)} - \frac{5}{18a^2\xi^2} \ln \left| \frac{(\xi+x)^2}{\xi^2-\xi x+x^2} \right| -$$

$$- \frac{5}{3\sqrt{3}a^2\xi^2} \operatorname{arctg} \frac{2x-\xi}{\sqrt{3}\xi}.$$

8.12. $$\int \frac{dx}{x^3(a+bx^3)^3} = -\frac{9a^2+32abx^3+20b^2x^6}{18a^3x^2(a+bx^3)^2} -$$

$$- \frac{10}{27a^3\xi^2} \ln \left| \frac{(\xi+x)^2}{\xi^2-\xi x+x^2} \right| - \frac{20}{9\sqrt{3}a^3\xi^2} \operatorname{arctg} \frac{2x-\xi}{\sqrt{3}\xi}.$$

8.13. $$\int \frac{dx}{x^3(a+bx^3)^m} = \frac{1}{3(m-1)ax^2(a+bx^3)^{m-1}} +$$

$$+ \frac{3m-1}{3(m-1)a} \int \frac{dx}{x^3(a+bx^3)^{m-1}} \quad (m \geqslant 2);$$

$$= \frac{-1}{2ax^2(a+bx^3)^{m-1}} - \frac{(3m-1)bx}{6(m-1)a^2(a+bx^3)^{m-1}} -$$

$$- \frac{(3m-1)(3m-4)b}{6(m-1)a^2} \int \frac{x\,dx}{(a+bx^3)^{m-1}} \quad \text{(см. 7.7)}.$$

8.14. $$\int \frac{dx}{x^4(a+bx^3)} = -\frac{1}{3ax^3} - \frac{b}{3a^2} \ln \left| \frac{bx^3}{a+bx^3} \right|.$$

8.15. $$\int \frac{dx}{x^4(a+bx^3)^2} = -\frac{a+2bx^3}{3a^2x^3(a+bx^3)} - \frac{2b}{3a^3} \ln \left| \frac{bx^3}{a+bx^3} \right|.$$

8.16. $$\int \frac{dx}{x^4(a+bx^3)^3} =$$

$$= -\frac{1}{3a^3x^3} - \frac{b}{6a^2(a+bx^3)^2} - \frac{2b}{a^3(a+bx^3)} - \frac{7b}{a^4} \ln \left| \frac{bx^3}{a+bx^3} \right|.$$

8.17. $$\int \frac{dx}{x^4(a+bx^3)^m} = \frac{1}{3(m-1)ax^3(a+bx^3)^{m-1}} +$$

$$+ \frac{3m}{3(m-1)a} \int \frac{dx}{x^4(a+bx^3)^{m-1}} \quad (m \geqslant 2).$$

8.18. $$\int \frac{dx}{x^n(a+bx^3)^m} = \frac{1}{3(m-1)ax^{n-1}(a+bx^3)^{m-1}} +$$

$$+ \frac{3m+n-4}{3(m-1)a} \int \frac{dx}{x^n(a+bx^3)^{m-1}} \quad (m \geqslant 2);$$

$$= \frac{-1}{(n-1)ax^{n-1}(a+bx^3)^{m-1}} -$$

$$- \frac{(3m+n-4)b}{(n-1)a} \int \frac{dx}{x^{n-3}(a+bx^3)^m} \quad (n \geqslant 2).$$

ТАБЛИЦА 9

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{(a+bx^4)^m}; \quad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

9.1. $$\int \frac{dx}{a+bx^4} =$$
$$= \begin{cases} \dfrac{\eta}{4\sqrt{2}\,a}\left[\ln\dfrac{x^2+\sqrt{2}\,\eta x+\eta^2}{x^2-\sqrt{2}\,\eta x+\eta^2}+2\operatorname{arctg}\dfrac{\sqrt{2}\,\eta x}{\eta^2-x^2}\right] & \text{при } ab>0; \\ -\dfrac{\eta}{4a}\left[\ln\left|\dfrac{x+\eta}{x-\eta}\right|+2\operatorname{arctg}\dfrac{x}{\eta}\right] & \text{при } ab<0. \end{cases}$$

9.2. $$\int \frac{dx}{(a+bx^4)^2} = \frac{x}{4a\,(a+bx^4)} + \frac{3}{4a}\int \frac{dx}{a+bx^4} \quad \text{(см. 9.1).}$$

9.3. $$\int \frac{dx}{(a+bx^4)^3} = \frac{11ax+7bx^5}{32a^2\,(a+bx^4)^2} + \frac{21}{32a^2}\int \frac{dx}{a+bx^4} \quad \text{(см. 9.1).}$$

9.4. $$\int \frac{dx}{(a+bx^4)^m} = \frac{x}{4\,(m-1)\,a\,(a+bx^4)^{m-1}} + \frac{4m-5}{4\,(m-1)\,a}\int \frac{dx}{(a+bx^4)^{m-1}} \quad (m\geqslant 2).$$

9.5. $$\int \frac{x\,dx}{a+bx^4} = \begin{cases} \dfrac{1}{2b\eta^2}\operatorname{arctg}\dfrac{x^2}{\eta^2} & \text{при } ab>0; \\ -\dfrac{1}{4\sqrt{|ab|}}\ln\left|\dfrac{a-x^2\sqrt{|ab|}}{a+x^2\sqrt{|ab|}}\right| & \text{при } ab<0. \end{cases}$$

9.6. $$\int \frac{x\,dx}{(a+bx^4)^2} = \frac{x^2}{4a\,(a+bx^4)} + \frac{1}{2a}\int \frac{x\,dx}{a+bx^4} \quad \text{(см. 9.5).}$$

9.7. $$\int \frac{x\,dx}{(a+bx^4)^3} = \frac{5ax^2+3bx^6}{16a^2\,(a+bx^4)^2} + \frac{3}{8a^2}\int \frac{x\,dx}{a+bx^4} \quad \text{(см. 9.5).}$$

9.8 $$\int \frac{x\,dx}{(a+bx^4)^m} = \frac{x^2}{4\,(m-1)\,a\,(a+bx^4)^{m-1}} + \frac{2m-3}{2\,(m-1)\,a}\int \frac{x\,dx}{(a+bx^4)^{m-1}} \quad (m\geqslant 2).$$

9.9. $$\int \frac{x^2\,dx}{a+bx^4} =$$
$$= \begin{cases} \dfrac{-1}{4\sqrt{2}\,b\eta}\left[\ln\dfrac{x^2+\sqrt{2}\,\eta x+\eta^2}{x^2-\sqrt{2}\,\eta x+\eta^2}-2\operatorname{arctg}\dfrac{\sqrt{2}\,\eta x}{\eta^2-x^2}\right] & \text{при } ab>0; \\ -\dfrac{1}{4b\eta}\left[\ln\left|\dfrac{x+\eta}{x-\eta}\right|-2\operatorname{arctg}\dfrac{x}{\eta}\right] & \text{при } ab<0. \end{cases}$$

9.10. $$\int \frac{x^2\,dx}{(a+bx^4)^2} = \frac{x^3}{4a\,(a+bx^4)} + \frac{1}{4a}\int \frac{x^2\,dx}{a+bx^4} \quad \text{(см. 9.9).}$$

9.11. $$\int \frac{x^2\,dx}{(a+bx^4)^3} = \frac{9ax^3+5bx^7}{32a^2\,(a+bx^4)^2} + \frac{5}{32a^2}\int \frac{x^2\,dx}{a+bx^4} \quad \text{(см. 9.9).}$$

9.12. $$\int \frac{x^n\,dx}{(a+bx^4)^m} = \frac{x^{n+1}}{4\,(m-1)\,a\,(a+bx^4)^{m-1}} + \frac{4m-n-5}{4\,(m-1)\,a}\int \frac{x^n\,dx}{(a+bx^4)^{m-1}} \quad (m\geqslant 2);$$
$$= \frac{x^{n-3}}{(n+1-4m)\,b\,(a+bx^4)^{m-1}} - \frac{(n-3)\,a}{(n+1-4m)\,b}\int \frac{x^{n-4}\,dx}{(a+bx^4)^m} \quad (n\neq 4m-1).$$

9.13. $$\int \frac{dx}{x\,(a+bx^4)} = \frac{1}{4a}\ln\left|\frac{x^4}{a+bx^4}\right|.$$

9.14. $$\int \frac{dx}{x\,(a+bx^4)^2} = \frac{1}{4a\,(a+bx^4)} + \frac{1}{4a^2}\ln\left|\frac{x^4}{a+bx^4}\right|.$$

9.15. $$\int \frac{dx}{x\,(a+bx^4)^3} = \frac{3+2bx^4}{8a\,(a+bx^4)^2} + \frac{1}{4a^3}\ln\left|\frac{x^4}{a+bx^4}\right|.$$

9.16. $$\int \frac{dx}{x^3\,(a+bx^4)} = -\frac{1}{ax} - \frac{b}{a}\int \frac{x^2\,dx}{a+bx^4} \quad \text{(см. 9.9).}$$

9.17. $$\int \frac{dx}{x^3\,(a+bx^4)^2} = -\frac{1}{a^2x} - \frac{bx^3}{4a^2\,(a+bx^4)} - \frac{5b}{4a^2}\int \frac{x^2\,dx}{a+bx^4} \quad \text{(см. 9.9).}$$

9.18. $$\int \frac{dx}{x^3\,(a+bx^4)^3} = -\frac{1}{ax\,(a+bx^4)^2} - \frac{9b}{a}\int \frac{x^2\,dx}{(a+bx^4)^3} \quad \text{(см. 9.11).}$$

9.19. $$\int \frac{dx}{x^n\,(a+bx^4)^m} = -\frac{1}{(n-1)\,ax^{n-1}\,(a+bx^4)^{m-1}} - \frac{(4m+n-5)\,b}{(n-1)\,a}\int \frac{dx}{x^{n-4}\,(a+bx^4)^m} \quad (n\geqslant 2);$$
$$= \frac{1}{a}\int \frac{dx}{x^n\,(a+bx^4)^{m-1}} - \frac{b}{a}\int \frac{dx}{x^{n-4}\,(a+bx^4)^m}.$$

ТАБЛИЦА 10

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n\,dx}{a+bx^m}, \quad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=5,\ 6,\ 7,\ \ldots \end{matrix}$$

10.1. $$\int \frac{dx}{1+x^{2k}} = -\frac{1}{2k}\sum_{\nu=0}^{k-1} \ln\left(x^2-2x\cos\frac{2\nu+1}{2k}\pi+1\right)\cos\frac{2\nu+1}{2k}\pi + \frac{1}{k}\sum_{\nu=0}^{k-1}\operatorname{arctg}\left(\frac{x\sin\frac{2\nu+1}{2k}\pi}{1-x\cos\frac{2\nu+1}{2k}\pi}\right)\sin\frac{2\nu+1}{2k}\pi.$$

10.2. $$\int \frac{dx}{1+x^{2k+1}} = \frac{1}{2k+1}\ln|1+x| - \frac{1}{2k+1}\sum_{\nu=0}^{k-1} \ln\left(x^2-2x\cos\frac{2\nu+1}{2k+1}\pi+1\right)\cos\frac{2\nu+1}{2k+1}\pi + \frac{2}{2k+1}\sum_{\nu=0}^{k-1}\operatorname{arctg}\left(\frac{x\sin\frac{2\nu+1}{2k+1}\pi}{1-x\cos\frac{2\nu+1}{2k+1}\pi}\right)\sin\frac{2\nu+1}{2k+1}\pi.$$

10.3. $$\int \frac{dx}{1-x^{2k}} = \frac{1}{2k}\ln\left|\frac{1+x}{1-x}\right| - \frac{1}{2k}\sum_{\nu=0}^{k-1} \ln\left(x^2+2x\cos\frac{2\nu+1}{2k}\pi+1\right)\cos\frac{\nu}{k}\pi + \frac{1}{k}\sum_{\nu=0}^{k-1}\operatorname{arctg}\left(\frac{x\sin\frac{2\nu+1}{2k}\pi}{1+x\cos\frac{2\nu+1}{2k}\pi}\right)\sin\frac{\nu}{k}\pi.$$

10.4. $$\int \frac{dx}{1-x^{2k+1}} = -\frac{1}{2k+1}\ln|1-x| + \frac{1}{2k+1}\sum_{\nu=0}^{k-1} \ln\left(x^2+2x\cos\frac{2\nu+1}{2k+1}\pi+1\right)\cos\frac{2\nu+1}{2k+1}\pi + \frac{2}{2k+1}\sum_{\nu=0}^{k-1}\operatorname{arctg}\left(\frac{x\sin\frac{2\nu+1}{2k+1}\pi}{1+x\cos\frac{2\nu+1}{2k+1}\pi}\right)\sin\frac{2\nu+1}{2k+1}\pi.$$

10.5. $$\int \frac{x^n\,dx}{1+x^{2k}} = \frac{1}{k}\sum_{\nu=1}^{k}\operatorname{arctg}\frac{x-\cos\frac{2\nu-1}{2k}\pi}{\sin\frac{2\nu-1}{2k}\pi}\cos\frac{(n+1)(2\nu-1)}{2k}\pi - \frac{1}{2k}\sum_{\nu=1}^{k}\ln\left(x^2-2x\cos\frac{2\nu-1}{2k}\pi+1\right)\cos\frac{(n+1)(2\nu-1)}{2k}\pi.$$

10.6. $$\int \frac{x^n\,dx}{1+x^{2k+1}} = (-1)^n\frac{\ln|1+x|}{2k+1} - \frac{1}{2k+1}\sum_{\nu=1}^{2k+1}\ln\left(x^2-2x\cos\frac{2\nu-1}{2k+1}\pi+1\right)\cos\frac{(n+1)(2\nu-1)}{2k+1}\pi + \frac{2}{2k+1}\sum_{\nu=1}^{2k+1}\operatorname{arctg}\frac{x-\cos\frac{2\nu-1}{2k+1}\pi}{\sin\frac{2\nu-1}{2k+1}\pi}\sin\frac{(n+1)(2\nu-1)}{2k+1}\pi.$$

10.7. $$\int \frac{x^n\,dx}{1-x^{2k}} = \frac{1}{2k}\{(-1)^n[\ln|1+x|]-\ln|1-x|\} + (-1)^n\frac{1}{2k}\sum_{\nu=1}^{k-1}\ln\left(x^2+2x\cos\frac{\nu}{k}\pi+1\right)\cos\frac{\nu(n+1)}{k}\pi + (-1)^n\frac{1}{k}\sum_{\nu=1}^{k-1}\operatorname{arctg}\frac{x+\cos\frac{\nu}{k}\pi}{\sin\frac{\nu}{k}\pi}\sin\frac{\nu(n+1)}{k}\pi.$$

10.8. $$\int \frac{x^n\,dx}{1-x^{2k+1}} = -\frac{1}{2k+1}\ln|1-x| + \frac{(-1)^n}{2k+1}\sum_{\nu=1}^{2k+1}\ln\left(x^2+2x\cos\frac{2\nu-1}{2k+1}\pi+1\right)\cos\frac{(n+1)(2\nu-1)}{2k+1}\pi + (-1)^n\frac{2}{2k+1}\sum_{\nu=1}^{2k+1}\operatorname{arctg}\frac{x+\cos\frac{2\nu-1}{2k+1}\pi}{\sin\frac{2\nu-1}{2k+1}\pi}\sin\frac{(n+1)(2\nu-1)}{2k+1}\pi.$$

10.9. $$\int \frac{x^n\,dx}{a+bx^m} = \begin{cases} \frac{1}{a}\sqrt[m]{\left(\frac{a}{b}\right)^{n+1}}\int\frac{t^n\,dt}{1+t^m}, & \text{где } t=\sqrt[m]{\frac{b}{a}}\,x \quad (ab>0) \\ \frac{1}{a}\sqrt[m]{\left(-\frac{a}{b}\right)^{n+1}}\int\frac{t^n\,dt}{1-t^m}, & \text{где } t=\sqrt[m]{-\frac{b}{a}}\,x \quad (ab<0). \end{cases}$$

ТАБЛИЦА 11

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n\,dx}{(ax^2+bx+c)^m}, \quad b^2-4ac\neq 0; \quad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

11.1. $$\int \frac{dx}{ax^2+bx+c}=\begin{cases} \dfrac{2}{\sqrt{-\delta}}\operatorname{arctg}\dfrac{2ax+b}{\sqrt{-\delta}} & \text{при } b^2<4ac; \\ \dfrac{1}{\sqrt{\delta}}\ln\left|\dfrac{2ax+b-\sqrt{\delta}}{2ax+b+\sqrt{\delta}}\right| & \text{при } b^2>4ac. \end{cases}$$

11.2. $$\int \frac{dx}{(ax^2+bx+c)^2}=\frac{-2ax-b}{\delta\,(ax^2+bx+c)}-\frac{2a}{\delta}\int\frac{dx}{ax^2+bx+c}$$

(см. 11.1).

11.3 $$\int \frac{dx}{(ax^2+bx+c)^3}=\frac{-2ax-b}{2\delta\,(ax^2+bx+c)^2}+\frac{3a\,(2ax+b)}{\delta^2\,(ax^2+bx+c)}+$$
$$+\frac{6a^2}{\delta^2}\int\frac{dx}{ax^2+bx+c}$$ (см. 11.1).

11.4. $$\int \frac{dx}{(ax^2+bx+c)^m}=\frac{-2ax-b}{(m-1)\,\delta\,(ax^2+bx+c)^{m-1}}-$$
$$-\frac{2\,(2m-3)\,a}{(m-1)\,\delta}\int\frac{dx}{(ax^2+bx+c)^{m-1}} \quad (m\geqslant 2).$$

11.5. $$\int \frac{x\,dx}{ax^2+bx+c}=$$
$$=\begin{cases} \dfrac{1}{2a}\ln|ax^2+bx+c|-\dfrac{b}{a\sqrt{-\delta}}\operatorname{arctg}\dfrac{2ax+b}{\sqrt{-\delta}} & (\delta<0); \\ \dfrac{1}{2a}\ln|ax^2+bx+c|-\dfrac{b}{2a\sqrt{\delta}}\ln\left|\dfrac{2ax+b-\sqrt{\delta}}{2ax+b+\sqrt{\delta}}\right| & (\delta>0) \end{cases}$$

11.6. $$\int \frac{x\,dx}{(ax^2+bx+c)^2}=\frac{bx+2c}{\delta\,(ax^2+bx+c)}+\frac{b}{\delta}\int\frac{dx}{ax^2+bx+c}$$

(см. 11.1).

11.7. $$\int \frac{x\,dx}{(ax^2+bx+c)^m}=-\frac{bx+2c}{(m-1)\,\delta\,(ax^2+bx+c)^{m-1}}-$$
$$-\frac{(2m-3)\,b}{(m-1)\,\delta}\int\frac{dx}{(ax^2+bx+c)^{m-1}} \quad (m\geqslant 2)$$ (см. 11.4).

11.8. $$\frac{x^2\,dx}{ax^2+bx+c}=\frac{x}{a}-\frac{b}{2a^2}\ln|ax^2+bx+c|+$$
$$+\frac{b^2-2ac}{2a^2}\int\frac{dx}{ax^2+bx+c}$$ (см. 11.1).

11.9. $$\int \frac{x^2\,dx}{(ax^2+bx+c)^2}=-\frac{(b^2-2ac)\,x+bc}{a\delta\,(ax^2+bx+c)}-\frac{2c}{\delta}\int\frac{dx}{ax^2+bx+c}$$

(см. 11.1)

11.10. $$\int \frac{x^2\,dx}{(ax^2+bx+c)^m}=-\frac{(b^2-2ac)\,x+bc}{(m-1)\,a\delta\,(ax^2+bx+c)^{m-1}}-$$
$$-\frac{(m-4)\,b^2+10ac}{(m+1)\,a\delta}\int\frac{dx}{(ax^2+bx+c)^{m-1}} \quad (m\geqslant 2)$$ (см. 11.4).

11.11. $$\int \frac{x^3\,dx}{ax^2+bx+c}=\frac{ax^2-bx}{2a^2}+\frac{b^2-ac}{2a^2}\ln|ax^2+bx+c|-$$
$$-\frac{b\,(b^2-3ac)}{2a^2}\int\frac{dx}{ax^2+bx+c}$$ (см. 11.1).

11.12. $$\int \frac{x^n\,dx}{ax^2+bx+c}=\frac{x^{n-1}}{(n-1)\,a}-\frac{c}{a}\int\frac{x^{n-2}\,dx}{ax^2+bx+c}-$$
$$-\frac{b}{a}\int\frac{x^{n-1}\,dx}{ax^2+bx+c} \quad (n\geqslant 2).$$

11.13. $$\int \frac{x^3\,dx}{(ax^2+bx+c)^m}=-\frac{x^2}{2\,(m-2)\,a\,(ax^2+bx+c)^{m-1}}+$$
$$+\frac{c}{(m-2)\,a}\int\frac{x\,dx}{(ax^2+bx+c)^m}+\frac{(m-3)\,b}{2\,(m-2)\,a}\int\frac{x^2\,dx}{(ax^2+bx+c)^m} \quad (m\neq 2).$$

11.14. $$\int \frac{x^n\,dx}{(ax^2+bx+c)^m}=-\frac{x^{n-1}}{(2m-n-1)\,a\,(ax^2+bx+c)^{m-1}}+$$
$$+\frac{(n-1)\,c}{(2m-n-1)\,a}\int\frac{x^{n-2}\,dx}{(ax^2+bx+c)^m}+$$
$$+\frac{(m-n)\,b}{(2m-n-1)\,a}\int\frac{x^{n-1}\,dx}{(ax^2+bx+c)^m} \quad (n\neq 2m-1).$$

11.15. $$\int \frac{x^{2m-1}\,dx}{(ax^2+bx+c)^m}=\frac{1}{a}\int\frac{x^{2m-3}\,dx}{(ax^2+bx+c)^{m-1}}-$$
$$-\frac{c}{a}\int\frac{x^{2m-3}\,dx}{(ax^2+bx+c)^m}-\frac{b}{a}\int\frac{x^{2m-2}\,dx}{(ax^2+bx+c)^m}.$$

ТАБЛИЦА 12

ИНТЕГРАЛЫ ВИДА

$$\int \frac{dx}{x^n (ax^2+bx+c)^m}, \quad b^2-4ac\neq 0; \quad \begin{matrix} n=1,\ 2,\ 3,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

12.1. $$\int \frac{dx}{x(ax^2+bx+c)} =$$

$$= \begin{cases} \dfrac{1}{2c}\ln\dfrac{x^2}{|ax^2+bx+c|} - \dfrac{b}{c\sqrt{-\delta}}\operatorname{arctg}\dfrac{2ax+b}{\sqrt{-\delta}} & (\delta<0); \\ \dfrac{1}{2c}\ln\dfrac{x^2}{|ax^2+bx+c|} - \dfrac{b}{2c\sqrt{\delta}}\ln\left|\dfrac{2ax+b-\sqrt{\delta}}{2ax+b+\sqrt{\delta}}\right| & (\delta>0) \end{cases}$$

12.2. $$\int \frac{dx}{x(ax^2+bx+c)^2} = \frac{abx-2ac+b^2}{c\delta(ax^2+bx+c)} + \frac{1}{2c}\ln\frac{x^2}{|ax^2+bx+c|} +$$
$$+ \frac{b(6ac-b^2)}{2c^2\delta}\int\frac{dx}{ax^2+bx+c} \quad \text{(см. 11.1).}$$

12.3. $$\int \frac{dx}{x(ax^2+bx+c)^3} = \frac{2ax^2+2bx+3c}{4c^2(ax^2+bx+c)^2} +$$
$$+ \frac{1}{2c^3}\ln\frac{x^2}{|ax^2+bx+c|} - \frac{b}{2c^3}\int\frac{dx}{ax^2+bx+c} -$$
$$- \frac{b}{2c^2}\int\frac{dx}{(ax^2+bx+c)^2} - \frac{b}{2c}\int\frac{dx}{(ax^2+bx+c)}$$
(см. 11.1, 11.2 и 11.3)

12.4. $$\int \frac{dx}{x(ax^2+bx+c)^m} = -\frac{1}{c}\int\frac{t^{2m-3}\,dt}{(a+bt+ct^2)^{m-1}} +$$
$$+ \frac{a}{c}\int\frac{t^{2m-3}\,dt}{(a+bt+ct^2)^m} + \frac{b}{c}\int\frac{t^{2m-2}\,dt}{(a+bt+ct^2)^m}, \text{ где } t=\frac{1}{x}$$
(см. 11.13).

12.5. $$\int \frac{dx}{x^2(ax^2+bx+c)} = -\frac{1}{cx} - \frac{b}{2c^2}\ln\frac{x^2}{|ax^2+bx+c|} +$$
$$+ \frac{b^2-2ac}{2c^2}\int\frac{dx}{ax^2+bx+c} \quad \text{(см. 11.1).}$$

12.6. $$\int \frac{dx}{x^2(ax^2+bx+c)^2} = \frac{2a^2cx-ab^2x+3abc-b^3}{c^2\delta(ax^2+bx+c)} - \frac{1}{c^2x} -$$
$$- \frac{b}{c^2}\ln\frac{x^2}{|ax^2+bx+c|} + \frac{b^4-6ab^2c+6a^2c^2}{c^3\delta}\int\frac{dx}{ax^2+bx+c}$$
(см. 11.1).

12.7. $$\int \frac{dx}{x^2(ax^2+bx+c)^3} = -\frac{1}{cx(ax^2+bx+c)^2} -$$
$$- \frac{3b}{c}\int\frac{dx}{x(ax^2+bx+c)^3} - \frac{5a}{c}\int\frac{dx}{(ax^2+bx+c)^3} \quad \text{(см. 12.3 и 11.3).}$$

12.8. $$\int \frac{dx}{x^2(ax^2+bx+c)^m} = -\frac{1}{cx(ax^2+bx+c)^{m-1}} -$$
$$- \frac{mb}{c}\int\frac{dx}{x(ax^2+bx+c)^m} - \frac{(2m-1)a}{c}\int\frac{dx}{(ax^2+bx+c)^m}$$
(см. 12.4 и 11.4).

12.9. $$\int \frac{dx}{x^3(ax^2+bx+c)} = \frac{b^2-ac}{2c^3}\ln\frac{x^2}{|ax^2+bx+c|} + \frac{2bx-c}{2c^2x^2} +$$
$$+ \frac{b(3c-b^2)}{2c^3}\int\frac{dx}{ax^2+bx+c} \quad \text{(см 11.1)}$$

12.10. $$\int \frac{dx}{x^3(ax^2+bx+c)^2} = \frac{3bx-c}{2c^2x^2(ax^2+bx+c)} +$$
$$+ \frac{3b^2-2ac}{c^2}\int\frac{dx}{x(ax^2+bx+c)^2} + \frac{9ab}{2c^2}\int\frac{dx}{(ax^2+bx+c)^2}$$
(см. 12.2 и 11.2).

12.11. $$\int \frac{dx}{x^3(ax^2+bx+c)^3} = \frac{4bx-c}{2c^2x^2(ax^2+bx+c)^2} +$$
$$+ \frac{6b^2-3ac}{c^2}\int\frac{dx}{x(ax^2+bx+c)^3} + \frac{10ab}{c^2}\int\frac{dx}{(ax^2+bx+c)^3}$$
(см. 12.3 и 11.3).

12.12. $$\int \frac{dx}{x^n(ax^2+bx+c)^m} = -\frac{1}{(n-1)cx^{n-1}(ax^2+bx+c)^{m-1}} -$$
$$- \frac{(n+m-2)b}{(n-1)c}\int\frac{dx}{x^{n-1}(ax^2+bx+c)^m} -$$
$$- \frac{(n+2m-3)a}{(n-1)c}\int\frac{dx}{x^{n-2}(ax^2+bx+c)^m} \qquad (n\geqslant 2).$$

ТАБЛИЦА 13

ИНТЕГРАЛЫ ВИДА

$$\int \frac{(\alpha x+\beta)^{\pm n}dx}{(ax^2+bx+c)^m}, \quad b^2-4ac\neq 0; \quad \begin{matrix} n=1,\ 2,\ 3,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots, \end{matrix}$$

13.1. $$\int \frac{\alpha x+\beta}{ax^2+bx+c}dx =$$

$$= \begin{cases} \dfrac{\alpha}{2a}\ln|ax^2+bx+c| + \dfrac{2a\beta-b\alpha}{a\sqrt{-\delta}}\operatorname{arctg}\dfrac{2ax+b}{\sqrt{-\delta}} & (\delta<0); \\ \dfrac{\alpha}{2a}\ln|ax^2+bx+c| + \dfrac{2a\beta-b\alpha}{2a\sqrt{\delta}}\ln\left|\dfrac{2ax+b-\sqrt{\delta}}{2ax+b+\sqrt{\delta}}\right| & (\delta>0). \end{cases}$$

13.2. $$\int \frac{(\alpha x+\beta)\,dx}{(ax^2+bx+c)^2} = \frac{b\alpha x+2c\alpha}{\delta(ax^2+bx+c)} + \frac{\alpha b}{\delta}\int\frac{dx}{ax^2+bx+c} + \beta\int\frac{dx}{(ax^2+bx+c)^2}$$ (см. 11.1 и 11.2).

13.3. $$\int \frac{(\alpha x+\beta)\,dx}{(ax^2+bx+c)^3} = -\frac{(b\alpha+2a\beta)x+b\beta+2c\alpha}{2\delta(ax^2+bx+c)^2} - \frac{6a\beta-3\alpha b}{2\delta}\int\frac{dx}{(ax^2+bx+c)^2}$$ (см. 11.2).

13.4. $$\int \frac{(\alpha x+\beta)^2\,dx}{ax^2+bx+c} = \frac{\alpha^2}{a}x + \frac{a\beta^2-c\alpha^2}{a}\int\frac{dx}{ax^2+bx+c} + \frac{\alpha}{a}(2a\beta-b\alpha)\int\frac{x\,dx}{ax^2+bx+c}$$ (см. 11.1 и 11.5).

13.5. $$\int \frac{(\alpha x+\beta)^n\,dx}{(ax^2+bx+c)^m} = \frac{\alpha(\alpha x+\beta)^{n-1}}{(n-2m+1)a(ax^2+bx+c)^{m-1}} - \frac{(n-1)(c\alpha^2-b\alpha\beta+a\beta^2)}{(n-2m+1)a}\int\frac{(\alpha x+\beta)^{n-2}}{(ax^2+bx+c)^m}dx - \frac{(n-m)(b\alpha-2a\beta)}{(n-2m+1)a}\int\frac{(\alpha x+\beta)^{n-1}}{(ax^2+bx+c)^m}dx \quad (n\neq 2m-1);$$

$$= \frac{(2a\beta-b\alpha-2ax)(\alpha x+\beta)^n}{(m-1)\delta(ax^2+bx+c)^{m-1}} + \frac{n(b\alpha-2a\beta)}{(m-1)\delta}\int\frac{(\alpha x+\beta)^{n-1}dx}{(ax^2+bx+c)^{m-1}} + \frac{2(n-2m+3)a}{(m-1)\delta}\int\frac{(\alpha x+\beta)^{n-1}\,dx}{(ax^2+bx+c)^{m-1}} \quad (m\neq 1).$$

13.6. $$\int \frac{dx}{(\alpha x+\beta)(ax^2+bx+c)} =$$

$$= \begin{cases} \dfrac{\alpha}{2(c\alpha^2-b\alpha\beta+a\beta^2)}\ln\left|\dfrac{(\alpha x+\beta)^2}{ax^2+bx+c}\right| - \dfrac{b\alpha-2a\beta}{(c\alpha^2-b\alpha\beta+a\beta^2)\sqrt{-\delta}}\operatorname{arctg}\dfrac{2ax+b}{\sqrt{-\delta}} & (\delta<0); \\ \dfrac{\alpha}{2(c\alpha^2-b\alpha\beta+a\beta^2)}\ln\left|\dfrac{(\alpha x+\beta)^2}{ax^2+bx+c}\right| - \dfrac{b\alpha-2a\beta}{2(c\alpha^2-b\alpha\beta+a\beta^2)\sqrt{\delta}}\ln\left|\dfrac{2ax+b-\sqrt{\delta}}{2ax+b+\sqrt{\delta}}\right| & (\delta>0). \end{cases}$$

13.7. $$\int \frac{dx}{(\alpha x+\beta)(ax^2+bx+c)^2} = \frac{1}{2(c\alpha^2-b\alpha\beta+a\beta^2)}\left[\frac{\alpha}{ax^2+bx+c} - 2a^2\int\frac{dx}{(\alpha x+\beta)(ax^2+bx+c)} - (b\alpha-2a\beta)\int\frac{dx}{(ax^2+bx+c)^2}\right]$$ (см. 13.6 и 11.2).

13.8. $$\frac{dx}{(\alpha x+\beta)^2(ax^2+bx+c)} = -\frac{1}{c\alpha^2-b\alpha\beta+a\beta^2}\left[\frac{\alpha}{\alpha x+\beta} + (\alpha b-2\alpha\beta)\int\frac{dx}{(\alpha x+\beta)(ax^2+bx+c)} + a\int\frac{dx}{ax^2+bx+c}\right]$$ (см. 13.6 и 11.1).

13.9. $$\int \frac{dx}{(\alpha x+\beta)^n(ax^2+bx+c)^m} =$$

$$= -\frac{\alpha}{(n-1)(c\alpha^2+b\alpha\beta+a\beta^2)(\alpha x+\beta)^{n-1}(ax^2+bx+c)^{m-1}} - \frac{(n+2m-3)a}{2(n-1)(c\alpha^2-\alpha\beta b+a\beta^2)}\int\frac{dx}{(\alpha x+\beta)^{n-1}(ax^2+bx+c)^m} - \frac{(n+m-2)(b\alpha-2a\beta)}{(n-1)(c\alpha^2-\alpha\beta b+a\beta^2)}\int\frac{dx}{(\alpha x+\beta)^{n-1}(ax^2+bx+c)^m} \quad (n\geqslant 2).$$

13.10. $$\int \frac{dx}{(\alpha x+\beta)^n(ax^2+bx+c)^m} =$$

$$= \frac{\alpha}{2(m-1)(c\alpha^2-\alpha\beta b+a\beta^2)(\alpha x+\beta)^{n-1}(ax^2+bx+c)^{m-1}} - \frac{b\alpha-2a\beta}{2(c\alpha^2-b\alpha\beta+a\beta^2)}\int\frac{dx}{(\alpha x+\beta)^{n-1}(ax^2+bx+c)^m} + \frac{(n+2m-3)\alpha^2}{2(m-1)(c\alpha^2-\alpha\beta b+a\beta^2)}\int\frac{dx}{(\alpha x+\beta)^n(ax^2+bx+c)^{m-1}} \quad (m\geqslant 2).$$

ТАБЛИЦА 14

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{(ax^{2k}+bx^k+c)^m}; \quad \begin{matrix} k=2,\ 3,\ 4,\ \ldots, \\ n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

14.1 $$\int \frac{dx}{ax^4+bx^2+c} =$$

$$= \begin{cases} \dfrac{2a}{\sqrt{\delta}}\left[\displaystyle\int \frac{dx}{2ax^2+b-\sqrt{\delta}} - \int \frac{dx}{2ax^2+b+\sqrt{\delta}}\right] & \text{при } b^2>4ac, \\ & \text{(см. 5.1 и 6.1);} \\ \dfrac{a}{8|a|\sqrt{|c|}\sqrt{2\sqrt{ac}-b}} \ln \dfrac{x^2\sqrt{|a|}+x\sqrt{2\sqrt{ac}-b}+\sqrt{|c|}}{x^2\sqrt{|a|}-x\sqrt{2\sqrt{ac}-b}+\sqrt{|c|}} + & \\ + \dfrac{a}{4|a|\sqrt{|c|}\sqrt{2\sqrt{ac}+b}} \operatorname{arctg} \dfrac{x^2\sqrt{|a|}-\sqrt{|c|}}{x\sqrt{2\sqrt{ac}-b}} & \text{при } b^2<4ac. \end{cases}$$

14.2 $$\int \frac{dx}{(ax^4+bx^2+c)^m} = \frac{abx^3+(b^2-2ac)x}{2(m-1)c\delta} \cdot \frac{1}{(ax^4+bx^2+c)^{m-1}} +$$
$$+ \frac{2(m-1)\delta+2ac-b^2}{2(m-1)c\delta} \int \frac{dx}{(ax^4+bx^2+c)^{m-1}} +$$
$$+ \frac{(4m-7)ab}{2(m-1)c\delta} \int \frac{x^2\,dx}{(ax^4+bx^2+c)^{m-1}} \quad (m \geqslant 2).$$

14.3. $$\int \frac{x\,dx}{ax^4+bx^2+c} = \begin{cases} \dfrac{1}{2\sqrt{\delta}} \ln \left|\dfrac{2ax^2+b-\sqrt{\delta}}{2ax^2+b+\sqrt{\delta}}\right| & \text{при } b^2>4ac; \\ \dfrac{1}{\sqrt{-\delta}} \operatorname{arctg} \dfrac{2ax^2+b}{\sqrt{-\delta}} & \text{при } b^2<4ac. \end{cases}$$

14.4. $$\int \frac{x^2\,dx}{ax^4+bx^2+c} =$$

$$= \begin{cases} \dfrac{b+\sqrt{\delta}}{\sqrt{\delta}} \displaystyle\int \frac{dx}{2ax^2+b+\sqrt{\delta}} - \frac{b-\sqrt{\delta}}{\sqrt{\delta}} \int \frac{dx}{2ax^2+b-\sqrt{\delta}} & \\ \text{при } b^2>4ac \text{ (см. 5.1 и 6.1),} & \\ \dfrac{1}{4a\lambda}\left[\displaystyle\int \frac{x\,dx}{x^2+2\lambda x+\lambda^2+\mu^2} - \int \frac{x\,dx}{x^2-2\lambda x+\lambda^2+\mu^2}\right], & \\ \text{где } \lambda = \sqrt{\frac{1}{2}\sqrt{\frac{c}{a}}-\frac{b}{4a}};\ \mu = \sqrt{\frac{1}{2}\sqrt{\frac{c}{a}}+\frac{b}{4a}} & \\ \text{при } b^2<4ac \text{ (см. 11.5).} & \end{cases}$$

14.5. $$\int \frac{x^n\,dx}{ax^4+bx^2+c} =$$

$$= \begin{cases} \dfrac{1}{4a} \ln|ax^4+bx^2+c| - \dfrac{b}{4a\sqrt{\delta}} \ln \left|\dfrac{2ax^2+b-\sqrt{\delta}}{2ax^2+b+\sqrt{\delta}}\right| & (\delta>0); \\ \dfrac{1}{4a} \ln|ax^4+bx^2+c| - \dfrac{b}{2a\sqrt{-\delta}} \operatorname{arctg} \dfrac{2ax^2+b}{\sqrt{-\delta}} & (\delta<0). \end{cases}$$

14.6. $$\int \frac{x^n\,dx}{(ax^4+bx^2+c)^m} = \frac{x^{n-3}}{(n+1-4m)a(ax^4+bx^2+c)^{m-1}} -$$
$$- \frac{(n-3)c}{(n+1-4m)a} \int \frac{x^{n-4}\,dx}{(ax^4+bx^2+c)^m} -$$
$$- \frac{(n-1-2m)b}{(n+1-4m)a} \int \frac{x^{n-2}\,dx}{(ax^4+bx^2+c)^m} \quad (n+1 \neq 4m);$$
$$= \frac{1}{a} \int \frac{x^{n-4}\,dx}{(ax^4+bx^2+c)^{m-1}} -$$
$$- \frac{c}{a} \int \frac{x^{n-4}\,dx}{(ax^4+bx^2+c)^m} - \frac{b}{a} \int \frac{x^{n-2}\,dx}{(ax^4+bx^2+c)^m}.$$

14.7. $$\int \frac{x^n\,dx}{(ax^{2k}+bx^k+c)^m} = \frac{x^{n+1-2k}}{(n+1-2km)a(ax^{2k}+bx^k+c)^{m-1}} -$$
$$- \frac{(n+1-2k)c}{(n+1-2km)a} \int \frac{x^{n-2k}\,dx}{(ax^{2k}+bx^k+c)^m} -$$
$$- \frac{(n+1-k-km)b}{(n+1-2km)a} \int \frac{x^{n-k}\,dx}{(ax^{2k}+bx^k+c)^m} \quad (n \neq 2km-1).$$

14.8. $$\int \frac{dx}{x(ax^4+bx^2+c)} =$$
$$= \frac{1}{4c} \ln \frac{x^4}{|ax^4+bx^2+c|} - \frac{b}{2c} \int \frac{x\,dx}{ax^4+bx^2+c} \text{ (см. 14.3).}$$

14.9. $$\int \frac{dx}{x^n(ax^4+bx^2+c)^m} = -\frac{1}{(n-1)cx^{n-1}(ax^4+bx^2+c)^{m-1}} -$$
$$- \frac{n+2m-3}{(n-1)c} \int \frac{dx}{x^{n-2}(ax^4+bx^2+c)^m} -$$
$$- \frac{n+2m-5}{(n-1)c} \int \frac{dx}{x^{n-4}(ax^4+bx^2+c)^m} \quad (n \geqslant 2).$$

14.10. $$\int \frac{dx}{x^n(ax^{2k}+bx^k+c)^m} = \frac{-1}{(n-1)cx^{n-1}(ax^{2k}+bx^k+c)^{m-1}} -$$
$$- \frac{(n-1-k+mk)b}{(n-1)c} \int \frac{dx}{x^{n-k}(ax^{2k}+bx^k+c)^m} -$$
$$- \frac{n-1-2k+2km}{(n-1)c} \int \frac{dx}{x^{n-2k}(ax^{2k}+bx^k+c)^m} \quad (n \geqslant 2).$$

ТАБЛИЦА 15

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n+\frac{1}{2}}\,dx}{(a\pm bx)^m}; \quad a>0,\ b>0,\quad n=0,\ 1,\ 2,\ldots,\quad m=1,\ 2,\ 3,\ldots$$

15.1. $$\int \frac{\sqrt{x}\,dx}{a+bx} = \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}}{b\sqrt{b}}\operatorname{arctg}\sqrt{\frac{bx}{a}}.$$

15.2. $$\int \frac{\sqrt{x}\,dx}{a-bx} = -\frac{2\sqrt{x}}{b} + \frac{\sqrt{a}}{b\sqrt{b}}\ln\left|\frac{\sqrt{a}+\sqrt{bx}}{\sqrt{a}-\sqrt{bx}}\right|.$$

15.3. $$\int \frac{\sqrt{x}\,dx}{(a+bx)^2} = -\frac{\sqrt{x}}{b(a+bx)} + \frac{1}{b\sqrt{ab}}\operatorname{arctg}\sqrt{\frac{bx}{a}}.$$

15.4. $$\int \frac{\sqrt{x}\,dx}{(a-bx)^2} = \frac{\sqrt{x}}{b(a-bx)} - \frac{1}{2b\sqrt{ab}}\ln\left|\frac{\sqrt{a}+\sqrt{bx}}{\sqrt{a}-\sqrt{bx}}\right|.$$

15.5. $$\int \frac{\sqrt{x}\,dx}{(a\pm bx)^m} = \frac{\pm x\sqrt{x}}{(m-1)a(a\pm bx)^{m-1}} + \frac{2m-5}{2(m-1)a}\int \frac{\sqrt{x}\,dx}{(a\pm bx)^{m-1}} \quad (m\geqslant 2).$$

15.6. $$\int \frac{x\sqrt{x}\,dx}{a+bx} = -\frac{6a\sqrt{x}+2bx\sqrt{x}}{3b^2} + \frac{2a\sqrt{a}}{b^2\sqrt{b}}\operatorname{arctg}\sqrt{\frac{bx}{a}}.$$

15.7. $$\int \frac{x\sqrt{x}\,dx}{a-bx} = -\frac{6a\sqrt{x}+2bx\sqrt{x}}{3b^2} + \frac{a\sqrt{a}}{b^2\sqrt{b}}\ln\left|\frac{\sqrt{a}+\sqrt{bx}}{\sqrt{a}-\sqrt{bx}}\right|.$$

15.8. $$\int \frac{x\sqrt{x}\,dx}{(a+bx)^2} = \frac{3a\sqrt{x}+2bx\sqrt{x}}{b^2(a+bx)} - \frac{3\sqrt{a}}{b^2\sqrt{b}}\operatorname{arctg}\sqrt{\frac{bx}{a}}.$$

15.9. $$\int \frac{x\sqrt{x}\,dx}{(a-bx)^2} = \frac{3a\sqrt{x}-2bx\sqrt{x}}{b^2(a-bx)} - \frac{3\sqrt{a}}{2b^2\sqrt{b}}\ln\left|\frac{\sqrt{a}+\sqrt{bx}}{\sqrt{a}-\sqrt{bx}}\right|.$$

15.10. $$\int \frac{x\sqrt{x}\,dx}{(a\pm bx)^m} = \pm\frac{x\sqrt{x}}{(m-1)b(a\pm bx)^{m-1}} \mp \frac{3}{2(m-1)b}\int \frac{\sqrt{x}\,dx}{(a\pm bx)^{m-1}} \quad (m\geqslant 2)$$ (см. 15.5).

15.11. $$\int \frac{x^n\sqrt{x}\,dx}{a\pm bx} = 2\sqrt{x}\sum_{\nu=0}^{n}\frac{(-1)^\nu a^\nu x^{n-\nu}}{(2n-2\nu+1)(\pm b)^{\nu+1}} + \frac{a^{n+1}}{(\mp b)^{n+1}}\int \frac{dx}{\sqrt{x}(a\pm bx)} \quad (n\geqslant 2)$$ (см. 15.13 или 15.14).

15.12. $$\int \frac{x^n\sqrt{x}\,dx}{(a\pm bx)^m} = -\frac{x^n\sqrt{x}\,dx}{(m-1)b(a\pm bx)^{m-1}} + \frac{2n+1}{2(m-1)b}\int \frac{x^n\sqrt{x}\,dx}{(a\pm bx)^{m-1}} \quad (m\geqslant 2).$$

15.13. $$\int \frac{dx}{\sqrt{x}(a+bx)} = \frac{2}{\sqrt{ab}}\operatorname{arctg}\sqrt{\frac{bx}{a}}.$$

15.14. $$\int \frac{dx}{\sqrt{x}(a-bx)} = \frac{1}{\sqrt{ab}}\ln\left|\frac{\sqrt{a}+\sqrt{bx}}{\sqrt{a}-\sqrt{bx}}\right|.$$

15.15. $$\int \frac{dx}{\sqrt{x}(a+bx)^2} = \frac{\sqrt{x}}{a(a+bx)} + \frac{1}{a\sqrt{ab}}\operatorname{arctg}\sqrt{\frac{bx}{a}}.$$

15.16. $$\int \frac{dx}{\sqrt{x}(a-bx)^2} = \frac{\sqrt{x}}{a(a-bx)} + \frac{1}{2a\sqrt{ab}}\ln\left|\frac{\sqrt{a}+\sqrt{bx}}{\sqrt{a}-\sqrt{bx}}\right|.$$

15.17. $$\int \frac{dx}{\sqrt{x}(a\pm bx)^m} = \frac{2\sqrt{x}}{a(a\pm bx)^{m-1}} \pm \frac{(2m-3)b}{a}\int \frac{\sqrt{x}\,dx}{(a\pm bx)^m}$$ (см. 15.5).

15.18. $$\int \frac{dx}{x\sqrt{x}(a+bx)} = -\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}}{a\sqrt{a}}\operatorname{arctg}\sqrt{\frac{bx}{a}}.$$

15.19. $$\int \frac{dx}{x\sqrt{x}(a-bx)} = -\frac{2}{a\sqrt{x}} + \frac{\sqrt{b}}{a\sqrt{a}}\ln\left|\frac{\sqrt{a}+\sqrt{bx}}{\sqrt{a}-\sqrt{bx}}\right|.$$

15.20. $$\int \frac{dx}{x\sqrt{x}(a+bx)^2} = -\frac{2a+3bx}{a^2(a+bx)\sqrt{x}} - \frac{3}{a^2}\sqrt{\frac{b}{a}}\operatorname{arctg}\sqrt{\frac{bx}{a}}.$$

15.21. $$\int \frac{dx}{x\sqrt{x}(a-bx)^2} = \frac{\sqrt{x}}{a(a-bx)} + \frac{1}{2a\sqrt{ab}}\ln\left|\frac{\sqrt{a}+\sqrt{bx}}{\sqrt{a}-\sqrt{bx}}\right|.$$

15.22. $$\int \frac{dx}{x^{n-\frac{1}{2}}(a\pm bx)^m} = -\frac{2\sqrt{x}}{(2n-3)ax^{n-1}(a\pm bx)^{m-1}} \mp \frac{(2m+2n-5)b}{(2n-3)a}\int \frac{dx}{x^{n-\frac{3}{2}}(a\pm bx)^m}.$$

ТАБЛИЦА 16

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{\sqrt{(a+bx)^m}}; \qquad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 3,\ 5,\ \ldots \end{matrix}$$

16.1. $$\int \frac{dx}{\sqrt{a+bx}} = \frac{2}{b}\sqrt{a+bx}.$$

16.2. $$\int \frac{dx}{\sqrt{(a+bx)^m}} = -\frac{2}{(m-2)\,b\sqrt{(a+bx)^{m-2}}}.$$

16.3. $$\int \frac{x\,dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{3b^2}(bx-2a).$$

16.4. $$\int \frac{x\,dx}{\sqrt{(a+bx)^m}} = \frac{2}{b^2\sqrt{(a+bx)^{m-2}}}\left(-\frac{a+bx}{m-4}+\frac{a}{m-2}\right).$$

16.5. $$\int \frac{x^2\,dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b^3}\left[\frac{(a+bx)^2}{5}-\frac{2a\,(a+bx)}{3}+a^2\right].$$

16.6 $$\int \frac{x^2\,dx}{\sqrt{(a+bx)^3}} = \frac{2}{b^3\sqrt{a+bx}}\left[\frac{(a+bx)^2}{3}-2a\,(a+bx)-a^2\right].$$

16.7 $$\int \frac{x^2\,dx}{\sqrt{(a+x)^m}} =$$
$$= \frac{2}{b^3\sqrt{(a+bx)^{m-2}}}\left[-\frac{(a+bx)^3}{m-6}+\frac{2\,(a+bx)}{m-4}-\frac{a^3}{m-2}\right].$$

16.8. $$\int \frac{x^3\,dx}{\sqrt{a+bx}} =$$
$$= \frac{2\sqrt{a+bx}}{b^4}\left[\frac{(a+bx)^3}{7}-\frac{3a\,(a+bx)^2}{5}+a^2\,(a+bx)-a^3\right].$$

16.9. $$\int \frac{x^3\,dx}{\sqrt{(a+bx)^3}} =$$
$$= \frac{2}{b^4\sqrt{a+bx}}\left[\frac{(a+bx)^3}{5}-a\,(a+bx)^2+3a^2\,(a+bx)+a^3\right].$$

16.10. $$\int \frac{x^3\,dx}{\sqrt{(a+bx)^m}} = \frac{2}{b^4\sqrt{(a+bx)^{m-2}}}\left[-\frac{(a+bx)^3}{m-8}+\right.$$
$$\left.+\frac{3a\,(a+bx)^2}{m-6}-\frac{3a^2\,(a+bx)}{m-4}+\frac{a^3}{m-2}\right].$$

16.11. $$\int \frac{x^n\,dx}{\sqrt{(a+bx)^m}} = \frac{2}{b^{n+1}\sqrt{(a+bx)^{m-2}}}\sum_{\nu=0}^{n}\frac{(-1)^\nu C_n^\nu (a+bx)^{n-\nu}a^\nu}{2n-2\nu-m+2}.$$

16.12. $$\int \frac{dx}{x\sqrt{a+bx}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|\dfrac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right| & \text{при } a>0; \\ \dfrac{2}{\sqrt{-a}}\operatorname{arctg}\dfrac{\sqrt{a+bx}}{\sqrt{-a}} & \text{при } a<0. \end{cases}$$

16.13. $$\int \frac{dx}{x\sqrt{(a+bx)^3}} = \frac{2}{a\sqrt{a+bx}}+\frac{1}{a}\int \frac{dx}{x\sqrt{a+bx}}$$ (см. 16.12).

16.14. $$\int \frac{dx}{x\sqrt{(a+bx)^m}} = \frac{2}{(m-2)\,a\sqrt{(a+bx)^{m-2}}}+$$
$$+\frac{1}{a}\int \frac{dx}{x\sqrt{(a+bx)^{m-2}}} \quad (m\geqslant 3).$$

16.15. $$\int \frac{dx}{x^2\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{ax}-\frac{b}{2a}\int \frac{dx}{x\sqrt{a+bx}}$$ (см. 16.12).

16.16. $$\int \frac{dx}{x^2\sqrt{(a+bx)^3}} = \frac{-a-3b}{a^2x\sqrt{a+bx}}-\frac{3b}{2a^2}\int \frac{dx}{x\sqrt{a+bx}}$$ (см. 16.12).

16.17. $$\int \frac{dx}{x^2\sqrt{(a+bx)^m}} =$$
$$= \frac{-1}{ax\sqrt{(a+bx)^{m-2}}}-\frac{mb}{2a}\int \frac{dx}{x\sqrt{(a+bx)^m}}$$ (см. 16.14).

16.18. $$\int \frac{dx}{x^3\sqrt{a+bx}} =$$
$$= \frac{3bx-2a}{4a^2x^2}\sqrt{a+bx}+\frac{3b^2}{8a^2}\int \frac{dx}{x\sqrt{a+bx}}$$ (см. 16.12).

16.19. $$\int \frac{dx}{x^n\sqrt{a+bx}} =$$
$$= -\frac{\sqrt{a+bx}}{(n-1)\,ax^{n-1}}-\frac{(2n-3)\,b}{2\,(n-1)\,a}\int \frac{dx}{x^{n-1}\sqrt{a+bx}} \quad (n\geqslant 2).$$

16.20. $$\int \frac{dx}{x^n\sqrt{(a+bx)^m}} = -\frac{1}{(n-1)\,x^{n-1}\sqrt{(a+bx)^m}}-$$
$$-\frac{mb}{2\,(n-1)}\int \frac{dx}{x^{n-1}\sqrt{(a+bx)^{m+2}}} \quad (n\geqslant 2);$$
$$= \frac{-2}{(m-2)\,bx^n\sqrt{(a+bx)^{m-2}}}-$$
$$-\frac{2n}{(m-2)\,b}\int \frac{dx}{x^{n+1}\sqrt{(a+bx)^{m-2}}} \quad (m\geqslant 3).$$

ТАБЛИЦА 17

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\sqrt{(a+bx)^m}\,dx: \qquad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 3,\ 5,\ \ldots \end{matrix}$$

17.1. $$\int \sqrt{a+bx}\,dx=\frac{2\sqrt{(a+bx)^3}}{3b}.$$

17.2. $$\int \sqrt{(a+bx)^m}\,dx=\frac{2\sqrt{(a+bx)^{m+2}}}{(m+2)\,b}.$$

17.3. $$\int x\sqrt{a+bx}\,dx=\frac{2}{b^2}\left[\frac{\sqrt{(a+bx)^5}}{5}-\frac{a\sqrt{(a+bx)^3}}{3}\right].$$

17.4. $$\int x\sqrt{(a+bx)^m}\,dx=\frac{2}{b^2}\left[\frac{\sqrt{(a+bx)^{m+4}}}{m+4}-\frac{a\sqrt{(a+bx)^{m+2}}}{m+2}\right].$$

17.5. $$\int x^2\sqrt{a+bx}\,dx=$$
$$=\frac{2}{b^3}\left[\frac{\sqrt{(a+bx)^7}}{7}-\frac{2a\sqrt{(a+bx)^5}}{5}+\frac{a^2\sqrt{(a+bx)^3}}{3}\right].$$

17.6. $$\int x^2\sqrt{(a+bx)^3}\,dx=$$
$$=\frac{2}{b^3}\left[\frac{\sqrt{(a+bx)^9}}{9}-\frac{2a\sqrt{(a+bx)^7}}{7}+\frac{a^2\sqrt{(a+bx)^5}}{5}\right].$$

17.7. $$\int x^2\sqrt{(a+bx)^m}\,dx=$$
$$=\frac{2}{b^3}\left[\frac{\sqrt{(a+bx)^{m+6}}}{m+6}-\frac{2a\sqrt{(a+bx)^{m+4}}}{m+4}+\frac{a^2\sqrt{(a+bx)^{m+2}}}{m+2}\right].$$

17.8. $$\int x^3\sqrt{a+bx}\,dx=$$
$$=\frac{2\sqrt{(a+bx)^3}}{b^4}\left[\frac{(a+bx)^3}{9}-\frac{3(a+bx)^2a}{7}+\frac{3(a+bx)a^2}{5}-\frac{a^3}{3}\right].$$

17.9. $$\int x^3\sqrt{(a+bx)^3}\,dx=$$
$$=\frac{2\sqrt{(a+bx)^5}}{b^4}\left[\frac{(a+bx)^3}{11}-\frac{3(a+bx)^2a}{9}+\frac{3(a+bx)a^2}{7}-\frac{a^3}{5}\right].$$

17.10. $$\int x^3\sqrt{(a+bx)^m}\,dx=\frac{2\sqrt{(a+bx)^{m+2}}}{b^4}\left[\frac{(a+bx)^3}{8+m}-\right.$$
$$\left.-\frac{3(a+bx)^2a}{6+m}+\frac{3(a+bx)a^2}{4+m}-\frac{a^3}{2+m}\right].$$

17.11. $$\int x^n\sqrt{(a+bx)^m}\,dx=$$
$$=\frac{2\sqrt{(a+bx)^{m+2}}}{b^{n+1}}\sum_{\nu=0}^{n}\frac{(-1)^\nu C_n^\nu(a+bx)^{n-\nu}a^\nu}{2n-2\nu+m+2}.$$

17.12. $$\int\frac{\sqrt{a+bx}}{x}\,dx=$$
$$=\begin{cases}2\sqrt{a+bx}+\sqrt{a}\,\ln\left|\dfrac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right| & \text{при } a>0;\\ 2\sqrt{a+bx}+\dfrac{2a}{\sqrt{-a}}\operatorname{arctg}\dfrac{\sqrt{a+bx}}{\sqrt{-a}} & \text{при } a<0.\end{cases}$$

17.13. $$\int\frac{\sqrt{(a+bx)^3}}{x}\,dx=\frac{2\sqrt{(a+bx)^3}}{3}+$$
$$+2a\sqrt{a+bx}+a^2\int\frac{dx}{x\sqrt{a+bx}}\quad(\text{см. } 16.12).$$

17.14. $$\int\frac{\sqrt{(a+bx)^m}}{x}\,dx=$$
$$=\sum_{\nu=0}^{\frac{m-3}{2}}\frac{(a+bx)^{\frac{m+1}{2}-\nu}}{(m-2\nu)}a^\nu+a^{\frac{m-1}{2}}\int\frac{\sqrt{a+bx}}{x}\,dx\quad(\text{см. } 17.12).$$

17.15. $$\int\frac{\sqrt{a+bx}}{x^2}\,dx=-\frac{\sqrt{a+bx}}{x}+\frac{b}{2}\int\frac{dx}{x\sqrt{a+bx}}\quad(\text{см. } 16.12).$$

17.16. $$\int\frac{\sqrt{(a+bx)^m}\,dx}{x^2}\,dx=$$
$$=-\frac{\sqrt{(a+bx)^{m+2}}}{ax}+\frac{mb}{2a}\int\frac{\sqrt{(a+bx)^m}}{x}\,dx\quad(\text{см. } 17.14).$$

17.17. $$\int\frac{\sqrt{a+bx}}{x^3}\,dx=$$
$$=-\frac{(2a+bx)\sqrt{a+bx}}{4ax^2}-\frac{b^2}{8a}\int\frac{dx}{x\sqrt{a+bx}}\quad(\text{см. } 16.12).$$

17.18. $$\int\frac{\sqrt{(a+bx)^m}}{x^3}\,dx=$$
$$=-\frac{\sqrt{(a+bx)^{m+2}}}{2ax^2}+\frac{(m-2)\,b}{4a}\int\frac{\sqrt{(a+bx)^m}}{x^2}\,dx.$$

17.19. $$\int\frac{\sqrt{a+bx}}{x^n}\,dx=$$
$$=-\frac{\sqrt{(a+bx)^3}}{(n-1)\,ax^{n-1}}+\frac{(5-2n)\,b}{2(n-1)\,a}\int\frac{\sqrt{a+bx}}{x^{n-1}}\,dx\quad(n\geqslant 2).$$

17.20. $$\int\frac{\sqrt{(a+bx)^m}}{x^n}\,dx=-\frac{\sqrt{(a+bx)^{m+2}}}{(n-1)\,ax^{n-1}}+$$
$$+\frac{(m-2n+4)\,b}{2(n-1)\,a}\int\frac{\sqrt{(a+bx)^m}}{x^{n-1}}\,dx\quad(n\geqslant 2).$$

ТАБЛИЦА 18

ИНТЕГРАЛЫ ВИДА

$$\int \sqrt{(a+bx)^{\pm n}\,(c+fx)^{\pm m}}\,dx;$$

$af-bc\neq 0$, $n=1, 3, 5, \ldots$, $m=1, 3, 5, \ldots$

18.1. $$\int \frac{dx}{\sqrt{(a+bx)\,(c+fx)}}=\begin{cases}\dfrac{-1}{\sqrt{-bf}}\arcsin\dfrac{2bfx+af+bc}{\Delta} & \text{при } bf<0;\\[2ex] \dfrac{2}{\sqrt{bf}}\ln\left|\sqrt{bf\,(a+bx)}+b\sqrt{c+fx}\right| & \text{при } bf>0.\end{cases}$$

18.2. $$\int \frac{dx}{\sqrt{(a+bx)\,(c+fx)^3}}=-\frac{2}{\Delta}\sqrt{\frac{a+bx}{c+fx}}.$$

18.3. $$\int \frac{dx}{\sqrt{(a+bx)\,(c+fx)^5}}=-\frac{2}{\Delta}\left[\frac{1}{3}\sqrt{\frac{a+bx}{(c+fx)^3}}+b\sqrt{\frac{a+bx}{c+fx}}\right].$$

18.4. $$\int \frac{dx}{\sqrt{(a+bx)\,(c+fx)^m}}=-\frac{2}{(m-2)\,\Delta}\left[\sqrt{\frac{a+bx}{(c+fx)^{m-2}}}+\frac{(m-3)\,b}{2}\int\frac{dx}{\sqrt{(a+bx)\,(c+fx)^{m-2}}}\right] \quad (m\geqslant 3).$$

18.5. $$\int\sqrt{\frac{a+bx}{c+fx}}\,dx=\begin{cases}\dfrac{\sqrt{(a+bx)\,(c+fx)}}{f}+\dfrac{\Delta}{2f\sqrt{-bf}}\arcsin\dfrac{2bfx+af+bc}{\Delta} & \text{при } bf<0;\\[2ex] \dfrac{\sqrt{(a+bx)\,(c+fx)}}{f}-\dfrac{\Delta}{f\sqrt{bf}}\ln\left|\sqrt{bf\,(a+bx)}+b\sqrt{c+fx}\right| & \text{при } bf>0.\end{cases}$$

18.6. $$\int\sqrt{\frac{(a+bx)^3}{c+fx}}\,dx=\frac{\sqrt{(a+bx)^3\,(c+fx)}}{2f}-\frac{3\Delta}{4f}\int\sqrt{\frac{a+bx}{c+fx}}\,dx$$ (см. 18.5).

18.7. $$\int\sqrt{\frac{(a+bx)^5}{c+fx}}\,dx=\frac{\sqrt{(a+bx)^5\,(c+fx)}}{3f}-\frac{5\Delta\sqrt{(a+bx)^3\,(c+fx)}}{4f}+\frac{15\Delta^2}{8f}\int\sqrt{\frac{a+bx}{c+fx}}\,dx$$ (см. 18.5).

18.8. $$\int\sqrt{\frac{(a+bx)^n}{c+fx}}\,dx=\frac{\sqrt{(a+bx)^n\,(c+fx)}}{(n+1)\,f}-\frac{n\Delta}{(n+1)\,f}\int\sqrt{\frac{(a+bx)^{n-2}}{c+fx}}\,dx.$$

18.9. $$\int\sqrt{\frac{a+bx}{(c+fx)^3}}\,dx=\frac{2}{f}\sqrt{\frac{a+bx}{c+fx}}-\frac{1}{f}\int\frac{dx}{\sqrt{(a+bx)\,(c+fx)}}$$ (см. 18.1).

18.10. $$\int\sqrt{\frac{a+bx}{(c+fx)^5}}\,dx=-\frac{1}{f}\sqrt{\frac{a+bx}{(c+fx)^3}}-\frac{\Delta}{f^2}\sqrt{\frac{a+bx}{c+fx}}+\frac{\Delta}{2f^2}\int\frac{dx}{\sqrt{(a+bx)\,(c+fx)}}$$ (см 18.1).

18.11. $$\int\sqrt{\frac{a+bx}{(c+fx)^m}}\,dx=-\frac{2}{(m-3)\,f}\sqrt{\frac{a+bx}{(c+fx)^{m-2}}}-\frac{\Delta}{(m-3)\,f}\int\frac{dx}{\sqrt{(a+bx)\,(c+fx)^m}}\ (m\geqslant 3)$$ (см. 18.4).

18.12. $$\int\sqrt{(a+bx)\,(c+fx)}\,dx=\begin{cases}\dfrac{af+bc+2bfx}{4bf}\sqrt{(a+bx)\,(c+fx)}+\dfrac{\Delta^2}{8bf\sqrt{-bf}}\arcsin\dfrac{af+bc+2bfx}{|af-bc|} & \text{при } bf<0;\\[2ex] \dfrac{af+bc+2bfx}{4bd}\sqrt{(a+bx)\,(c+dx)}-\dfrac{\Delta^2}{4bf\sqrt{bf}}\ln\left|\sqrt{bf\,(a+bx)}+b\sqrt{c+fx}\right| & \text{при } bf>0.\end{cases}$$

18.13. $$\int\sqrt{(a+bx)^2\,(c+fx)}\,dx=\frac{\sqrt{(a+bx)^5\,(c+fx)}}{3b}+\frac{\Delta}{6b}\int\sqrt{\frac{(a+bx)^3}{c+fx}}\,dx$$ (см. 18.6).

18.14. $$\int\sqrt{(a+bx)^5\,(c+fx)}\,dx=\frac{\sqrt{(a+bx)^7\,(c+fx)}}{4b}+\frac{\Delta}{8b}\int\sqrt{\frac{(a+bx)^5}{c+fx}}\,dx$$ (см. 18.7).

18.15. $$\int\sqrt{(a+bx)^n\,(c+fx)}\,dx=\frac{2\sqrt{(a+bx)^{n+2}\,(c+fx)}}{(n+3)\,b}+\frac{\Delta}{(n+3)\,b}\int\sqrt{\frac{(a+bx)^n}{c+fx}}\,dx$$ (см. 18.8).

ТАБЛИЦА 19

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n\,dx}{\sqrt{(a^2+b^2x^2)^m}};\quad a>0,\quad b>0,\quad n=0,\ 1,\ 2,\ \ldots,\quad m=1,\ 3,\ 5,\ \ldots$$

19.1. $$\int \frac{dx}{\sqrt{a^2+b^2x^2}} = \frac{1}{b}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.2. $$\int \frac{dx}{\sqrt{(a^2+b^2x^2)^3}} = \frac{x}{a^2\sqrt{a^2+b^2x^2}}.$$

19.3. $$\int \frac{dx}{\sqrt{(a^2+b^2x^2)^5}} = \frac{1}{a^4}\left[\frac{x}{\sqrt{a^2+b^2x^2}} - \frac{b^2x^3}{3\sqrt{(a^2+b^2x^2)^3}}\right].$$

19.4. $$\int \frac{dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m-1}}\sum_{\nu=0}^{\frac{m-3}{2}} \frac{(-1)^\nu C^\nu_{\frac{m-3}{2}} b^{2\nu}x^{2\nu+1}}{(2\nu+1)\sqrt{(a^2+b^2x^2)^{2\nu+1}}} \quad (m\geqslant 3).$$

19.5. $$\int \frac{x\,dx}{\sqrt{a^2+b^2x^2}} = \frac{1}{b^2}\sqrt{a^2+b^2x^2}.$$

19.6. $$\int \frac{x\,dx}{\sqrt{(a^2+b^2x^2)^3}} = -\frac{1}{b^2\sqrt{a^2+b^2x^2}}.$$

19.7. $$\int \frac{x\,dx}{\sqrt{(a^2+b^2x^2)^m}} = -\frac{1}{(m-2)\,b^2\sqrt{(a^2+b^2x^2)^{m-2}}} \quad (m\geqslant 3).$$

19.8. $$\int \frac{x^2\,dx}{\sqrt{a^2+b^2x^2}} = \frac{x\sqrt{a^2+b^2x^2}}{2b^2} - \frac{a^2}{2b^3}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.9. $$\int \frac{x^2\,dx}{\sqrt{(a^2+b^2x^2)^3}} = -\frac{x}{b^2\sqrt{a^2+b^2x^2}} + \frac{1}{b^3}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.10. $$\int \frac{x^2\,dx}{\sqrt{(a^2+b^2x^2)^5}} = \frac{1}{3a^2}\frac{x^3}{\sqrt{(a^2+b^2x^2)^3}}.$$

19.11. $$\int \frac{x^2\,dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m-3}}\sum_{\nu=0}^{\frac{m-5}{2}} \frac{(-1)^\nu C^\nu_{\frac{m-5}{2}} b^{2\nu}x^{2\nu+3}}{(2\nu+3)\sqrt{(a^2+b^2x^2)^{2\nu+3}}} \quad (m\geqslant 5).$$

19.12. $$\int \frac{x^3\,dx}{\sqrt{a^2+b^2x^2}} = \frac{\sqrt{(a^2+b^2x^2)^3}}{3b^4} - \frac{a^2}{b^4}\sqrt{a^2+b^2x^2}.$$

19.13. $$\int \frac{x^3\,dx}{\sqrt{(a^2+b^2x^2)^3}} = \frac{\sqrt{a^2+b^2x^2}}{b^4} + \frac{a^2}{b^4\sqrt{a^2-b^2x^2}}.$$

19.14. $$\int \frac{x^3\,dx}{\sqrt{(a^2+b^2x^2)^m}} = -\frac{1}{(m-4)\,b^4\sqrt{(a^2+b^2x^2)^{m-4}}} + \frac{a^2}{(m-2)\,b^4\sqrt{(a^2+b^2x^2)^{m-2}}}.$$

19.15. $$\int \frac{x^4\,dx}{\sqrt{a^2+b^2x^2}} = \frac{x^3\sqrt{a^2+b^2x^2}}{4b^2} - \frac{3a^2x}{8b^4}\sqrt{a^2+b^2x^2} + \frac{3a^4}{8b^5}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.16. $$\int \frac{x^4\,dx}{\sqrt{(a^2+b^2x^2)^3}} = \frac{x\sqrt{a^2+b^2x^2}}{2b^4} + \frac{a^2x}{b^4\sqrt{a^2+b^2x^2}} - \frac{3a^2}{2b^5}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.17. $$\int \frac{x^4\,dx}{\sqrt{(a^2+b^2x^2)^5}} = -\frac{3a^2x+4b^2x^3}{3b^4\sqrt{(a^2+b^2x^2)^3}} + \frac{1}{b^5}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.18. $$\int \frac{x^4\,dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m-5}}\sum_{\nu=0}^{\frac{m-7}{2}} \frac{(-1)^\nu C^\nu_{\frac{m-7}{2}} b^{2\nu}x^{2\nu+5}}{(2\nu+5)\sqrt{(a^2+b^2x^2)^{2\nu+5}}} \quad (m\geqslant 7).$$

19.19. $$\int \frac{x^{2k+1}dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{2}\int \frac{t^k\,dt}{\sqrt{(a^2+b^2t)^m}},\ \text{где } t=x^2 \quad (\text{см. } 16.11).$$

19.20. $$\int \frac{x^{2k}\,dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m-2k-1}}\sum_{\nu=0}^{\frac{m-2k-3}{2}} \frac{(-1)^\nu C^\nu_{\frac{m-2k-3}{2}} b^{2\nu}x^{2\nu+2k+1}}{(2\nu+2k+1)\sqrt{(a^2+b^2x^2)^{2\nu+2k+1}}} \quad (m\geqslant 2k+3).$$

19.21. $$\int \frac{x^n\,dx}{\sqrt{(a^2+b^2x^2)^m}} = -\frac{x^{n-1}}{(m-2)\,b^2\sqrt{(a^2+b^2x^2)^{m-2}}} + \frac{n-1}{(m-2)\,b^2}\int \frac{x^{n-2}\,dx}{\sqrt{(a^2+b^2x^2)^{m-2}}}.$$

ТАБЛИЦА 20

ИНТЕГРАЛЫ ВИДА

$$\int \frac{dx}{x^n\sqrt{(a^2+b^2x^2)^m}};\quad a>0,\quad n=1,\ 2,\ 3,\ \ldots,\quad b>0,\quad m=1,\ 3,\ 5,\ \ldots$$

20.1. $$\int \frac{dx}{x\sqrt{a^2+b^2x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

20.2. $$\int \frac{dx}{x\sqrt{(a^2+b^2x^2)^3}} = \frac{1}{a^2\sqrt{a^2+b^2x^2}} - \frac{1}{a^3}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

20.3. $$\int \frac{dx}{x\sqrt{(a^2+b^2x^2)^m}} = \sum_{\nu=1}^{\frac{m-1}{2}} \frac{1}{(m-2\nu)\,a^{2\nu}\sqrt{(a^2+b^2x^2)^{m-2\nu}}} - \frac{1}{a^m}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right| \quad (m\geqslant 3)$$

20.4. $$\int \frac{dx}{x^2\sqrt{a^2+b^2x^2}} = -\frac{\sqrt{a^2+b^2x^2}}{a^2x}.$$

20.5. $$\int \frac{dx}{x^2\sqrt{(a^2+b^2x^2)^3}} = -\frac{a^2+2b^2x^2}{a^4x\sqrt{a^2+b^2x^2}}.$$

20.6. $$\int \frac{dx}{x^2\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m+1}}\sum_{\nu=0}^{\frac{m-1}{2}} \frac{(-1)^\nu C^\nu_{\frac{m-1}{2}} b^{2\nu}x^{2\nu-1}}{(2\nu-1)\sqrt{(a^2+b^2x^2)^{2\nu-1}}}.$$

20.7. $$\int \frac{dx}{x^3\sqrt{a^2+b^2x^2}} = -\frac{\sqrt{a^2+b^2x^2}}{2a^2x^2} + \frac{b^2}{2a^3}\ln\left|\frac{a+\sqrt{a+b^2x^2}}{bx}\right|.$$

20.8. $$\int \frac{dx}{x^3\sqrt{(a^2+b^2x^2)^3}} = -\frac{a^2+3b^2x^2}{2a^4x^2\sqrt{a^2+b^2x^2}} + \frac{3b^2}{2a^5}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

20.9. $$\int \frac{dx}{x^3\sqrt{(a^2+b^2x^2)^m}} = -\frac{1}{(m-2)\,b^2x^4\sqrt{(a^2+b^2x^2)^{m-2}}} - \frac{4}{(m-2)\,b^2}\int \frac{dx}{x^3\sqrt{(a^2+b^2x^2)^{m-2}}} \quad (m\geqslant 3).$$

20.10. $$\int \frac{dx}{x^4\sqrt{a^2+b^2x^2}} = \frac{2b^2x^2-a^2}{a^4x}\sqrt{a^2+b^2x^2}.$$

20.11. $$\int \frac{dx}{x^4\sqrt{(a^2+b^2x^2)^3}} = -\frac{8b^4x^4+4a^2b^2x^2-a^4}{3a^6x^3\sqrt{a^2+b^2x^2}}.$$

20.12. $$\int \frac{dx}{x^4\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m+3}} + \sum_{\nu=0}^{\frac{m+1}{2}} \frac{(-1)^\nu}{2\nu-3} C^\nu_{\frac{m+1}{2}} b^{2\nu}\left(\frac{x}{\sqrt{a^2+b^2x^2}}\right)^{2\nu-3}.$$

20.13. $$\int \frac{dx}{x^5\sqrt{a^2+b^2x^2}} = \frac{3b^2x^2-2a^2}{8a^4x^4}\sqrt{a^2+b^2x^2} - \frac{3b^4}{8a^5}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

20.14. $$\int \frac{dx}{x^5\sqrt{(a^2+b^2x^2)^3}} = \frac{15b^4x^4+5a^2b^2x^2-2a^4}{8a^6x^4\sqrt{a^2+b^2x^2}} - \frac{15b^4}{8a^7}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

20.15. $$\int \frac{dx}{x^5\sqrt{(a^2+b^2x^2)^m}} = -\frac{1}{(m-2)\,b^2x^6\sqrt{(a^2+b^2x^2)^{m-2}}} - \frac{6}{(m-2)\,b^2}\int \frac{dx}{x^7\sqrt{(a^2+b^2x^2)^{m-2}}} \quad (m\geqslant 3).$$

20.16. $$\int \frac{dx}{x^6\sqrt{a^2+b^2x^2}} = \frac{1}{a^6}\left[-\frac{b^4\sqrt{a^2+b^2x^2}}{x} + \frac{2b^2\sqrt{(a^2+b^2x^2)^3}}{3x^3} - \frac{\sqrt{(a^2+b^2x^2)^5}}{5x^5}\right].$$

20.17. $$\int \frac{dx}{x^6\sqrt{(a^2+b^2x^2)^3}} = \frac{1}{a^6}\left[-\frac{\sqrt{(a^2+b^2x^2)^5}}{5x^5} + \frac{b^2\sqrt{(a^2+b^2x^2)^3}}{x^3} - \frac{3b^4\sqrt{a^2+b^2x^2}}{x} - \frac{b^6x}{\sqrt{a^2+b^2x^2}}\right].$$

20.18. $$\int \frac{dx}{x^6\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m+5}} - \sum_{\nu=0}^{\frac{m+3}{2}} \frac{(-1)^\nu}{2\nu-5} C^\nu_{\frac{m+3}{2}} b^{2\nu}\left(\frac{x}{\sqrt{a^2+b^2x^2}}\right)^{2\nu-5}.$$

20.19. $$\int \frac{dx}{x^n\sqrt{(a^2+b^2x^2)^m}} = -\frac{1}{(m-2)\,b^2x^{n+1}\sqrt{(a^2+b^2x^2)^{m-2}}} - \frac{n+1}{(m-2)\,b^2}\int \frac{dx}{x^{n+2}\sqrt{(a^2+b^2x^2)^{m-2}}} \quad (m\geqslant 3).$$

ТАБЛИЦА 21

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\sqrt{(a^2+b^2x^2)^m}\,dx;\quad \begin{matrix} a>0, & n=0,\ 1,\ 2,\ \ldots,\\ b>0, & m=1,\ 3,\ 5,\ \ldots \end{matrix}$$

21.1. $$\int \sqrt{a^2+b^2x^2}\,dx=\frac{x\sqrt{a^2+b^2x^2}}{2}+\frac{a^2}{2b}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

21.2. $$\int \sqrt{(a^2+b^2x^2)^3}\,dx=$$
$$=\frac{5a^2x+2b^2x^3}{8}\sqrt{a^2+b^2x^2}+\frac{3a^4}{8b}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|$$

21.3. $$\int \sqrt{(a^2+b^2x^2)^m}\,dx=$$
$$=\frac{x}{m+1}\sqrt{(a^2+b^2x^2)^m}+\frac{ma^2}{m+1}\int\sqrt{(a^2+b^2x^2)^{m-2}}\,dx$$

21.4. $$\int x\sqrt{(a^2+b^2x^2)^m}\,dx=\frac{\sqrt{(a^2+b^2x^2)^{m+2}}}{(m+2)\,b^2}.$$

21.5. $$\int x^2\sqrt{a^2+b^2x^2}\,dx=$$
$$=\frac{a^2x+2b^2x^3}{8b^2}\sqrt{a^2+b^2x^2}-\frac{a^4}{8b^3}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|$$

21.6. $$\int x^2\sqrt{(a^2+b^2x^2)^3}\,dx=\frac{x\sqrt{(a^2+b^2x^2)^5}}{6b^2}-\frac{a^2x\sqrt{(a^2+b^2x^2)^3}}{24b^2}-$$
$$-\frac{a^4x\sqrt{a^2+b^2x^2}}{16b^2}-\frac{a^6}{16b^3}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|$$

21.7. $$\int x^2\sqrt{(a^2+b^2x^2)^m}\,dx=\frac{x\sqrt{(a^2+b^2x^2)^{m+2}}}{(m+3)\,b^2}-$$
$$-\frac{a^2}{(m+3)\,b^2}\int\sqrt{(a^2+b^2x^2)^m}\,dx\quad\text{(см. 21.3)}$$

21.8. $$\int x^3\sqrt{a^2+b^2x^2}\,dx=\frac{\sqrt{(a^2+b^2x^2)^5}}{5b^4}-\frac{a^2\sqrt{(a^2+b^2x^2)^3}}{3b^4}.$$

21.9. $$\int x^3\sqrt{(a^2+b^2x^2)^m}\,dx=\frac{\sqrt{(a^2+b^2x^2)^{m+4}}}{(m+4)\,b^4}-\frac{a^2\sqrt{(a^2+b^2x^2)^{m+2}}}{(m+2)\,b^4}.$$

21.10. $$\int x^4\sqrt{a^2+b^2x^2}\,dx=\frac{x^3\sqrt{(a^2+b^2x^2)^3}}{6b^2}-\frac{a^2x\sqrt{(a^2+b^2x^2)^3}}{8b^4}+$$
$$+\frac{a^4x\sqrt{a^2+b^2x^2}}{16b^4}+\frac{a^6}{16b^5}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|$$

21.11. $$\int x^n\sqrt{(a^2+b^2x^2)^m}\,dx=\frac{x^{n-1}\sqrt{(a^2+b^2x^2)^{m+2}}}{(m+n+1)\,b^2}-$$
$$-\frac{(n-1)\,a^2}{(m+n+1)\,b^2}\int x^{n-2}\sqrt{(a^2+b^2x^2)^m}\,dx.$$

21.12. $$\int\frac{\sqrt{a^2+b^2x^2}}{x}\,dx=\sqrt{a^2+b^2x^2}-a\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

21.13. $$\int\frac{\sqrt{(a^2+b^2x^2)^3}}{x}\,dx=$$
$$=\frac{4a^2+b^2x^2}{3}\sqrt{a^2+b^2x^2}-a^3\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

21.14. $$\int\frac{\sqrt{(a^2+b^2x^2)^m}}{x}\,dx=\frac{\sqrt{(a^2+b^2x^2)^m}}{m}+a^2\int\frac{\sqrt{(a^2+b^2x^2)^{m-2}}}{x}\,dx.$$

21.15. $$\int\frac{\sqrt{a^2+b^2x^2}}{x^2}\,dx=-\frac{\sqrt{a^2+b^2x^2}}{x}+b\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

21.16. $$\int\frac{\sqrt{(a^2+b^2x^2)^3}}{x^2}\,dx=$$
$$=\frac{b^2x^2-2a^2}{2x}\sqrt{a^2+b^2x^2}+\frac{3ba^2}{2}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

21.17. $$\int\frac{\sqrt{(a^2+b^2x^2)^m}}{x^2}\,dx=$$
$$=-\frac{\sqrt{(a^2+b^2x^2)^m}}{x}+mb^2\int x\sqrt{(a^2+b^2x^2)^{m-2}}\,dx\quad\text{(см. 21.4)}.$$

21.18. $$\int\frac{\sqrt{a^2+b^2x^2}}{x^3}\,dx=-\frac{\sqrt{a^2+b^2x^2}}{2x^2}-\frac{b^2}{2a}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

21.19. $$\int\frac{\sqrt{(a^2+b^2x^2)^3}}{x^3}\,dx=$$
$$=\frac{2b^2x^2-a^2}{2x^2}\sqrt{a^2+b^2x^2}-\frac{3}{2}ab^2\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|.$$

21.20. $$\int\frac{\sqrt{a^2+b^2x^2}}{x^4}\,dx=-\frac{\sqrt{(a^2+b^2x^2)^3}}{3a^2b^2x^3}.$$

21.21. $$\int\frac{\sqrt{(a^2+b^2x^2)^m}}{x^n}\,dx=$$
$$=a^2\int\frac{\sqrt{(a^2+b^2x^2)^{m-2}}}{x^n}\,dx+b^2\int\frac{\sqrt{(a^2+b^2x^2)^{m-2}}}{x^{n-2}}\,dx,$$

ТАБЛИЦА 22

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n\,dx}{\sqrt{(a^2-b^2x^2)^m}}; \quad \begin{matrix} a>0, & n=0,\ 1,\ 2,\ \ldots, \\ b>0, & m=1,\ 3,\ 5,\ \ldots \end{matrix}$$

22.1. $$\int \frac{dx}{\sqrt{a^2-b^2x^2}} = \frac{1}{b}\arcsin\frac{bx}{a}.$$

22.2. $$\int \frac{dx}{\sqrt{(a^2-b^2x^2)^3}} = \frac{x}{a^2\sqrt{a^2-b^2x^2}}.$$

22.3 $$\int \frac{dx}{\sqrt{(a^2-b^2x^2)^5}} = \frac{3a^2-2b^2x^2}{3a^4\sqrt{(a^2-b^2x^2)^3}}.$$

22.4 $$\int \frac{dx}{\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{a^{m-1}}\sum_{\nu=0}^{\frac{m-3}{2}} \frac{C^{\nu}_{\frac{m-3}{2}}\, b^{2\nu}x^{2\nu+1}}{(2\nu+1)\sqrt{(a^2-b^2x^2)^{2\nu+1}}} \quad (m\geqslant 3).$$

22.5. $$\int \frac{x\,dx}{\sqrt{a^2-b^2x^2}} = -\frac{\sqrt{a^2-b^2x^2}}{b^2}.$$

22.6. $$\int \frac{x\,dx}{\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{(m-2)\,b^2\sqrt{(a^2-b^2x^2)^{m-2}}}.$$

22.7. $$\int \frac{x^2\,dx}{\sqrt{a^2-b^2x^2}} = -\frac{x\sqrt{a^2-b^2x^2}}{2b^2} + \frac{a^2}{2b^3}\arcsin\frac{bx}{a}.$$

22.8. $$\int \frac{x^2\,dx}{\sqrt{(a^2-b^2x^2)^3}} = \frac{x}{b^2\sqrt{a^2-b^2x^2}} - \frac{1}{b^3}\arcsin\frac{bx}{a}.$$

22.9. $$\int \frac{x^2\,dx}{\sqrt{(a^2-b^2x^2)^5}} = \frac{x^3}{3a^2\sqrt{(a^2-b^2x^2)^3}}.$$

22.10. $$\int \frac{x^2\,dx}{\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{a^{m-3}}\sum_{\nu=0}^{\frac{m-5}{2}} \frac{C^{\nu}_{\frac{m-5}{2}}\, b^{2\nu}x^{2\nu+3}}{(2\nu+3)\sqrt{(a^2-b^2x^2)^{2\nu+3}}} \quad (m\geqslant 5).$$

22.11. $$\int \frac{x^3\,dx}{\sqrt{a^2-b^2x^2}} = -\frac{2a^2+b^2x^2}{3b^4}\sqrt{a^2-b^2x^2}.$$

22.12. $$\int \frac{x^3\,dx}{\sqrt{(a^2-b^2x^2)^3}} = \frac{2a^2-b^2x^2}{b^4\sqrt{a^2-b^2x^2}}.$$

22.13. $$\int \frac{x^3\,dx}{\sqrt{(a^2-b^2x^2)^m}} = -\frac{1}{(m-4)\,b^4\sqrt{(a^2-b^2x^2)^{m-4}}} + \frac{a^2}{(m-2)\,b^4\sqrt{(a^2-b^2x^2)^{m-2}}}.$$

22.14. $$\int \frac{x^4\,dx}{\sqrt{a^2-b^2x^2}} = -\frac{2b^2x^3+3a^2x}{8b^4}\sqrt{a^2-b^2x^2} + \frac{3a^4}{8b^5}\arcsin\frac{bx}{a}.$$

22.15. $$\int \frac{x^4\,dx}{\sqrt{(a^2-b^2x^2)^3}} = \frac{x\sqrt{a^2-b^2x^2}}{2b^4} + \frac{a^2x}{b^4\sqrt{a^2-b^2x^2}} - \frac{a^2}{b^5}\arcsin\frac{bx}{a}.$$

22.16. $$\int \frac{x^4\,dx}{\sqrt{(a^2-b^2x^2)^5}} = -\frac{x}{b^4\sqrt{a^2-b^2x^2}} + \frac{x^3}{3b^2\sqrt{(a^2-b^2x^2)^3}} + \frac{1}{b^5}\arcsin\frac{bx}{a}.$$

22.17. $$\int \frac{x^4\,dx}{\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{a^{m-5}}\sum_{\nu=0}^{\frac{m-7}{2}} \frac{C^{\nu}_{\frac{m-7}{2}}\, b^{2\nu}x^{2\nu+5}}{(2\nu+5)\sqrt{(a^2-b^2x^2)^{2\nu+5}}} \quad (m\geqslant 7).$$

22.18. $$\int \frac{x^n\,dx}{\sqrt{a^2-b^2x^2}} = -\frac{x^{n-1}\sqrt{a^2-b^2x^2}}{b^2} + \frac{n-1}{b^2}\int x^{n-2}\sqrt{a^2-b^2x^2}\,dx.$$

22.19. $$\int \frac{x^{2k+1}\,dx}{\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{2}\int \frac{t^k\,dt}{\sqrt{(a^2-b^2t)^m}}, \text{ где } t=x^2 \quad \text{(см. 16.11).}$$

22.20. $$\int \frac{x^{2k}\,dx}{\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{a^{m-2k-1}}\sum_{\nu=0}^{\frac{m-2k-3}{2}} \frac{C^{\nu}_{\frac{m-2k-3}{2}}\, x^{2\nu+2k+1}}{(2\nu+2k+1)\sqrt{(a^2-b^2x^2)^{2\nu+2k+1}}} \quad (m\geqslant 2k+3).$$

22.21. $$\int \frac{x^n\,dx}{\sqrt{(a^2-b^2x^2)^m}} = \frac{x^{n-1}}{(m-2)\,b^2\sqrt{(a^2-b^2x^2)^{m-2}}} - \frac{n-1}{(m-2)\,b^2}\int \frac{x^{n-2}\,dx}{\sqrt{(a^2-b^2x^2)^{m-2}}} \quad (m\geqslant 3).$$

ТАБЛИЦА 23

ИНТЕГРАЛЫ ВИДА

$$\int \frac{dx}{x^n \sqrt{(a^2-b^2x^2)^m}};\quad a>0,\ n=0,\ 1,\ 2,\ \ldots,\quad b>0,\ m=1,\ 3,\ 5,\ \ldots$$

23.1. $$\int \frac{dx}{x\sqrt{a^2-b^2x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|.$$

23.2. $$\int \frac{dx}{x\sqrt{(a^2-b^2x^2)^3}} = \frac{1}{a^2\sqrt{a^2-b^2x^2}} - \frac{1}{a^3}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|.$$

23.3. $$\int \frac{dx}{x\sqrt{(a^2-b^2x^2)^m}} = \sum_{\nu=1}^{\frac{m-1}{2}} \frac{1}{(m-2\nu)\,a^{2\nu}\sqrt{(a^2-b^2x^2)^{m-2\nu}}} - \frac{1}{a^m}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right| \quad (m\geqslant 3)$$

23.4. $$\int \frac{dx}{x^2\sqrt{a^2-b^2x^2}} = -\frac{\sqrt{a^2-b^2x^2}}{a^2x}.$$

23.5. $$\int \frac{dx}{x^2\sqrt{(a^2-b^2x^2)^3}} = \frac{2b^2x^2-a^2}{a^4x\sqrt{a^2-b^2x^2}}.$$

23.6. $$\int \frac{dx}{x^2\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{a^{m+1}}\sum_{\nu=0}^{\frac{m-1}{2}} \frac{C^{\nu}_{\frac{m-1}{2}}\, b^{2\nu}x^{2\nu-1}}{(2\nu-1)\sqrt{(a^2-b^2x^2)^{2\nu-1}}}.$$

23.7. $$\int \frac{dx}{x\sqrt{a^2-b^2x^2}} = -\frac{\sqrt{a^2-b^2x^2}}{2a^2x^2} - \frac{b^2}{2a^3}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|.$$

23.8. $$\int \frac{dx}{x^3\sqrt{(a^2-b^2x^2)^3}} = \frac{3b^2x^2-a^2}{2a^4x^2\sqrt{a^2-b^2x^2}} - \frac{3b^2}{2a^5}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|$$

23.9. $$\int \frac{dx}{x^2\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{(m-2)\,b^2x^1\sqrt{(a^2-b^2x^2)^{m-2}}} + \frac{4}{(m-2)\,b^2}\int \frac{dx}{x^5\sqrt{(a^2-b^2x^2)^{m-2}}} \quad (m\geqslant$$

23.10. $$\int \frac{dx}{x^4\sqrt{a^2-b^2x^2}} = -\frac{a^2+2b^2x^2}{3a^4x^3}\sqrt{a^2-b^2x^2}.$$

23.11. $$\int \frac{dx}{x^4\sqrt{(a^2-b^2x^2)^3}} = -\frac{1}{a^6}\left[-\frac{b^4x}{\sqrt{a^2-b^2x^2}} + \frac{2b^2\sqrt{a^2-b^2x^2}}{x} + \frac{\sqrt{(a^2-b^2x^2)^3}}{3x^3}\right].$$

23.12. $$\int \frac{dx}{x^4\sqrt{(a^2-b^2x^2)^m}} = \frac{-1}{a^{m+3}}\left[\frac{\sqrt{(a^2-b^2x^2)^3}}{3x^3} + \frac{(m+1)\,b^2\sqrt{a^2-b^2x^2}}{2x} - \sum_{\nu=2}^{\frac{m+1}{2}} \frac{(-1)^{\nu}}{2\nu-3} C^{\nu}_{\frac{m+1}{2}}\, b^{2\nu}\left(\frac{x}{\sqrt{a^2-b^2x^2}}\right)^{2\nu-3}\right]$$
$(m\geqslant 3).$

23.13. $$\int \frac{dx}{x^5\sqrt{a^2-b^2x^2}} = -\frac{\sqrt{a^2-b^2x^2}}{4a^2x^4} - \frac{3b^2}{8}\frac{\sqrt{a^2-b^2x^2}}{a^4x^2} - \frac{3b^4}{8a^5}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|.$$

23.14. $$\int \frac{dx}{x^5\sqrt{(a^2-b^2x^2)^3}} = -\frac{1}{4a^2x^4\sqrt{a^2-b^2x^2}} - \frac{5b^2}{8a^4x^2\sqrt{a^2-b^2x^2}} + \frac{15b^4}{8a^6\sqrt{a^2-b^2x^2}} - \frac{15b^4}{8a^7}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|.$$

23.15. $$\int \frac{dx}{x^5\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{(m-2)\,b^2x^6\sqrt{(a^2-b^2x^2)^{m-2}}} + \frac{6}{(m-2)\,b^2}\int \frac{dx}{x^7\sqrt{(a^2-b^2x^2)^{m-2}}} \quad (m\geqslant 3).$$

23.16. $$\int \frac{dx}{x^6\sqrt{a^2-b^2x^2}} = -\frac{1}{a^6}\left[\frac{b^4\sqrt{a^2-b^2x^2}}{x} + \frac{2b^2\sqrt{(a^2-b^2x^2)^3}}{3x^3} + \frac{\sqrt{(a^2-b^2x^2)^5}}{5x^5}\right].$$

23.17. $$\int \frac{dx}{x^n\sqrt{(a^2-b^2x^2)^m}} = \frac{1}{(m-2)\,b^2x^{n+1}\sqrt{(a^2-b^2x^2)^{m-2}}} + \frac{n+1}{(m-2)\,b^2}\int \frac{dx}{x^{n+2}\sqrt{(a^2-b^2x^2)^{m-2}}} \quad (m\geqslant 3).$$

ТАБЛИЦА 24

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\sqrt{(a^2-b^2x^2)^m}\,dx; \qquad \begin{matrix} a>0,\ n=0,\ 1,\ 2,\ \ldots, \\ b>0,\ m=1,\ 3,\ 5,\ \ldots \end{matrix}$$

24.1. $\int \sqrt{a^2-b^2x^2}\,dx = \frac{x\sqrt{a^2-b^2x^2}}{2} + \frac{a^2}{2b}\arcsin\frac{bx}{a}$.

24.2. $\int \sqrt{(a^2-b^2x^2)^3}\,dx = \frac{5a^2x-2b^2x^3}{8}\sqrt{a^2-b^2x^2} + \frac{3a^4}{8b}\arcsin\frac{bx}{a}$.

24.3. $\int \sqrt{(a^2-b^2x^2)^m}\,dx = \frac{x}{m+1}\sqrt{(a^2-b^2x^2)^m} + \frac{ma^2}{m+1}\int \sqrt{(a^2-b^2x^2)^{m-2}}\,dx \quad (m \geqslant 3)$.

24.4. $\int x\sqrt{a^2-b^2x^2}\,dx = -\frac{\sqrt{(a^2-b^2x^2)^3}}{3b^2}$.

24.5. $\int x\sqrt{(a^2-b^2x^2)^m}\,dx = -\frac{\sqrt{(a^2-b^2x^2)^{m+2}}}{(m+2)\,b^2}$.

24.6. $\int x^2\sqrt{a^2-b^2x^2}\,dx = \frac{2b^2x^3-a^2x}{8b^2}\sqrt{a^2-b^2x^2} + \frac{a^4}{8b^3}\arcsin\frac{bx}{a}$.

24.7 $\int x^2\sqrt{(a^2-b^2x^2)^3}\,dx = -\frac{x\sqrt{(a^2-b^2x^2)^5}}{6b^2} + \frac{a^2x\sqrt{(a^2-b^2x^2)^3}}{24b^2} + \frac{a^4x\sqrt{a^2-b^2x^2}}{16b^2} + \frac{a^6}{16b^3}\arcsin\frac{bx}{a}$.

24.8. $\int x^2\sqrt{(a^2-b^2x^2)^m}\,dx = -\frac{x\sqrt{(a^2-b^2x^2)^{m+2}}}{(m+3)\,b^2} + \frac{a^2}{(m+3)\,b^2}\int \sqrt{(a^2-b^2x^2)^m}\,dx$ (см. 24.3).

24.9. $\int x^3\sqrt{(a^2-b^2x^2)^m}\,dx = \frac{\sqrt{(a^2-b^2x^2)^{m+4}}}{(m+4)\,b^4} - \frac{a^2\sqrt{(a^2-b^2x^2)^{m+2}}}{(m+2)\,b^4}$.

24.10. $\int x^5\sqrt{a^2-b^2x^2}\,dx = -\frac{x^3\sqrt{(a^2-b^2x^2)^3}}{6b^2} - \frac{a^2x\sqrt{(a^2-b^2x^2)^3}}{8b^4} + \frac{a^4x\sqrt{a^2-b^2x^2}}{16b^4} + \frac{a^6}{16b^5}\arcsin\frac{bx}{a}$.

24.11. $\int x^n\sqrt{(a^2-b^2x^2)^m}\,dx = -\frac{x^{n-1}\sqrt{(a^2-b^2x^2)^{m+2}}}{(m+n+1)\,b^2} + \frac{(n-1)\,a^2}{(m+n+1)\,b^2}\int x^{n-2}\sqrt{(a^2-b^2x^2)^m}\,dx$.

24.12. $\int \frac{\sqrt{a^2-b^2x^2}}{x}\,dx = \sqrt{a^2-b^2x^2} - a\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|$.

24.13. $\int \frac{\sqrt{(a^2-b^2x^2)^3}}{x}\,dx = \frac{4a^2-b^2x^2}{3}\sqrt{a^2-b^2x^2} - a^3\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|$.

24.14. $\int \frac{\sqrt{(a^2-b^2x^2)^m}}{x}\,dx = \frac{1}{m}\sqrt{(a^2-b^2x^2)^m} + a^2b^2\int \frac{\sqrt{(a^2-b^2x^2)^{m-2}}}{x}\,dx$.

24.15. $\int \frac{\sqrt{a^2-b^2x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-b^2x^2}}{x} - b\arcsin\frac{x}{a}$.

24.16. $\int \frac{\sqrt{(a^2-b^2x^2)^3}}{x^2}\,dx = -\frac{\sqrt{(a^2-b^2x^2)^3}}{x} - \frac{3b^2x}{2}\sqrt{a^2-b^2x^2} - \frac{3a^2b}{2}\arcsin\frac{bx}{a}$.

24.17. $\int \frac{\sqrt{(a^2-b^2x^2)^m}}{x^2}\,dx = a^2\int \frac{\sqrt{(a^2-b^2x^2)^{m-2}}}{x^2}\,dx - b^2\int \sqrt{(a^2-b^2x^2)^{m-2}}\,dx$ (см. 24.3).

24.18. $\int \frac{\sqrt{a^2-b^2x^2}}{x^3}\,dx = -\frac{\sqrt{a^2-b^2x^2}}{2x^2} + \frac{b^2}{2a}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|$.

24.19. $\int \frac{\sqrt{(a^2-b^2x^2)^3}}{x^3}\,dx = -\frac{\sqrt{(a^2-b^2x^2)^3}}{2x^2} - \frac{3b^2\sqrt{a^2-b^2x^2}}{2} + \frac{3ab^2}{2}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|$.

24.20. $\int \frac{\sqrt{a^2-b^2x^2}}{x^4}\,dx = -\frac{\sqrt{(a^2-b^2x^2)^3}}{3a^2x^3}$.

24.21. $\int \frac{\sqrt{(a^2-b^2x^2)^3}}{x^4}\,dx = -\frac{\sqrt{(a^2-b^2x^2)^3}}{3x^3} + \frac{b^2\sqrt{a^2-b^2x^2}}{x} + b^3\arcsin\frac{bx}{a}$.

24.22. $\int \frac{\sqrt{(a^2-b^2x^2)^m}}{x^n}\,dx = a^2\int \frac{\sqrt{(a^2-b^2x^2)^{m-2}}}{x^n}\,dx - b^2\int \frac{\sqrt{(a^2-b^2x^2)^{m-2}}}{x^{n-2}}\,dx \quad (m \geqslant 3)$.

ТАБЛИЦА 25

ИНТЕГРАЛЫ ВИДА

$\int x^{\pm n}\sqrt{(b^2x^2-a^2)^{\pm m}}\,dx$; $\quad a>0,\ b>0,\ n=0,\ 1,\ 2,\ \ldots,\ m=1,\ 3,\ 5,\ \ldots$

25.1. $$\int\frac{dx}{\sqrt{b^2x^2-a^2}}=\frac{1}{b}\ln|bx+\sqrt{b^2x^2-a^2}|.$$

25.2. $$\int\frac{dx}{\sqrt{(b^2x^2-a^2)^m}}=\frac{(-1)^{\frac{m-1}{2}}}{a^{m-1}}\sum_{\nu=0}^{\frac{m-3}{2}}\frac{(-1)^\nu C_{\frac{m-3}{2}}^{\nu}\,b^{2\nu}x^{2\nu+1}}{(2\nu+1)\sqrt{(b^2x^2-a^2)^{2\nu+1}}}$$ $(m\geqslant 3)$.

25.3. $$\int\frac{x\,dx}{\sqrt{(b^2x^2-a^2)^m}}=-\frac{1}{(m-2)\,b^2\sqrt{(b^2x^2-a^2)^{m-2}}}.$$

25.4. $$\int\frac{x^2\,dx}{\sqrt{b^2x^2-a^2}}=\frac{x^2\sqrt{b^2x^2-a^2}}{2b}+\frac{a^2}{2b^3}\ln|bx+\sqrt{b^2x^2-a^2}|.$$

25.5. $$\int\frac{x^2\,dx}{\sqrt{(b^2x^2-a^2)^3}}=-\frac{x}{b^2\sqrt{b^2x^2-a^2}}+\frac{1}{b^3}\ln|bx\sqrt{b^2x^2-a^2}|.$$

25.6. $$\int\frac{x^2\,dx}{\sqrt{(b^2x^2-a^2)^m}}=\frac{(-1)^{\frac{m-3}{2}}}{a^{m-3}}\sum_{\nu=0}^{\frac{m-5}{2}}\frac{(-1)^\nu C_{\frac{m-5}{2}}^{\nu}\,b^{2\nu}x^{2\nu+3}}{(2\nu+3)\sqrt{(b^2x^2-a^2)^{2\nu+3}}}$$ $(m\geqslant 5)$.

25.7. $$\int\frac{x^n\,dx}{\sqrt{b^2x^2-a^2}}=\frac{x^{n-1}\sqrt{b^2x^2-a^2}}{b^2}-\frac{n-1}{b^2}\int x^{n-2}\sqrt{b^2x^2-a^2}\,dx.$$

25.8. $$\int\frac{x^n\,dx}{\sqrt{(b^2x^2-a^2)^m}}=-\frac{x^{n-1}}{(m-2)\,b^2\sqrt{(b^2x^2-a^2)^{m-2}}}+\frac{n-1}{(m-2)\,b^2}\int\frac{x^{n-2}\,dx}{\sqrt{(b^2x^2-a^2)^{m-2}}}$$ $(m\geqslant 3)$.

25.9. $$\int\frac{dx}{x\sqrt{b^2x^2-a^2}}=\frac{1}{a}\arccos\left|\frac{a}{bx}\right|.$$

25.10. $$\int\frac{dx}{x\sqrt{(b^2x^2-a^2)^3}}=-\frac{1}{a^2\sqrt{b^2x^2-a^2}}-\frac{1}{a^3}\arccos\left|\frac{a}{bx}\right|.$$

25.11. $$\int\frac{dx}{x\sqrt{(b^2x^2-a^2)^m}}=\sum_{\nu=1}^{\frac{m-1}{2}}\frac{(-1)^\nu}{(m-2\nu)\,a^{2\nu}\sqrt{(b^2x^2-a^2)^{m-2\nu}}}-\frac{(-1)^{\frac{m-1}{2}}}{a^m}\arccos\left|\frac{a}{bx}\right|$$ $(m\geqslant 3)$.

25.12. $$\int\frac{dx}{x^2\sqrt{b^2x^2-a^2}}=\frac{\sqrt{b^2x^2-a^2}}{a^2x}.$$

25.13. $$\int\frac{dx}{x^2\sqrt{(b^2x^2-a^2)^3}}=-\frac{1}{a^4}\left[\frac{\sqrt{b^2x^2-a^2}}{x}+\frac{xb^2}{\sqrt{b^2x^2-a^2}}\right].$$

25.14. $$\int\frac{dx}{x^2\sqrt{(b^2x^2-a^2)^m}}=\frac{(-1)^{\frac{m+1}{2}}}{a^{m+1}}\sum_{\nu=0}^{\frac{m-1}{2}}\frac{(-1)^\nu C_{\frac{m-1}{2}}^{\nu}\,x^{2\nu-1}b^{2\nu}}{(2\nu-1)\sqrt{(b^2x^2-a^2)^{2\nu-1}}}.$$

25.15. $$\int\frac{dx}{x^n\sqrt{(b^2x^2-a^2)^m}}=-\frac{1}{(m-2)\,b^2x^{n+1}\sqrt{(b^2x^2-a^2)^{m-2}}}-\frac{n+1}{(m-2)\,b^2}\int\frac{dx}{x^{n+1}\sqrt{(b^2x^2-a^2)^{m-2}}}$$ $(m\geqslant 3)$.

25.16. $$\int\sqrt{b^2x^2-a^2}\,dx=\frac{x\sqrt{b^2x^2-a^2}}{2}-\frac{a^2}{2b}\ln|bx+\sqrt{b^2x^2-a^2}|.$$

25.17. $$\int\sqrt{(b^2x^2-a^2)^3}\,dx=\frac{2b^2x^2-5a^2}{8}x\sqrt{b^2x^2-a^2}+\frac{3a^4}{8b}\ln|bx+\sqrt{b^2x^2-a^2}|.$$

25.18. $$\int\sqrt{(b^2x^2-a^2)^m}\,dx=\frac{x}{m+1}\sqrt{(b^2x^2-a^2)^m}-\frac{ma^2}{m+1}\int\sqrt{(b^2x^2-a^2)^{m-2}}\,dx.$$

25.19. $$\int x^n\sqrt{(b^2x^2-a^2)^m}\,dx=\frac{x^{n-1}\sqrt{(b^2x^2-a^2)^{m+2}}}{(m+n+1)\,b^2}+\frac{(n-1)\,a^2}{(m+n+1)\,b}\int x^{n-2}\sqrt{(b^2x^2-a^2)^m}\,dx.$$

25.20. $$\int\frac{\sqrt{b^2x^2-a^2}}{x}\,dx=\sqrt{b^2x^2-a^2}-a\arccos\left|\frac{a}{bx}\right|.$$

25.21. $$\int\frac{\sqrt{(b^2x^2-a^2)^3}}{x}\,dx=\frac{\sqrt{(b^2x^2-a^2)^3}}{3}-a^2\sqrt{b^2x^2-a^2}+a^3\arccos\left|\frac{a}{bx}\right|.$$

25.22. $$\int\frac{\sqrt{b^2x^2-a^2}}{x^2}\,dx=-\frac{\sqrt{b^2x^2-a^2}}{x}+b\ln|bx+\sqrt{b^2x^2-a^2}|.$$

25.23. $$\int\frac{\sqrt{(b^2x^2-a^2)^m}}{x^n}\,dx=-a^2\int\frac{\sqrt{(b^2x^2-a^2)^{m-2}}}{x^n}\,dx+b^2\int\frac{\sqrt{(b^2x^2-a^2)^{m-2}}}{x^{n-2}}\,dx.$$

ТАБЛИЦА 26

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{\sqrt{(ax^2+bx+c)^m}}; \qquad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 3,\ 5,\ \ldots \end{matrix}$$

26.1. $$\int \frac{dx}{\sqrt{x^2+px+q}} = \ln\left|2x+p+2\sqrt{x^2+px+q}\right|.$$

26.2. $$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| & \text{при } a>0,\ b^2\neq 4ac; \\ \dfrac{1}{\sqrt{a}}\ln|2ax+b| & \text{при } a>0,\ b^2=4ac; \\ -\dfrac{1}{\sqrt{-a}}\arcsin\dfrac{2ax+b}{\sqrt{b^2-4ac}} & \text{при } a<0,\ b^2>4ac. \end{cases}$$

26.3. $$\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{4ax+2b}{(4ac-b^2)\sqrt{ax^2+bx+c}}.$$

26.4. $$\int \frac{dx}{\sqrt{(ax^2+bx+c)^5}} = \frac{4ax+2b}{3(4ac-b^2)\sqrt{ax^2+bx+c}}\left(\frac{1}{\sqrt{ax^2+bx+c}}+\frac{8a}{4ac-b^2}\right).$$

26.5. $$\int \frac{dx}{\sqrt{(ax^2+bx+c)^m}} = \frac{4ax+2b}{(m-2)(4ac-b^2)\sqrt{(ax^2+bx+c)^{m-2}}} + \frac{4(m-3)a}{(m-2)(4ac-b^2)}\int \frac{dx}{\sqrt{(ax^2+bx+c)^{m-2}}} \quad (m\geqslant 3).$$

26.6. $$\int \frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{ax^2+bx+c}} \quad \text{(см. 26.2).}$$

26.7. $$\int \frac{x\,dx}{\sqrt{(ax^2+bx+c)^3}} = -\frac{2bx+4c}{(4ac-b^2)\sqrt{ax^2+bx+c}}.$$

26.8. $$\int \frac{x\,dx}{\sqrt{(ax^2+bx+c)^m}} = -\frac{1}{(m-2)a\sqrt{(ax^2+bx+c)^{m-2}}} - \frac{b}{2a}\int \frac{dx}{\sqrt{(ax^2+bx+c)^m}} \quad \text{(см. 26.5).}$$

26.9. $$\int \frac{x^2\,dx}{\sqrt{ax^2+bx+c}} = \frac{2ax-3b}{4a^2}\sqrt{ax^2+bx+c} + \frac{3b^2-4ac}{8a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}} \quad \text{(см. 26.2).}$$

26.10. $$\int \frac{x^3\,dx}{\sqrt{ax^2+bx+c}} = \frac{8a^2x^2-10abx+15b^2-8ac}{24a^3}\sqrt{ax^2+bx+c} - \frac{5b^3-12bc}{16a^3}\int \frac{dx}{\sqrt{ax^2+bx+c}} \quad \text{(см. 26.2).}$$

26.11. $$\int \frac{x^n\,dx}{\sqrt{(ax^2+bx+c)^m}} = \frac{1}{a}\int \frac{x^{n-2}\,dx}{\sqrt{(ax^2+bx+c)^{m-2}}} - \frac{c}{a}\int \frac{x^{n-2}\,dx}{\sqrt{(ax^2+bx+c)^m}} - \frac{b}{a}\int \frac{x^{n-1}\,dx}{\sqrt{(ax^2+bx+c)^m}}.$$

26.12. $$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = \begin{cases} -\dfrac{1}{\sqrt{c}}\ln\left|\dfrac{bx+2c+2\sqrt{c}\sqrt{ax^2+bx+c}}{x}\right| & \text{при } c>0,\ b^2\neq 4ac; \\ -\dfrac{1}{\sqrt{c}}\ln\left|\dfrac{bx+2c}{x}\right| & \text{при } c>0,\ b^2=4ac; \\ \dfrac{1}{\sqrt{-c}}\arcsin\dfrac{bx+2c}{x\sqrt{b^2-4ac}} & \text{при } c<0,\ b^2>4ac. \end{cases}$$

26.13. $$\int \frac{dx}{x\sqrt{(ax^2+bx+c)^m}} = \frac{1}{(m-2)c\sqrt{(ax^2+bx+c)^{m-2}}} + \frac{1}{c}\int \frac{dx}{x\sqrt{(ax^2+bx+c)^{m-2}}} - \frac{b}{2c}\int \frac{dx}{\sqrt{(ax^2+bx+c)^m}} \quad \text{(см. 26.5).}$$

26.14. $$\int \frac{dx}{x^2\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{ax} - \frac{b}{2c}\int \frac{dx}{x\sqrt{ax^2+bx+c}} \quad \text{(см. 26.12).}$$

26.15. $$\int \frac{dx}{x^n\sqrt{(ax^2+bx+c)^m}} = -\frac{1}{(n-1)cx^{n-1}\sqrt{(ax^2+bx+c)^{m-2}}} - \frac{(2n+m-4)b}{2(n-1)c}\int \frac{dx}{x^{n-1}\sqrt{(ax^2+bx+c)^m}} - \frac{(n+m-3)a}{(n-1)c}\int \frac{dx}{x^{n-2}\sqrt{(ax^2+bx+c)^m}} \quad (n\geqslant 2).$$

ТАБЛИЦА 27

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\sqrt{(ax^2+bx+c)^m}\,dx \qquad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 3,\ 5,\ \ldots \end{matrix}$$

27.1. $$\int \sqrt{x^2+px+q}\,dx=\frac{2x+p}{4}\sqrt{x^2+px+q}+\frac{4q-p^2}{8}\ln\left|2\sqrt{x^2+px+q}+2x+p\right|.$$

27.2. $$\int \sqrt{ax^2+bx+c}\,dx=$$
$$=\begin{cases}\dfrac{2ax+b}{4a}\sqrt{ax^2+bx+c}+\dfrac{4ac-b^2}{8a\sqrt{a}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| & \text{при } a>0,\ b^2\neq 4ac;\\ \dfrac{2ax+b}{4a}\sqrt{ax^2+bx+c} & \text{при } a>0,\ b^2=4ac;\\ \dfrac{2ax+b}{4a}\sqrt{ax^2+bx+c}+\dfrac{b^2-4ac}{8a\sqrt{-a}}\arcsin\dfrac{2ax+b}{\sqrt{b^2-4ac}} & \text{при } a<0,\ b^2>4ac.\end{cases}$$

27.3. $$\int \sqrt{(ax^2+bx+c)^m}\,dx=\frac{2ax+b}{2(m+1)a}\sqrt{(ax^2+bx+c)^m}+\frac{m(4ac-b^2)}{4(m+1)a}\int \sqrt{(ax^2+bx+c)^{m-2}}\,dx.$$

27.4. $$\int x\sqrt{ax^2+bx+c}\,dx=$$
$$=\frac{\sqrt{(ax^2+bx+c)^3}}{3a}-\frac{b}{2a}\int \sqrt{ax^2+bx+c}\,dx \quad (\text{см. } 27.2).$$

27.5. $$\int x^2\sqrt{ax^2+bx+c}\,dx=\frac{6ax-5b}{24a^2}\sqrt{(ax^2+bx+c)^3}+\frac{5b^2-4ac}{16a^2}\int \sqrt{ax^2+bx+c}\,dx \quad (\text{см. } 27.2).$$

27.6. $$\int x^3\sqrt{ax^2+bx+c}\,dx=-\frac{7b^3-12bc}{32a^3}\int \sqrt{ax^2+bx+c}\,dx+\frac{48a^2x^2-42abx+35b^2-32ac}{240a^3}\sqrt{(ax^2+bx+c)^3} \quad (\text{см. } 27.2).$$

27.7. $$\int x^n\sqrt{(ax^2+bx+c)^m}\,dx=\frac{x^{n-1}\sqrt{(ax^2+bx+c)^{m+2}}}{(m+n+1)a}-\frac{(n-1)c}{(m+n+1)a}\int x^{n-2}\sqrt{(ax^2+bx+c)^m}\,dx-\frac{(2n+m)b}{(m+n+1)2a}\int x^{n-1}\sqrt{(ax^2+bx+c)^m}\,dx.$$

27.8. $$\int \frac{\sqrt{ax^2+bx+c}}{x}\,dx=\sqrt{ax^2+bx+c}+\frac{b}{2}\int \frac{dx}{\sqrt{ax^2+bx+c}}+c\int \frac{dx}{x\sqrt{ax^2+bx+c}} \quad (\text{см. } 26.2 \text{ и } 26.12).$$

27.9. $$\int \frac{\sqrt{(ax^2+bx+c)^m}}{x}\,dx=\frac{1}{m}\sqrt{(ax^2+bx+c)^m}+\frac{b}{2}\int \sqrt{(ax^2+bx+c)^{m-2}}\,dx+c\int \frac{\sqrt{(ax^2+bx+c)^{m-2}}}{x}\,dx \quad (\text{см. } 27.3).$$

27.10. $$\int \frac{\sqrt{ax^2+bx+c}}{x^2}\,dx=-\frac{\sqrt{ax^2+bx+c}}{x}+\frac{b}{2}\int \frac{dx}{x\sqrt{ax^2+bx+c}}+a\int \frac{dx}{\sqrt{ax^2+bx+c}} \quad (\text{см. } 26.12 \text{ и } 26.2).$$

27.11. $$\int \frac{\sqrt{ax^2+bx+c}}{x^3}\,dx=-\frac{2c+bx}{4cx^2}\sqrt{ax^2+bx+c}-\frac{b^2-4ac}{8c}\int \frac{dx}{x\sqrt{ax^2+bx+c}} \quad (\text{см. } 26.12).$$

27.12. $$\int \frac{\sqrt{(ax^2+bx+c)^m}}{x^n}\,dx=-\frac{\sqrt{(ax^2+bx+c)^{m+2}}}{(n-1)cx^{n-1}}+\frac{(m-2n+1)b}{2(n-1)c}\int \frac{\sqrt{(ax^2+bx+c)^m}}{x^{n-1}}\,dx+\frac{(m-n+3)a}{(n-1)c}\int \frac{\sqrt{(ax^2+bx+c)^m}}{x^{n-2}}\,dx \quad (n\geqslant 2).$$

ТАБЛИЦА 28

ИНТЕГРАЛЫ ВИДА

$$\int \frac{(A+Bx)\,dx}{(\alpha x^2+\beta x+\gamma)\sqrt{(ax^2+bx+c)^r}};\quad r=-1,\ 1,\ 3.$$

28.1. $$\int \frac{dx}{(x+p)\sqrt{ax^2+bx+c}} =$$

$$= \begin{cases} -\dfrac{1}{2\sqrt{c-bp+ap^2}} \ln \left| \dfrac{\sqrt{c-bp+ap^2}+\sqrt{ax^2+bx+c}}{x+p} + \right. \\ \qquad \left. + \dfrac{b-2ap}{2\sqrt{c-bp+ap^2}} \right| + \\ + \dfrac{1}{2\sqrt{c-bp+ap^2}} \ln \left| \dfrac{\sqrt{c-bp+ap^2}-\sqrt{ax^2+bx+c}}{x+p} + \right. \\ \qquad \left. + \dfrac{b-2ap}{2\sqrt{c-bp+ap^2}} \right| \\ \qquad \text{при } bp<c+ap^2; \\ \dfrac{\arcsin \dfrac{(b-2ap)x-bp+2c}{(x+p)\sqrt{b^2-4ac}}}{\sqrt{bp-c-ap^2}} \quad \text{при } bp>c+ap^2 \text{ и } b^2>4ac; \\ -\dfrac{2\sqrt{ax^2+bx+c}}{(b-2ap)(x+p)} \quad \text{при } bp=c+ap^2. \end{cases}$$

28.2. $$\int \frac{dx}{(x+p)\sqrt{(ax^2+bx+c)^3}} = \frac{1}{c-bp+ap^2}\int \frac{dx}{\sqrt{ax^2+bx+c}} -$$
$$-\frac{b-2ap}{2}\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}} + \int \frac{dx}{(x+p)\sqrt{ax^2+bx+c}}$$
(см. 26.2, 26.3 и 28.1).

28.3. $$\int \frac{dx}{(\alpha x^2+\beta x+\gamma)\sqrt{ax^2+bx+c}} =$$
$$= \frac{1}{\sqrt{\beta^2-4\alpha\gamma}} \left[\int \frac{dx}{(x+M)\sqrt{ax^2+bx+c}} - \right.$$
$$\left. - \int \frac{dx}{(x+N)\sqrt{ax^2+bx+c}} \right] \quad \text{(см. 28.1)},$$

где $M=\dfrac{\beta-\sqrt{\beta^2-4\alpha\gamma}}{2\alpha}$, $N=\dfrac{\beta+\sqrt{\beta^2-4\alpha\gamma}}{2\alpha}$ при $\beta^2>4\alpha\gamma$.

28.4. $$\int \frac{dx}{(\alpha x^2+\gamma)\sqrt{ax^2+c}} =$$

$$= \begin{cases} \dfrac{1}{2\sqrt{\gamma}\sqrt{\gamma a-\alpha c}} \ln \left| \dfrac{\sqrt{\gamma}\sqrt{ax^2+c}+x\sqrt{\gamma a+\alpha c}}{\sqrt{\gamma}\sqrt{ax^2+c}-x\sqrt{\gamma a-\alpha c}} \right| \\ \qquad \text{при } \gamma a>\alpha c,\ \gamma>0; \\ \dfrac{1}{\sqrt{\gamma}\sqrt{\alpha c-\gamma a}} \operatorname{arctg} \dfrac{x\sqrt{\alpha c-\gamma a}}{\sqrt{\gamma}\sqrt{ax^2+c}} \quad \text{при } \gamma a<\alpha c,\ \gamma>0. \end{cases}$$

28.5. $$\int \frac{dx}{(x^2+p^2)\sqrt{ax^2+bx+c}} = \begin{cases} \dfrac{1}{(M^2+N^2)p}[M\varphi - N\ln r] \\ \qquad \text{при } p^2<1; \\ -\dfrac{1}{(M^2+N^2)p}[N\varphi + M\ln r] \\ \qquad \text{при } p^2>1, \end{cases}$$

где $\sqrt{c-ap^2-bpi} = M+Ni$; $\dfrac{b-2api}{2\sqrt{c-ap^2-bpi}} = L+Ri$;

$$\frac{x\sqrt{ax^2+bx+c}+Mx+Np}{x^2+p^2}+L = r\cos\varphi;$$

$$\frac{-p\sqrt{ax^2+bx+c}-Mp+Nx}{x^2+p^2}+R = r\sin\varphi.$$

28.6. $$\int \frac{(A+Bx)\,dx}{(x+p)(x+q)\sqrt{ax^2+bx+c}} =$$
$$= \frac{A-pB}{q-p}\int \frac{dx}{(x+p)\sqrt{ax^2+bx+c}} +$$
$$+ \frac{A-qB}{p-q}\int \frac{dx}{(x+q)\sqrt{ax^2+bx+c}} \quad (p\neq q) \text{ (см. 28.1).}$$

28.7. $$\int \frac{\sqrt{ax^2+bx+c}}{x+p}\,dx = a\int \frac{x\,dx}{\sqrt{ax^2+bx+c}} +$$
$$+ (b-ap)\int \frac{dx}{\sqrt{ax^2+bx+c}} + (c-bp+ap^2)\int \frac{dx}{(x+p)\sqrt{ax^2+bx+c}}$$
(см. 26.6, 26.2 и 28.1).

28.8. $$\int \frac{\sqrt{ax^2+bx+c}}{(x+p)(x+q)}\,dx = \frac{1}{q-p}\int \frac{\sqrt{ax^2+bx+c}}{x+p}\,dx +$$
$$+ \frac{1}{p-q}\int \frac{\sqrt{ax^2+bx+c}}{x+q}\,dx \quad (p\neq q) \text{ (см. 28.7).}$$

ТАБЛИЦА 29

ИНТЕГРАЛЫ ВИДА

$$\int x^n \sin^m px\,dx, \quad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

29.1. $\int \sin px\,dx = -\frac{1}{p}\cos px.$

29.2. $\int \sin^2 px\,dx = \frac{x}{2} - \frac{\sin 2px}{4p}.$

29.3. $\int \sin^3 px\,dx = \frac{\cos^3 px}{3p} - \frac{\cos px}{p}.$

29.4. $\int \sin^m px\,dx = -\frac{\sin^{m-1} px\cos px}{mp} + \frac{m-1}{m}\int \sin^{m-2} px\,dx.$

29.5. $\int x\sin px\,dx = \frac{1}{p^2}\sin px - \frac{x}{p}\cos px.$

29.6. $\int x\sin^2 px\,dx = \frac{x^2}{4} - \frac{x\sin 2px}{4p} - \frac{\cos 2px}{8p^2}.$

29.7. $\int x\sin^3 px\,dx = \frac{x\cos 3px}{12p} - \frac{\sin 3px}{36p^2} - \frac{3}{4p}x\cos px + \frac{3}{4p^2}\sin px.$

29.8. $\int x\sin^m px\,dx =$

$$= \frac{\sin^{m-1} px}{m^2p^2}(\sin px - mpx\cos px) + \frac{m-1}{m}\int x\sin^{m-2} px\,dx$$

29.9. $\int x^2\sin px\,dx = \frac{2x\sin px}{p^2} - \frac{p^2x^2-2}{p^3}\cos px$

29.10. $\int x^2\sin^2 px\,dx = \frac{x^3}{6} - \frac{2p^2x^2-1}{8p^3}\sin 2px - \frac{x\cos 2px}{4p^2}.$

29.11. $\int x^2\sin^m px\,dx = \frac{x\sin^{m-1} px}{m^2p^2}[2\sin px - mpx\cos px] +$

$$+\frac{m-1}{m}\int x^2\sin^{m-2} px\,dx - \frac{2}{m^2p^2}\int \sin^m px\,dx \quad \text{(см. 29.4).}$$

29.12. $\int x^3\sin px\,dx = \frac{3p^2x^2-6}{p^4}\sin px + \frac{6x-p^2x^3}{p^3}\cos px.$

29.13. $\int x^3\sin^2 px\,dx = \frac{x^4}{8} + \frac{3x-2p^2x^3}{8p^3}\sin 2px - \frac{6x^2p^2-3}{16p^4}\cos 2px.$

29.14. $\int x^3\sin^m px\,dx = \frac{x^2\sin^{m-1} px}{m^2p^2}[3\sin px - mpx\cos px] +$

$$+\frac{m-1}{m}\int x^3\sin^{m-2} px\,dx - \frac{6}{m^2p^2}\int x\sin^m px\,dx \quad \text{(см. 29.8).}$$

29.15. $\int x^4\sin px\,dx =$

$$= \frac{1}{p^4}(4p^2x^2-24x)\sin px - \frac{1}{p^5}(p^4x^4-12p^2x^2+24)\cos px.$$

29.16. $\int x^n\sin px\,dx =$

$$= -\frac{x^n}{p}\cos px + \frac{nx^{n-1}}{p^2}\sin px - \frac{n(n-1)}{p^2}\int x^{n-2}\sin px\,dx.$$

29.17. $\int P_n(x)\sin px\,dx =$

$$= -\cos px\sum_{\nu=0}^{E_1}\frac{(-1)^\nu}{p^{2\nu}}P_n^{(2\nu)}(x) + \sin px\sum_{\nu=1}^{E_2}\frac{(-1)^{\nu-1}}{p^{2\nu}}P_n^{(2\nu-1)}(x),$$

где $E_1=\frac{n}{2}$ и $E_2=\frac{n}{2}$ при n четном;

$E_1=\frac{n-1}{2}$ и $E_2=\frac{n+1}{2}$ при n нечетном.

29.18. $\int x^n\sin^m px\,dx = \frac{x^{n-1}\sin^{m-1} px}{m^2p^2}(n\sin px - mpx\cos px) +$

$$+\frac{m-1}{m}\int x^n\sin^{m-2} px\,dx - \frac{n(n-1)}{m^2p^2}\int x^{n-2}\sin^m px\,dx.$$

ТАБЛИЦА 30

ИНТЕГРАЛЫ ВИДА

$$\int \frac{\sin^m px}{x^n}\,dx, \quad \int \frac{x^n\,dx}{\sin^m px}, \quad \int \frac{x^n \sin^r x}{(a+b\sin x)^m}\,dx;$$

$n=0,\ 1,\ 2,\ \ldots,\quad m=1,\ 2,\ 3,\ \ldots,\quad r=0,\ 1.$

30.1.* $$\int \frac{\sin x}{x}\,dx = x - \frac{x^3}{3\cdot 3!} + \frac{x^5}{5\cdot 5!} - \frac{x^7}{7\cdot 7!} + \ldots + \frac{(-1)^{n-1}x^{2n-1}}{(2n-1)\,(2n-1)!} + \ldots$$

30.2.* $$\int \frac{\sin px}{x^n}\,dx = -\frac{\sin px}{(n-1)\,x^{n-1}} + \frac{p^{n-1}}{n-1}\int \frac{\cos t\,dt}{t^{n-1}}, \quad \text{где } t = px$$

$(n \geqslant 2)$ (см. 33.2).

30.3. $$\int \frac{\sin^m px}{x^n}\,dx = -\frac{\sin^{m-1} px\,[(n-2)\sin px + mpx\cos px]}{(n-1)\,(n-2)\,x^{n-1}} - \frac{m^2p^2}{(n-1)\,(n-2)}\int \frac{\sin^m px\,dx}{x^{n-2}} + \frac{m\,(m-1)\,p^2}{(n-1)\,(n-2)}\int \frac{\sin^{m-2} px\,dx}{x^{n-2}}$$

$(n \geqslant 3)$.

30.4. $$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| = -\frac{1}{2}\ln\left|\frac{1+\cos x}{1-\cos x}\right|.$$

30.5. $$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x.$$

30.6 $$\int \frac{dx}{\sin^3 x} = -\frac{\cos x}{2\sin^2 x} + \frac{1}{2}\ln\left|\operatorname{tg}\frac{x}{2}\right|.$$

30.7. $$\int \frac{dx}{\sin^m px} = -\frac{\cos px}{(m-1)\,p\sin^{m-1} px} + \frac{m-2}{m-1}\int \frac{dx}{\sin^{m-2} px} \quad (m \geqslant 2).$$

30.8.* $$\int \frac{x\,dx}{\sin x} = x + \frac{x^3}{3\cdot 3!} + \frac{7x^5}{3\cdot 5\cdot 5!} + \frac{31x^7}{3\cdot 7\cdot 7!} + \frac{127x^9}{3\cdot 5\cdot 9!} + \ldots + \frac{2\,(2^{2n-1}-1)}{(2n+1)!}B_n x^{2n+1} + \ldots$$

30.9. $$\int \frac{x\,dx}{\sin^2 x} = -x\operatorname{ctg} x + \ln|\sin x|.$$

30.10.* $$\int \frac{x\,dx}{\sin^3 x} = -\frac{x\cos x}{2\sin^2 x} - \frac{1}{2\sin x} + \frac{1}{2}\int \frac{x\,dx}{\sin x}$$ (см. 30.8).

30.11. $$\int \frac{x\,dx}{\sin^m px} = -\frac{x\cos px}{(m-1)\sin^{m-1} px} - \frac{1}{(m-1)\,(m-2)\,p\sin^{m-2} px} + \frac{m-2}{m-1}\int \frac{x\,dx}{\sin^{m-2} px} \quad (m \geqslant 3).$$

30.12. $$\int \frac{x^n\,dx}{\sin^m px} = -\frac{x^{n-1}}{(m-1)\,(m-2)\,p^2\sin^{m-1} px}[n\sin px + (m-2)\,px\cos px] + \frac{m-2}{m-1}\int \frac{x^n\,dx}{\sin^{m-2} px} + \frac{n\,(n-1)}{(m-1)\,(m-2)\,p^2}\int \frac{x^{n-2}\,dx}{\sin^{m-2} px} \quad (m \geqslant 3).$$

30.13. $$\int \frac{dx}{1\pm\sin x} = \mp\operatorname{tg}\left(\frac{\pi}{4}\mp\frac{x}{2}\right).$$

30.14. $$\int \frac{dx}{a+b\sin x} = \begin{cases} \dfrac{2}{\sqrt{a^2+b^2}}\operatorname{arctg}\dfrac{a\operatorname{tg}\frac{x}{2}+b}{\sqrt{a^2-b^2}} & \text{при } a^2 > b^2, \\ \dfrac{1}{\sqrt{b^2-a^2}}\ln\left|\dfrac{a\operatorname{tg}\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\operatorname{tg}\frac{x}{2}+b+\sqrt{b^2-a^2}}\right| & \text{при } a^2 < b^2. \end{cases}$$

30.15 $$\int \frac{dx}{(1+\sin x)^2} = -\frac{1}{2}\operatorname{tg}\left(\frac{\pi}{4}-\frac{x}{2}\right) - \frac{1}{6}\operatorname{tg}^3\left(\frac{\pi}{4}-\frac{x}{2}\right).$$

30.16. $$\int \frac{dx}{(1-\sin x)^2} = \frac{1}{2}\operatorname{ctg}\left(\frac{\pi}{4}-\frac{x}{2}\right) + \frac{1}{6}\operatorname{ctg}^3\left(\frac{\pi}{4}-\frac{x}{2}\right).$$

30.17. $$\int \frac{bx}{(a+b\sin x)^2} = \frac{b\cos x}{(a^2-b^2)\,(a+b\sin x)} + \frac{a}{a^2-b^2}\int \frac{dx}{a+b\sin x}$$

$(a^2 \neq b^2)$ (см. 30.14).

30.18. $$\int \frac{x\,dx}{1\pm\sin x} = \mp x\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right) + 2\ln\left|\cos\left(\frac{\pi}{4}\mp\frac{x}{2}\right)\right|.$$

30.19. $$\int \frac{\sin x\,dx}{1\pm\sin x} = \pm x + \operatorname{tg}\left(\frac{\pi}{4}\mp\frac{x}{2}\right).$$

30.20. $$\int \frac{\sin x\,dx}{a+b\sin x} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+b\sin x} \quad (a^2 \neq b^2)$$ (см. 30.14).

30.21 $$\int \frac{\sin x\,dx}{(1\pm\sin x)^2} = -\frac{1}{2}\operatorname{tg}\left(\frac{\pi}{4}\mp\frac{x}{2}\right) + \frac{1}{6}\operatorname{tg}^3\left(\frac{\pi}{4}\mp\frac{x}{2}\right).$$

30.22. $$\int \frac{\sin x\,ax}{(a+b\sin x)^2} = \frac{a\cos x}{(b^2-a^2)\,(a+b\sin x)} + \frac{b}{b^2-a^2}\int \frac{dx}{a+b\sin x}$$

$(a^2 \neq b^2)$ (см. 30.14).

ТАБЛИЦА 31

ИНТЕГРАЛЫ ВИДА

$$\int R(\sin px,\ \sin qx,\ \sqrt{a^2 \pm b^2 \sin^2 x})\,dx.$$

31.1 $$\int \sin px \sin qx\,dx = \frac{\sin(q-p)x}{2(q-p)} - \frac{\sin(q+p)x}{2(q+p)} \quad (p^2 \neq q^2).$$

31.2. $$\int \sin px \sin^n x\,dx = -\frac{\sin^n x \cos px}{p} + \frac{n}{2p}\int \sin^{n-1} x \cos(p-1)x\,dx + \frac{n}{2p}\int \sin^{n-1} x \cos(p+1)x\,dx \quad \text{(см. 35.8)}.$$

31.3. $$\int \frac{\sin x\,dx}{\sqrt{a^2+b^2\sin^2 x}} = -\frac{1}{b}\arcsin\frac{b\cos x}{\sqrt{a^2+b^2}}.$$

31.4 $$\int \frac{\sin x\,dx}{\sqrt{a^2-b^2\sin^2 x}} = -\frac{1}{b}\ln\left|b\cos x + \sqrt{a^2-b^2\sin^2 x}\right|.$$

31.5. $$\int \sin x \sqrt{a^2+b^2\sin^2 x}\,dx = -\frac{\cos x}{2}\sqrt{a^2+b^2\sin^2 x} - \frac{a^2+b^2}{2b}\arcsin\frac{b\cos x}{\sqrt{a^2+b^2}}.$$

31.6. $$\int \sin x \sqrt{a^2-b^2\sin^2 x}\,dx = -\frac{\cos x}{2}\sqrt{a^2-b^2\sin^2 x} - \frac{a^2-b^2}{2b}\ln\left|b\cos x + \sqrt{a^2-b^2\sin^2 x}\right|.$$

31.7 $$\int \frac{\sin^n x\,dx}{\sin(2k+1)x} = \frac{1}{2k+1}\sum_{\nu=0}^{2k}(-1)^{k+\nu}\cos^n\frac{2\nu+1}{2(2k+1)}\pi \ln\left|\frac{\sin\left[\frac{(\nu-k)\pi}{2(2k+1)}+\frac{x}{2}\right]}{\sin\left[\frac{\nu+k+1}{2(2k+1)}\pi-\frac{x}{2}\right]}\right|$$

31.8. $$\int \frac{\sin^{2l} x\,dx}{\sin 2kx} = \frac{(-1)^k}{2k}\left[\ln\cos x + \sum_{\nu=1}^{k-1}(-1)^\nu \cos^{2l}\frac{\nu\pi}{2k}\ln\left(\cos^2 x - \sin^2\frac{\nu\pi}{2k}\right)\right]$$

31.9. $$\int \frac{\sin^{2l+1} x}{\sin 2kx}\,dx = \frac{(-1)^k}{2k}\left\{\ln\operatorname{tg}\left(\frac{\pi}{4}-\frac{x}{2}\right) + \sum_{\nu=1}^{k-1}(-1)^\nu \cos^{2l+1}\frac{\nu\pi}{2k}\ln\left[\operatorname{tg}\left(\frac{k+\nu}{4k}\pi-\frac{x}{2}\right)\operatorname{tg}\left(\frac{k-\nu}{4k}\pi-\frac{x}{2}\right)\right]\right\}.$$

31.10. $$\int \frac{\sin 2x}{\sin x}\,dx = 2\sin x.$$

31.11. $$\int \frac{\sin 2x}{\sin^2 x}\,dx = 2\ln\sin x.$$

31.12. $$\int \frac{\sin 2x}{\sin^3 x}\,dx = -\frac{2}{\sin x}.$$

31.13. $$\int \frac{\sin 2x\,dx}{\sin^n x} = -\frac{2}{(n-2)\sin^{n-2} x} \quad (n \geqslant 3).$$

31.14. $$\int \frac{\sin x}{\sin 2x}\,dx = \frac{1}{2}\ln\left|\operatorname{ctg}\left(\frac{x}{2}-\frac{\pi}{4}\right)\right|.$$

31.15. $$\int \frac{\sin^2 x}{\sin 2x}\,dx = -\frac{1}{2}\ln|\cos x|.$$

31.16. $$\int \frac{\sin^3 x}{\sin 2x}\,dx = -\frac{1}{2}\ln\left|\operatorname{ctg}\left(\frac{x}{2}-\frac{\pi}{4}\right)\right| - \frac{1}{2}\sin x.$$

31.17. $$\int \frac{\sin^n x}{\sin 2x}\,dx = \frac{1}{2}\int \frac{t^{n-1}\,dt}{1-t^2}, \text{ где } t = \sin x \quad (n \geqslant 2) \text{ (см. 6.13)}.$$

31.18. $$\int \frac{\sin 3x}{\sin x}\,dx = x + \sin 2x.$$

31.19. $$\int \frac{\sin 3x}{\sin^2 x}\,dx = 3\ln\left|\operatorname{tg}\frac{x}{2}\right| + 4\cos x.$$

31.20. $$\int \frac{\sin 3x}{\sin^3 x}\,dx = -3\operatorname{ctg} x - 4x.$$

31.21. $$\int \frac{\sin kx}{\sin^n x}\,dx = \sum_{\nu=0}^{E}(-1)^\nu C_k^{2\nu+1}\int \frac{\cos^{k-2\nu-1} x}{\sin^{n-2\nu-1} x}\,dx,$$

где $E = \begin{cases} \frac{k}{2}, & \text{если } k \text{ четное,} \\ \frac{k+1}{2}, & \text{если } k \text{ нечетное.} \end{cases}$

ТАБЛИЦА 32

ИНТЕГРАЛЫ ВИДА

$$\int x^n \cos^m px\,dx;\quad \begin{matrix} m=1,\ 2,\ 3,\ \ldots, \\ n=0,\ 1,\ 2,\ \ldots \end{matrix}$$

32.1. $\int \cos px\,dx = \frac{1}{p}\sin px.$

32.2. $\int \cos^2 px\,dx = \frac{x}{2} + \frac{\sin 2px}{4p}.$

32.3. $\int \cos^3 px\,dx = \frac{1}{p}\sin px - \frac{\sin^3 px}{3p}.$

32.4 $\int \cos^m px\,dx = \frac{\sin px \cos^{m-1} px}{mp} + \frac{m-1}{m}\int \cos^{m-2} px\,dx.$

32.5. $\int x\cos px\,dx = \frac{1}{p^2}\cos px + \frac{x}{p}\sin px.$

32.6. $\int x\cos^2 px\,dx = \frac{x^2}{4} + \frac{x\sin 2px}{4p} + \frac{\cos 2px}{8p^2}.$

32.7. $\int x\cos^3 px\,dx = \frac{x\sin 3px}{12p} + \frac{\cos 3px}{36p^2} + \frac{3}{4p}x\sin px + \frac{3}{4p^2}\cos px.$

32.8. $\int x\cos^m px\,dx =$

$$= \frac{\cos^{m-1} x}{m^2p^2}[\cos px + pm\,x\sin px] + \frac{m-1}{m}\int x\cos^{m-2} px\,dx.$$

32.9. $\int x^2\cos px\,dx = \frac{2x\cos px}{p^2} + \frac{p^2x^2-2}{p^3}\sin px.$

32.10. $\int x^2\cos^2 px\,dx = \frac{x^3}{6} + \frac{2p^2x^2-1}{8p^3}\sin 2px + \frac{x\cos 2px}{4p^2}.$

32.11. $\int x^2\cos^m px\,dx = \frac{x\cos^{m-1} px}{m^2p^2}[2\cos px + mpx\sin px] +$

$$+\frac{m-1}{m}\int x^2\cos^{m-2} px\,dx - \frac{2}{m^2p^2}\int \cos^m px\,dx \quad \text{(см. 32.4).}$$

32.12. $\int x^3\cos px\,dx = \frac{3p^2x^2-6}{p^4}\cos px + \frac{p^2x^3-6x}{p^3}\sin px.$

32.13. $\int x^3\cos^2 px\,dx = \frac{x^4}{8} + \frac{2p^2x^2-3x}{8p^3}\sin 2px + \frac{6p^2x^2-3}{16p^4}\cos 2px.$

32.14. $\int x^3\cos^m px\,dx = \frac{x^2\cos^{m-1} px}{m^2p^2}[3\cos px + mpx\sin px] +$

$$+\frac{m-1}{m}\int x^3\cos^{m-2} px\,dx - \frac{6}{m^2p^2}\int x\cos^m px\,dx \quad \text{(см. 32.8).}$$

32.15. $\int x^4\cos px\,dx =$

$$= \frac{1}{p^4}(4p^2x^3 - 24x)\cos px + \frac{1}{p^5}(p^4x^4 - 12p^2x^2 + 24)\sin px.$$

32.16. $\int x^4\cos^2 px\,dx =$

$$= \frac{x^5}{10} + \frac{2p^4x^4 - 6p^2x^2 + 3}{8p^5}\sin 2px + \frac{2p^2x^3 - 3x}{4p^4}\cos 2px.$$

32.17. $\int x^4\cos^m px\,dx = \frac{x^3\cos^{m-1} px}{m^2p^2}(4\cos px + mpx\sin px) +$

$$+\frac{m-1}{m}\int x^4\cos^{m-2} px\,dx - \frac{12}{m^2p^2}\int x^2\cos^m px\,dx \quad \text{(см. 32.12).}$$

32.18. $\int x^n\cos px\,dx =$

$$= \frac{x^n}{p}\sin px + \frac{nx^{n-1}}{p^2}\cos px - \frac{n(n-1)}{p^2}\int x^{n-2}\cos px\,dx.$$

32.19. $\int P_n(x)\cos px\,dx =$

$$= \sin px\sum_{\nu=0}^{E_1}\frac{(-1)^\nu}{p^{2\nu}}P_n^{(2\nu)}(x) + \cos px\sum_{\nu=1}^{E_2}\frac{(-1)^{k-1}}{p^{2\nu}}P_n^{(2\nu-1)}(x),$$

где $E_1 = \frac{n}{2}$ и $E_2 = \frac{n}{2}$ при n четном;

$E_1 = \frac{n-1}{2}$ и $E_2 = \frac{n+1}{2}$ при n нечетном.

32.20. $\int x^n\cos^m px\,dx = \frac{x^{n-1}\cos^{m-1} px}{m^2p^2}(n\cos px + mpx\sin px) +$

$$+\frac{m-1}{m}\int x^n\cos^{m-2} px\,dx - \frac{n(n-1)}{m^2p^2}\int x^{n-2}\cos^m px\,dx.$$

ТАБЛИЦА 33

ИНТЕГРАЛЫ ВИДА

$$\int \frac{\cos^m px}{x^n}\,dx, \quad \int \frac{x^n\,dx}{\cos^m px}, \quad \int \frac{x^n \cos^r x}{(a+b\cos x)^m}\,dx;$$

$m = 1, 2, 3, \ldots, \quad n = 0, 1, 2, \ldots, \quad r = 0, 1.$

33.1.* $$\int \frac{\cos x}{x}\,dx = \ln x - \frac{x^2}{2\cdot 2!} + \frac{x^4}{4\cdot 4!} - \frac{x^6}{6\cdot 6!} + \frac{x^8}{8\cdot 8!} - \ldots$$

$$\ldots + (-1)^k \frac{x^{2k}}{2k\,(2k!)} + \ldots$$

33.2.* $$\int \frac{\cos px}{x^n}\,dx = -\frac{\cos px}{(n-1)\,x^{n-1}} - \frac{p^{n-1}}{n-1}\int \frac{\sin t\,dt}{t^{n-1}}, \quad \text{где } t = px$$

$(n \geqslant 2)$ (см. 30.2)

33.3. $$\int \frac{\cos^m px}{x^n}\,dx = -\frac{\cos^{m-1} px\,[(n-2)\cos px - mpx \sin px]}{(n-1)(n-2)\,x^{n-1}} -$$

$$- \frac{m^2p^2}{(n-1)(n-2)}\int \frac{\cos^m px\,dx}{x^{n-2}} + \frac{m(m-1)p^2}{(n-1)(n-2)}\int \frac{\cos^{m-2} px\,dx}{x^{n-2}} \quad (n \geqslant 3)$$

33.4. $$\int \frac{dx}{\cos x} = \ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|.$$

33.5 $$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x.$$

33.6. $$\int \frac{dx}{\cos^3 x} = \frac{\sin x}{2\cos^2 x} + \frac{1}{2}\ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|.$$

33.7. $$\int \frac{dx}{\cos^m px} = \frac{\sin px}{(m-1)\,p\cos^{m-1} px} + \frac{m-2}{m-1}\int \frac{dx}{\cos^{m-2} px} \quad (m \geqslant 2).$$

33.8.* $$\int \frac{x\,dx}{\cos x} = \frac{x^2}{2} + \frac{x^4}{4\cdot 2!} + \frac{5x^6}{6\cdot 4!} + \frac{61x^8}{8\cdot 6!} + \ldots + \frac{E_n x^{2n+2}}{(2n+2)(2n)!} + \ldots$$

33.9. $$\int \frac{x\,dx}{\cos^2 x} = x\operatorname{tg} x + \ln|\cos x|.$$

33.10.* $$\int \frac{x\,dx}{\cos^3 x} = \frac{x\sin x}{2\cos^2 x} - \frac{1}{2\cos x} + \frac{1}{2}\int \frac{x\,dx}{\cos x} \quad \text{(см. 33.8)}.$$

33.11. $$\int \frac{x\,dx}{\cos^m px} = \frac{x\sin px}{p\,(m-1)\cos^{m-1} px} -$$

$$- \frac{1}{(m-1)(m-2)p^2\cos^{m-2} px} + \frac{m-2}{m-1}\int \frac{x\,dx}{\cos^{m-2} px} \quad (m \geqslant 3).$$

33.12. $$\int \frac{x^n\,dx}{\cos^m px} =$$

$$= -\frac{x^{n-1}}{(m-1)(m-2)p^2\cos^{m-1} px}\{n\cos px - (m-2)px\sin px\} +$$

$$+ \frac{m-2}{m-1}\int \frac{x^n\,dx}{\cos^{m-2} px} + \frac{n(n-1)}{(m-1)(m-2)p^2}\int \frac{x^{n-2}\,dx}{\cos^{m-2} px} \quad (m \geqslant 3).$$

33.13. $$\int \frac{dx}{1 \pm \cos x} = \pm\operatorname{tg}\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4} - \frac{x}{2}\right)\right].$$

33.14. $$\int \frac{dx}{a + b\cos x} =$$

$$= \begin{cases} \dfrac{2}{\sqrt{a^2-b^2}}\operatorname{arctg}\dfrac{(a-b)\operatorname{tg}\frac{x}{2}}{\sqrt{a^2-b^2}} & \text{при } a^2 > b^2, \\ \dfrac{1}{\sqrt{b^2-a^2}}\ln\left|\dfrac{(b-a)\operatorname{tg}\frac{x}{2} + \sqrt{b^2-a^2}}{(b-a)\operatorname{tg}\frac{x}{2} - \sqrt{b^2-a^2}}\right| & \text{при } a^2 < b^2. \end{cases}$$

33.15. $$\int \frac{dx}{(1+\cos x)^2} = \frac{1}{2}\operatorname{tg}\frac{x}{2} + \frac{1}{6}\operatorname{tg}^3\frac{x}{2}.$$

33.16. $$\int \frac{dx}{(1-\cos x)^2} = -\frac{1}{2}\operatorname{ctg}\frac{x}{2} - \frac{1}{6}\operatorname{ctg}^3\frac{x}{2}.$$

33.17. $$\int \frac{dx}{(a+b\cos x)^2} = \frac{b\sin x}{(b^2-a^2)(a+b\cos x)} - \frac{a}{b^2-a^2}\int \frac{dx}{a+b\cos x}$$

$(a^2 \neq b^2)$ (см. 33.14).

33.18. $$\int \frac{x\,dx}{1 \pm \cos x} =$$

$$= \pm x\operatorname{tg}\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4} - \frac{x}{2}\right)\right] + 2\ln\left|\cos\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4} - \frac{x}{2}\right)\right]\right|.$$

33.19. $$\int \frac{\cos x\,dx}{1 \pm \cos x} = \pm x \mp \operatorname{tg}\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4} - \frac{x}{2}\right)\right].$$

33.20. $$\int \frac{\cos x\,dx}{a + b\cos x} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+b\cos x} \quad (a^2 \neq b^2) \text{ (см. 33.14)}.$$

33.21. $$\int \frac{\cos x\,dx}{(1 \pm \cos x)^2} =$$

$$= \frac{1}{2}\operatorname{tg}\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4} - \frac{x}{2}\right)\right] - \frac{1}{6}\operatorname{tg}^3\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4} - \frac{x}{2}\right)\right].$$

33.22. $$\int \frac{\cos x\,dx}{(a+b\cos x)^2} = \frac{a\sin x}{(a^2-b^2)(a+b\cos x)} - \frac{b}{a^2-b^2}\int \frac{dx}{a+b\cos x}$$

$(a^2 \neq b^2)$ (см. 33.14)

ТАБЛИЦА 34

ИНТЕГРАЛЫ ВИДА

$$\int R(\cos px,\ \cos qx,\ \sqrt{a^2 \pm b^2 \sin^2 x})\,dx.$$

34.1. $$\int \cos px \cos qx\,dx = \frac{\sin(p+q)x}{2(p+q)} + \frac{\sin(p-q)x}{2(p-q)} \quad (p^2 \neq q^2).$$

34.2. $$\int \cos px \cos^n x\,dx = \frac{\cos^n x \sin px}{p} + \frac{n}{2p}\int \cos^{n-1} x \cos(p-1)x\,dx - \frac{n}{2p}\int \cos^{n-1} x \cos(p+1)x\,dx.$$

34.3. $$\int \frac{\cos x\,dx}{\sqrt{a^2+b^2\cos^2 x}} = \frac{1}{b}\arcsin\frac{b\sin x}{\sqrt{a^2+b^2}}.$$

34.4. $$\int \frac{\cos x\,dx}{\sqrt{a^2-b^2\cos^2 x}} = \frac{1}{b}\ln\left|b\sin x + \sqrt{a^2-b^2\cos^2 x}\right|.$$

34.5. $$\int \cos x \sqrt{a^2+b^2\cos^2 x}\,dx = \frac{\sin x}{2}\sqrt{a^2+b^2\cos^2 x} + \frac{a^2+b^2}{2b}\arcsin\frac{b\sin x}{\sqrt{a^2+b^2}}.$$

34.6. $$\int \cos x \sqrt{a^2-b^2\cos^2 x}\,dx = \frac{\sin x}{2}\sqrt{a^2-b^2\cos^2 x} + \frac{a^2-b^2}{2b}\ln\left|b\sin x + \sqrt{a^2-b^2\cos^2 x}\right|.$$

34.7. $$\int \frac{\cos^n x}{\cos kx}\,dx = \frac{1}{k}\sum_{\nu=0}^{k-1}(-1)^\nu \cos^n\frac{2\nu+1}{2k}\pi \ln\left|\frac{\sin\left[\frac{2\nu+1}{4k}\pi + \frac{x}{2}\right]}{\sin\left[\frac{2\nu+1}{4k}\pi - \frac{x}{2}\right]}\right|.$$

34.8. $$\int \frac{\cos 2x}{\cos x}\,dx = 2\sin x - \ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|.$$

34.9. $$\int \frac{\cos 2x}{\cos^2 x}\,dx = 2x - \operatorname{tg} x.$$

34.10. $$\int \frac{\cos 2x}{\cos^3 x}\,dx = -\frac{\sin x}{2\cos^2 x} + \frac{3}{2}\ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|.$$

34.11. $$\int \frac{\cos 2x}{\cos^n x}\,dx = -\frac{\sin x}{(n-1)\cos^{n-1} x} + \frac{n}{n-1}\int \frac{dx}{\cos^{n-2} x} \quad (n \geqslant 2)$$ (см. 33.7).

34.12. $$\int \frac{\cos x}{\cos 2x}\,dx = \frac{1}{2\sqrt{2}}\ln\left|\frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x}\right|.$$

34.13. $$\int \frac{\cos^2 x\,dx}{\cos 2x} = \frac{x}{2} - \frac{1}{4}\ln\left|\frac{1-\operatorname{tg} x}{1+\operatorname{tg} x}\right|.$$

34.14. $$\int \frac{\cos^3 x\,dx}{\cos 2x} = \frac{1}{2}\sin x + \frac{1}{4\sqrt{2}}\ln\left|\frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x}\right|.$$

34.15. $$\int \frac{\cos^n x}{\cos 2x}\,dx = \frac{1}{2}\int \cos^{n-2} x\,dx + \frac{1}{2}\int \frac{\cos^{n-2} x}{\cos 2x}\,dx \quad (n \geqslant 2)$$ (см. 32.4).

34.16. $$\int \frac{\cos 3x}{\cos x}\,dx = \sin 2x - x.$$

34.17. $$\int \frac{\cos 3x}{\cos^2 x}\,dx = 4\sin x - 3\ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|.$$

34.18. $$\int \frac{\cos 3x}{\cos^3 x}\,dx = 4x - 3\operatorname{tg} x.$$

34.19. $$\int \frac{\cos 3x}{\cos^n x}\,dx = 4\int \frac{dx}{\cos^{n-3} x} - 3\int \frac{dx}{\cos^{n-1} x}$$ (см. 33.7).

34.20. $$\int \frac{\cos kx}{\cos^n x}\,dx = \sum_{\nu=0}^{E}(-1)^\nu \frac{k}{k-\nu}2^{k-2\nu-1}C_{n-\nu}^{\nu}\int \cos^{k-2\nu-n} x\,dx,$$

где $E = \begin{cases} \frac{k}{2}, & \text{если } k \text{ четное;} \\ \frac{k-1}{2}, & \text{если } k \text{ нечетное.} \end{cases}$

34.21. $$\int \frac{\cos x}{\cos 3x}\,dx = \frac{\sqrt{3}}{6}\ln\left|\frac{\sin\left(\frac{x}{2}+\frac{\pi}{12}\right)\sin\left(\frac{x}{2}-\frac{5\pi}{12}\right)}{\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)\sin\left(\frac{x}{2}+\frac{5\pi}{12}\right)}\right|.$$

34.22. $$\int \frac{\cos^2 x}{\cos 3x}\,dx = \frac{1}{4}\ln\left|\frac{\sin\left(\frac{x}{2}+\frac{\pi}{12}\right)\sin\left(\frac{x}{2}+\frac{5\pi}{12}\right)}{\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)\sin\left(\frac{x}{2}-\frac{5\pi}{12}\right)}\right|.$$

ТАБЛИЦА 35

ИНТЕГРАЛЫ ВИДА

$$\int \sin^{\pm m} px \cos^{\pm n} qx\, dx; \quad \begin{matrix} n = 1, 2, 3, \ldots \\ m = 1, 2, 3, \ldots \end{matrix}$$

35.1. $\int \sin x \cos x\, dx = \frac{\sin^2 x}{2}$.

35.2. $\int \sin x \cos^n x\, dx = -\frac{\cos^{n+1} x}{n+1}$.

35.3. $\int \sin^m x \cos x\, dx = \frac{\sin^{m+1} x}{m+1}$.

35.4. $\int \sin^2 x \cos^2 x\, dx = \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right)$.

35.5. $\int \sin^2 x \cos^3 x\, dx = \frac{\sin^3 x \cos^2 x}{5} + \frac{2}{15}\sin^3 x$.

35.6. $\int \sin^m x \cos^n x\, dx =$

$$= \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x\, dx$$

35.7. $\int \sin px \cos^n x\, dx =$

$$= -\frac{\cos^n x \cos px}{p+n} + \frac{n}{p+n}\int \cos^{n-1} x \sin(p-1)x\, dx$$

35.8. $\int \sin^m x \cos qx\, dx =$

$$= \frac{\sin^m x \sin qx}{m+q} - \frac{m}{m+q}\int \sin^{m-1} x \sin(q-1)x\, dx \quad \text{(см. 31.2)}.$$

35.9 $\int \frac{dx}{\sin x \cos x} = \ln|\operatorname{tg} x|$.

35.10. $\int \frac{dx}{\sin x \cos^n x} = \frac{1}{(n-1)\sin^{n-1} x} + \int \frac{dx}{\sin x \cos^{n-2} x} \quad (n \geqslant 2)$.

35.11. $\int \frac{dx}{\sin^m x \cos x} = \frac{-1}{(m-1)\sin^{m-1} x} + \int \frac{dx}{\sin^{m-2} x \cos x} \quad (m \geqslant 2)$

35.12. $\int \frac{dx}{\sin^m x \cos^n x} = \frac{1}{(n-1)\sin^{m-1} x \cos^{n-1} x} +$

$$+ \frac{m+n-2}{n-1}\int \frac{dx}{\sin^m x \cos^{n-2} x} \quad (n \geqslant 2),$$

$$= -\frac{1}{(m-1)\sin^{m-1} x \cos^{n-2} x} +$$

$$+ \frac{m+n-2}{m-1}\int \frac{dx}{\sin^{m-2} x \cos^n x} \quad (m \geqslant 2).$$

35.13. $\int \frac{\sin 2x}{\cos x}\, dx = -2\cos x$.

35.14. $\int \frac{\sin 2x}{\cos^2 x}\, dx = -2\ln|\cos x|$.

35.15. $\int \frac{\sin 3x}{\cos x}\, dx = 2\sin^2 x + \ln|\cos x|$.

35.16. $\int \frac{\sin 3x}{\cos^2 x}\, dx = -4\cos x - \frac{1}{\cos x}$.

35.17. $\int \frac{\sin 3x}{\cos^3 x}\, dx = -\frac{1}{2\cos^2 x} - 4\ln|\cos x|$.

35.18. $\int \frac{\sin 3x}{\cos^n x}\, dx = \frac{4}{(n-3)\cos^{n-3} x} - \frac{1}{(n-1)\cos^{n-1} x} \quad (n \geqslant 4)$.

35.19 $\int \frac{\cos 2x}{\sin x}\, dx = 2\cos x + \ln\left|\operatorname{tg}\frac{x}{2}\right|$

35.20. $\int \frac{\cos 2x}{\sin^3 x}\, dx = -\operatorname{ctg} x - 2x$.

35.21. $\int \frac{\cos 3x}{\sin x}\, dx = -2\sin^2 x + \ln|\sin x|$.

35.22. $\int \frac{\cos 3x}{\sin^2 x}\, dx = -4\sin x - \frac{1}{\sin x}$.

35.23. $\int \frac{\cos 3x}{\sin^3 x}\, dx = -\frac{1}{2\sin^2 x} - 4\ln|\sin x|$.

35.24. $\int \frac{\cos 3x}{\sin^m x}\, dx = \frac{4}{(m-3)\sin^{m-3} x} - \frac{1}{(m-1)\sin^{m-1} x} \quad (m \geqslant 4)$.

35.25. $\int \frac{\sin px}{\cos^n x}\, dx = 2\int \frac{\sin(p-1)x}{\cos^{n-1} x}\, dx - \int \frac{\sin(p-2)x}{\cos^n x}\, dx$.

35.26. $\int \frac{\cos qx}{\sin^m x}\, dx = -2\int \frac{\sin(q-1)x}{\sin^{m-1} x}\, dx + \int \frac{\cos(q-2)x}{\sin^m x}\, dx$ (см. 31.21).

ТАБЛИЦА 36

ИНТЕГРАЛЫ ВИДА

$$\int R(\sin x,\ \cos x)\,dx.$$

36.1. $\displaystyle\int \frac{\sin x\,dx}{a+b\cos x} = -\frac{1}{b}\ln|a+b\cos x|.$

36.2. $\displaystyle\int \frac{\sin x\,dx}{(a+b\cos x)^n} = \frac{1}{(n-1)\,b\,(a+b\cos x)^{n-1}} \qquad (n\geqslant 2).$

36.3. $\displaystyle\int \frac{\sin x\,dx}{\cos x\,(1\pm\cos x)} = \ln\left|\frac{1\pm\cos x}{\cos x}\right|.$

36.4. $\displaystyle\int \frac{\sin x\,dx}{\cos x\,(1\pm\sin x)} = \frac{1}{2\,(1\pm\sin x)} \pm \frac{1}{2}\ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|.$

36.5. $\displaystyle\int \frac{\sin x\,dx}{(a+b\cos x)\,(\alpha+\beta\cos x)} = \frac{1}{a\beta-b\alpha}\ln\left|\frac{a+b\cos x}{\alpha+\beta\cos x}\right|$

$(a\beta-\alpha b\neq 0).$

36.6 $\displaystyle\int \frac{\alpha+\beta\sin x}{a+b\cos x}\,dx = -\frac{\beta}{b}\ln|a+b\cos x| + \alpha\int\frac{dx}{a+b\cos x}$

(см. 33.14).

36.7. $\displaystyle\int \frac{\cos x\,dx}{a+b\sin x} = \frac{1}{b}\ln|a+b\sin x|.$

36.8. $\displaystyle\int \frac{\cos x\,dx}{(a+b\sin x)^n} = -\frac{1}{(n-1)\,b\,(a+b\sin x)^{n-1}} \qquad (n\geqslant 2).$

36.9. $\displaystyle\int \frac{\cos x\,dx}{\sin x\,(1\pm\sin x)} = \ln\left|\frac{\sin x}{1\pm\sin x}\right|.$

36.10. $\displaystyle\int \frac{\cos x\,dx}{\sin x\,(1\pm\cos x)} = -\frac{1}{2\,(1\pm\cos x)} \pm \frac{1}{2}\ln\left|\operatorname{tg}\frac{x}{2}\right|.$

36.11. $\displaystyle\int \frac{\cos x\,dx}{(a+b\sin x)\,(\alpha+\beta\sin x)} = \frac{1}{a\beta-b\alpha}\ln\left|\frac{\alpha+\beta\sin x}{a+b\sin x}\right|$

$(a\beta-\alpha b\neq 0).$

36.12. $\displaystyle\int \frac{\alpha+\beta\cos x}{a+b\sin x}\,dx = \frac{\beta}{b}\ln|a+b\sin x| + \alpha\int\frac{dx}{a+b\sin x}$ (см. 30.14).

36.13. $\displaystyle\int \frac{dx}{\sin x\,(1\pm\cos x)} = \pm\frac{1}{2\,(1\pm\cos x)} + \frac{1}{2}\ln\left|\operatorname{tg}\frac{x}{2}\right|.$

36.14. $\displaystyle\int \frac{dx}{\cos x\,(1\pm\sin x)} = \mp\frac{1}{2\,(1\pm\sin x)} + \frac{1}{2}\ln\left|\operatorname{tg}\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|.$

36.15. $\displaystyle\int \frac{dx}{\sin x\pm\cos x} = \frac{1}{\sqrt{2}}\ln\left|\operatorname{tg}\left(\frac{x}{2}\pm\frac{\pi}{8}\right)\right|.$

36.16. $\displaystyle\int \frac{dx}{a\cos x+b\sin x} = \frac{1}{\sqrt{a^2+b^2}}\ln\left|\operatorname{tg}\frac{x+\operatorname{arccos}\dfrac{b}{\sqrt{a^2+b^2}}}{2}\right|.$

36.17. $\displaystyle\int \frac{dx}{(a\cos x+b\sin x)^n} = \int\frac{d(x-\varphi)}{[\varrho\cos(x-\varphi)]^n},$

где $a=\varrho\cos\varphi$, $b=\varrho\sin\varphi$ (см. 33.7)

36.18. $\displaystyle\int \frac{dx}{1+\cos x\pm\sin x} = \pm\ln\left|1\pm\operatorname{tg}\frac{x}{2}\right|.$

36.19. $\displaystyle\int \frac{dx}{a+b\cos x+c\sin x} = \int\frac{dt}{a+\sqrt{b^2+c^2}\sin t},$

где $t=x+\operatorname{arctg}\dfrac{b}{c}$ (см. 30.14).

36.20. $\displaystyle\int \frac{dx}{(a+b\cos x+c\sin x)^n} = \int\frac{d(x-\varphi)}{[a+\varrho\cos(x-\varphi)]^n},$

где $b=\varrho\cos\varphi$, $c=\varrho\sin\varphi$

36.21. $\displaystyle\int \frac{\sin x\,dx}{\sin x\pm\cos x} = \frac{x}{2}\mp\frac{1}{2}\ln|\sin x\pm\cos x|.$

36.22. $\displaystyle\int \frac{\cos x\,dx}{\sin x\pm\cos x} = \pm\frac{x}{2}+\frac{1}{2}\ln|\sin x\pm\cos x|.$

36.23. $\displaystyle\int \frac{dx}{a^2\cos^2 x+b^2\sin^2 x} = \frac{1}{ab}\operatorname{arctg}\left(\frac{b}{a}\operatorname{tg}x\right) \qquad (a>0,\ b>0).$

36.24. $\displaystyle\int \frac{dx}{a^2\cos^2 x-b^2\sin^2 x} = \frac{1}{2ab}\ln\left|\frac{b\operatorname{tg}x+a}{b\operatorname{tg}x-a}\right|.$

36.25. $\displaystyle\int \frac{\sin x\cos x\,dx}{a\cos^2 x+b\sin^2 x} = \frac{1}{2\,(b-a)}\ln|a\cos^2 x+b\sin^2 x) \quad (a\neq b).$

ТАБЛИЦА 37

ИНТЕГРАЛЫ ВИДА

$$\int F(x,\ \operatorname{tg} x,\ \operatorname{ctg} x)\,dx.$$

37.1. $\int \operatorname{tg} x\,dx = -\ln|\cos x|.$

37.2. $\int \operatorname{tg}^2 x\,dx = \operatorname{tg} x - x.$

37.3. $\int \operatorname{tg}^3 x\,dx = \frac{\operatorname{tg}^2 x}{2} + \ln|\cos x|.$

37.4. $\int \operatorname{tg}^n x\,dx = \frac{\operatorname{tg}^{n-1} x}{n-1} - \int \operatorname{tg}^{n-2} x\,dx \quad (n \geqslant 2).$

37.5. $\int \operatorname{ctg} x\,dx = \ln|\sin x|.$

37.6. $\int \operatorname{ctg}^2 x\,dx = -\operatorname{ctg} x - x.$

37.7. $\int \operatorname{ctg}^3 x\,dx = -\frac{\operatorname{ctg}^2 x}{2} - \ln|\sin x|.$

37.8. $\int \operatorname{ctg}^n x\,dx = -\frac{\operatorname{ctg}^{n-1} x}{n-1} - \int \operatorname{ctg}^{n-2} x\,dx \quad (n \geqslant 2).$

37.9. $\int \frac{dx}{\operatorname{tg} x \pm 1} = \pm\frac{x}{2} + \frac{1}{2}\ln|\sin x \pm \cos x|.$

37.10. $\int \frac{dx}{a + b\operatorname{tg} x} = \frac{1}{a^2+b^2}(b\ln|a + b\operatorname{tg} x| + b\ln|\cos x| + ax).$

37.11. $\int \frac{\operatorname{tg} x\,dx}{\operatorname{tg} x \pm 1} = \frac{x}{2} \mp \frac{1}{2}\ln|\sin x \pm \cos x|.$

37.12. $\int \frac{\operatorname{tg} x\,dx}{a + b\operatorname{tg} x} = \frac{1}{a^2+b^2}(bx - a\ln|a\cos x + b\sin x|).$

37.13. $\int \frac{dx}{1+\operatorname{tg}^2 x} = \frac{x}{2} + \frac{1}{4}\sin 2x.$

37.14. $\int \frac{dx}{a^2 + b^2\operatorname{tg}^2 x} = \frac{1}{a^2-b^2}\left[x - \left|\frac{b}{a}\right|\operatorname{arctg}\left(\left|\frac{b}{a}\right|\operatorname{tg} x\right)\right] \quad (a^2 \neq b^2)$

37.15. $\int \frac{dx}{a^2 - b^2\operatorname{tg}^2 x} = \frac{1}{a^2+b^2}\left(x + \frac{b}{2a}\ln\left|\frac{a + b\operatorname{tg} x}{a - b\operatorname{tg} x}\right|\right).$

37.16. $\int \frac{\operatorname{tg} x\,dx}{1+\operatorname{tg}^2 x} = -\frac{\cos^2 x}{2}.$

37.17. $\int \frac{\operatorname{tg} x\,dx}{1 + a^2\operatorname{tg}^2 x} = \frac{\ln(\cos^2 x + a^2\sin^2 x)}{2(a^2-1)} \quad (a \neq 1).$

37.18. $\int \frac{dx}{\operatorname{ctg} x \pm 1} = \frac{x}{2} \pm \frac{1}{2}\ln|\sin x \pm \cos x|.$

37.19. $\int \frac{dx}{a + b\operatorname{ctg} x} = \int \frac{\operatorname{tg} x\,dx}{a\operatorname{tg} x + b}$ (см. 37.12).

37.20. $\int \frac{\operatorname{ctg} x\,dx}{\operatorname{ctg} x \pm 2} = \pm\int \frac{dx}{\operatorname{tg} x \pm 1}$ (см. 37.9).

37.21. $\int \frac{\operatorname{ctg} x\,dx}{a + b\operatorname{ctg} x} = \int \frac{dx}{a\operatorname{tg} x + b}$ (см. 37.10).

37.22. $\int \frac{dx}{1+\operatorname{ctg}^2 x} = \frac{x}{2} - \frac{1}{4}\sin 2x.$

37.23. $\int \frac{dx}{a^2 + b^2\operatorname{ctg}^2 x} = \frac{1}{a^2-b^2}\left[x - \left|\frac{b}{a}\right|\operatorname{arctg}\left(-\left|\frac{b}{a}\right|\operatorname{ctg} x\right)\right]$ $(a^2 \neq b^2).$

37.24. $\int \frac{dx}{a^2 - b^2\operatorname{ctg}^2 x} = \frac{1}{a^2+b^2}\left(x + \frac{b}{2a}\ln\left|\frac{a - b\operatorname{ctg} x}{a + b\operatorname{ctg} x}\right|\right).$

37.25. $\int \frac{\operatorname{ctg} x\,dx}{1+\operatorname{ctg}^2 x} = \frac{\sin^2 x}{2}.$

37.26. $\int \frac{\operatorname{ctg} x\,dx}{1 + a^2\operatorname{ctg}^2 x} = \frac{1}{a^2}\int \frac{\operatorname{tg} x\,dx}{1 + \frac{1}{a^2}\operatorname{tg}^2 x}$ (см. 37.17).

37.27. $\int \frac{\operatorname{tg} x\,dx}{1 \pm \operatorname{ctg} x} = \mp\frac{x}{2} + \frac{1}{2}\ln\left|\frac{\sin x \pm \cos x}{\cos^2 x}\right|.$

37.28. $\int \frac{\operatorname{ctg} x\,dx}{1 \pm \operatorname{tg} x} = \mp\frac{x}{2} + \frac{1}{2}\ln|\sin^2 x(\sin x \pm \cos x)|.$

37.29.* $\int x\operatorname{tg} x\,dx = \frac{x^3}{3} + \frac{x^5}{15} + \ldots + \frac{2^{2n}(2^{2n}-1)B_n}{(2n+1)!}x^{2n+1} + \ldots$

37.30.* $\int x\operatorname{ctg} x\,dx = x - \frac{x^3}{9} - \frac{x^5}{225} - \ldots - \frac{2^{2n}B_n}{(2n+1)!}x^{2n+1} - \ldots$

37.31. $\int \frac{\operatorname{tg} x\,dx}{\sqrt{a + b\operatorname{tg}^2 x}} = \frac{1}{\sqrt{b-a}}\arccos\left(\frac{\sqrt{b-a}}{\sqrt{b}}\cos x\right).$

ТАБЛИЦА 38

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\arcsin\frac{x}{a}\,dx,\quad \int x^{\pm n}\arccos\frac{x}{a}\,dx;\quad n=0,\ 1,\ 2,\ldots$$

38.1. $\int \arcsin\frac{x}{a}\,dx = x\arcsin\frac{x}{a} + \sqrt{a^2-x^2}.$

38.2. $\int \left(\arcsin\frac{x}{a}\right)^2 dx = x\left(\arcsin\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2-x^2}\arcsin\frac{x}{a}.$

38.3. $\int \left(\arcsin\frac{x}{a}\right)^3 dx =$

$$= \left(\arcsin\frac{x}{a}\right)^3\left[x + \frac{3\sqrt{a^2-x^2}}{\arcsin\frac{x}{a}} - \frac{6x}{\left(\arcsin\frac{x}{a}\right)^2} - \frac{6\sqrt{a^2-x^2}}{\left(\arcsin\frac{x}{a}\right)^3}\right].$$

38.4. $\int \left(\arcsin\frac{x}{a}\right)^n dx = a\int t^n\cos t\,dt$, где $t=\arcsin\frac{x}{a}$ (см. 32.16).

38.5. $\int x\arcsin\frac{x}{a}\,dx = \frac{2x^2-a^2}{4}\arcsin\frac{x}{a} + \frac{x}{4}\sqrt{a^2-x^2}.$

38.6. $\int x^2\arcsin\frac{x}{a}\,dx = \frac{x^3}{3}\arcsin\frac{x}{a} + \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2}.$

38.7. $\int x^3\arcsin\frac{x}{a}\,dx = \frac{8x^4-3a^4}{32}\arcsin\frac{x}{a} + \frac{2x^3+3xa^2}{32}\sqrt{a^2-x^2}.$

38.8. $\int x^4\arcsin\frac{x}{a}\,dx = \frac{x^5}{5}\arcsin\frac{x}{a} + \frac{3x^4+4x^2a^2+8a^4}{75}\sqrt{a^2-x^2}.$

38.9. $\int x^n\arcsin\frac{x}{a}\,dx = \frac{x^{n+1}}{n+1}\arcsin\frac{x}{a} - \frac{1}{n+1}\int\frac{x^{n+1}}{\sqrt{a^2-x^2}}\,dx.$

38.10.* $\int \frac{1}{x}\arcsin\frac{x}{a}\,dx =$

$$= \frac{x}{a} + \frac{1\cdot x^3}{2\cdot 3\cdot 3a^3} + \frac{1\cdot 3x^5}{2\cdot 4\cdot 5\cdot 5a^5} + \frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7\cdot 7a^7} + \ldots$$

38.11. $\int \frac{1}{x^n}\arcsin\frac{x}{a}\,dx =$

$$= -\frac{\arcsin\frac{x}{a}}{(n-1)x^{n-1}} + \frac{1}{n-1}\int\frac{dx}{x^{n-1}\sqrt{a^2-x^2}} \quad (n\geqslant 2).$$

38.12. $\int \arccos\frac{x}{a}\,dx = x\arccos\frac{x}{a} - \sqrt{a^2-x^2}.$

38.13. $\int \left(\arccos\frac{x}{a}\right)^2 dx = x\left(\arccos\frac{x}{2}\right)^2 - 2x - 2\sqrt{a^2-x^2}\arccos\frac{x}{a}.$

38.14. $\int \left(\arccos\frac{x}{a}\right)^3 dx =$

$$= \left[3\left(\arccos\frac{x}{a}\right)^2 - 6\right]\sqrt{a^2-x^2} - \left[\left(\arccos\frac{x}{a}\right)^3 - 6\arccos\frac{x}{a}\right]x.$$

38.15. $\int \left(\arccos\frac{x}{a}\right)^n dx = -a\int t^n\sin t\,dt$, где $t=\arccos\frac{x}{a}$ (см. 29.16).

38.16. $\int x\arccos\frac{x}{a}\,dx = \frac{2x^2-a^2}{4}\arccos\frac{x}{a} - \frac{x}{4}\sqrt{a^2-x^2}.$

38.17. $\int x^2\arccos\frac{x}{a}\,dx = \frac{x^3}{3}\arccos\frac{x}{a} - \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2}.$

38.18. $\int x^3\arccos\frac{x}{a}\,dx = \frac{8x^4-3a^4}{32}\arccos\frac{x}{a} - \frac{3xa^2+2x^3}{32}\sqrt{a^2-x^2},$

38.19. $\int x^4\arccos\frac{x}{a}\,dx = \frac{x^5}{5}\arccos\frac{x}{a} - \frac{1}{75}(3x^4+4x^2a^2+8a^4)\sqrt{a^2-x^2}.$

38.20. $\int x^n\arccos\frac{x}{a}\,dx = \frac{x^{n+1}}{n+1}\arccos\frac{x}{a} + \frac{1}{n+1}\int\frac{x^{n+1}}{\sqrt{a^2-x^2}}\,dx$

(см. 22.18).

38.21.* $\int \frac{1}{x}\arccos\frac{x}{a}\,dx =$

$$= \frac{\pi}{2}\ln|x| - \frac{x}{a} - \frac{1}{2\cdot 3\cdot 3}\frac{x^3}{a^3} - \frac{1\cdot 3}{2\cdot 4\cdot 5\cdot 5}\frac{x^5}{a^5} - \ldots$$

38.22. $\int \frac{1}{x^n}\arccos\frac{x}{a}\,dx = \frac{\arccos\frac{x}{a}}{(n-1)x^{n-1}} - \frac{1}{n-1}\int\frac{dx}{x^{n-1}\sqrt{a^2-x^2}}$ $(n\geqslant 2)$

(см. табл. 23).

ТАБЛИЦА 39

ИНТЕГРАЛЫ ВИДА

$$\int F\left(x,\ \operatorname{arctg}\frac{x}{a}\right)dx, \qquad \int F\left(x,\ \operatorname{arcctg}\frac{x}{a}\right)dx.$$

39.1. $\int \operatorname{arctg}\frac{x}{a}\,dx = x\operatorname{arctg}\frac{x}{a} - \frac{a}{2}\ln(a^2+x^2).$

39.2. $\int\left(\operatorname{arctg}\frac{x}{a}\right)^2 dx = x\left(\operatorname{arctg}\frac{x}{a}\right)^2 - 2a\int\frac{x\operatorname{arctg}\frac{x}{a}}{a^2+x^2}\,dx$ (см. 39.11).

39.3. $\int x\operatorname{arctg}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\operatorname{arctg}\frac{x}{a} - \frac{ax}{2}.$

39.4. $\int x^2\operatorname{arctg}\frac{x}{a}\,dx = \frac{x^3}{3}\operatorname{arctg}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(a^2+x^2).$

39.5. $\int x^3\operatorname{arctg}\frac{x}{a}\,dx = \frac{1}{4}(x^4-a^4)\operatorname{arctg}\frac{x}{a} - \frac{ax^3}{12} + \frac{a^3x}{4}.$

39.6. $\int x^n\operatorname{arctg}\frac{x}{a}\,dx = \frac{x^{n+1}}{n+1}\operatorname{arctg}\frac{x}{a} - \frac{a}{n+1}\int\frac{x^{n+1}}{a^2+x^2}\,dx$ (см. 5.15)

39.7.* $\int\frac{1}{x}\operatorname{arctg}\frac{x}{a}\,dx = \sum_{\nu=0}^{\infty}(-1)^\nu\frac{x^{2\nu+1}}{(2\nu+1)^2\,a^{2\nu+1}}.$

39.8. $\int\frac{1}{x^2}\operatorname{arctg}\frac{x}{a} = -\frac{1}{x}\operatorname{arctg}\frac{x}{a} - \frac{1}{2a}\ln\frac{a^2+x^2}{x^2}.$

39.9. $\int\frac{1}{x^3}\operatorname{arctg}\frac{x}{a}\,dx = -\frac{a^2+x^2}{2a^2x^2}\operatorname{arctg}\frac{x}{a} - \frac{1}{2ax}.$

39.10. $\int\frac{1}{x^n}\operatorname{arctg}\frac{x}{a}\,dx = -\frac{1}{(n-1)\,x^{n-1}}\operatorname{arctg}\frac{x}{a} + \frac{a}{n-1}\int\frac{dx}{x^{n-1}(a^2+x^2)}$ $(n\geqslant 2)$ (см. 5.23).

39.11.* $\int\frac{x\operatorname{arctg}x}{1+x^2}\,dx = \frac{1}{2}\operatorname{arctg}x\ln(1+x^2) - \frac{x^3}{6} - \frac{x^5}{60} - \frac{x^7}{325} - \ldots - \frac{2^{n-1}(2^{2n}-1)\,B_n}{n(2n+1)!}\,x^{2n+1} - \ldots$

39.12. $\int\frac{x^2\operatorname{arctg}x}{1+x^2}\,dx = x\operatorname{arctg}x - \frac{1}{2}\ln(1-x^2) - \frac{1}{2}(\operatorname{arctg}x)^2.$

39.13.* $\int\frac{x^3\operatorname{arctg}x}{1+x^2}\,dx = -\frac{1}{2}x + \frac{1}{2}(1+x^2)\operatorname{arctg}x - \int\frac{x\operatorname{arctg}x}{1+x^2}\,dx$ (см. 39.11.)

39.14. $\int\frac{x^4\operatorname{arctg}x}{1+x^2}\,dx = -\frac{1}{6}x^2 + \frac{2}{3}\ln(1+x^2) + \left(\frac{x^3}{6} - x\right)\operatorname{arctg}x + \frac{1}{2}(\operatorname{arctg}x)^2.$

39.15. $\int\frac{x\operatorname{arctg}x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}\operatorname{arctg}x + \sqrt{2}\operatorname{arctg}\frac{x\sqrt{2}}{\sqrt{1-x^2}} - \arcsin x.$

39.16. $\int\frac{\operatorname{arctg}x}{(\alpha+\beta x)^2}\,dx = \frac{1}{\alpha^2+\beta^2}\left[\ln\left|\frac{\alpha+\beta x}{\sqrt{1+x^2}}\right| - \frac{\beta-\alpha x}{\alpha+\beta x}\operatorname{arctg}x\right].$

39.17. $\int\operatorname{arcctg}\frac{x}{a}\,dx = x\operatorname{arcctg}\frac{x}{a} + \frac{a}{2}\ln(a^2+x^2).$

39.18. $\int x^n\operatorname{arcctg}\frac{x}{a}\,dx = \frac{x^{n+1}}{n+1}\operatorname{arcctg}\frac{x}{a} + \frac{a}{n+1}\int\frac{x^{n+1}}{a^2+x^2}\,dx$ (см. 5.15).

39.19.* $\int\frac{1}{x}\operatorname{arcctg}\frac{x}{a}\,dx = \frac{\pi}{2}\ln x - \sum_{\nu=0}^{\infty}(-1)^\nu\frac{x^{2\nu+1}}{(2\nu+1)\,a^{2\nu+1}}.$

39.20. $\int\frac{1}{x^n}\operatorname{arcctg}\frac{x}{a}\,dx = -\frac{1}{(n-1)\,x^{n-1}}\operatorname{arcctg}\frac{x}{a} - \frac{a}{n+1}\int\frac{dx}{x^{n-1}(a^2+x^2)}$ $(n\geqslant 2)$ (см. 5.23).

39.21.* $\int\frac{x\operatorname{arcctg}x}{1+x^2}\,dx = \frac{\pi}{4}\ln(1+x^2) - \int\frac{x\operatorname{arctg}x}{1+x^2}\,dx$ (см. 39.11).

39.22. $\int F(\operatorname{arcctg}x)\,dx = \int F\left(\frac{\pi}{2} - \operatorname{arctg}x\right)dx.$

ТАБЛИЦА 40
ИНТЕГРАЛЫ ВИДА

$$\int F\left(x,\ \operatorname{arcsec}\frac{x}{a}\right)dx, \qquad \int F\left(x,\ \operatorname{arccosec}\frac{x}{a}\right)dx.$$

40.1. $$\int \operatorname{arcsec}\frac{x}{a}\,dx = \begin{cases} x\operatorname{arcsec}\frac{x}{a} - a\ln\left|x+\sqrt{x^2-a^2}\right| \\ \qquad \text{при } 0<\operatorname{arcsec}\frac{x}{a}<\frac{\pi}{2}; \\ x\operatorname{arcsec}\frac{x}{a} + a\ln\left|x+\sqrt{x^2-a^2}\right| \\ \qquad \text{при } \frac{\pi}{2}<\operatorname{arcsec}\frac{x}{a}<\pi. \end{cases}$$

40.2. $$\int x\operatorname{arcsec}\frac{x}{a}\,dx = \begin{cases} \frac{x^2}{2}\operatorname{arcsec}\frac{x}{a} - \frac{a}{2}\sqrt{x^2-a^2} \\ \qquad \text{при } 0<\operatorname{arcsec}\frac{x}{a}<\frac{\pi}{2}; \\ \frac{x^2}{2}\operatorname{arcsec}\frac{x}{a} + \frac{a}{2}\sqrt{x^2-a^2} \\ \qquad \text{при } \frac{\pi}{2}<\operatorname{arcsec}\frac{x}{a}<\pi. \end{cases}$$

40.3. $$\int x^2\operatorname{arcsec}\frac{x}{a}\,dx = \begin{cases} \frac{x^3}{3}\operatorname{arcsec}\frac{x}{a} - \frac{ax}{6}\sqrt{x^2-a^2} - \\ \qquad -\frac{a^3}{6}\ln\left|x+\sqrt{x^2-a^2}\right| \\ \qquad \text{при } 0<\operatorname{arcsec}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^3}{3}\operatorname{arcsec}\frac{x}{a} + \frac{ax}{6}\sqrt{x^2-a^2} + \\ \qquad +\frac{a^3}{6}\ln\left|x+\sqrt{x^2-a^2}\right| \\ \qquad \text{при } \frac{\pi}{2}<\operatorname{arcsec}\frac{x}{a}<\pi. \end{cases}$$

40.4. $$\int x^n\operatorname{arcsec}\frac{x}{a}\,dx = \begin{cases} \frac{x^{n+1}}{n+1}\operatorname{arcsec}\frac{x}{a} - \frac{a}{n+1}\int\frac{x^n dx}{\sqrt{x^2-a^2}} \\ \qquad \text{при } 0<\operatorname{arcsec}\frac{x}{a}<\frac{\pi}{2}; \\ \frac{x^{n+1}}{n+1}\operatorname{arcsec}\frac{x}{a} + \frac{a}{n+1}\int\frac{x^n\,dx}{\sqrt{x^2-a^2}} \\ \qquad \text{при } \frac{\pi}{2}<\operatorname{arcsec}\frac{x}{a}<\pi \quad (\text{см. } 25.7 \end{cases}$$

40.5.* $$\int\frac{1}{x}\operatorname{arcsec}\frac{x}{a}\,dx = \frac{\pi}{2}\ln|x| + \frac{a}{x} + \frac{a^3}{2\cdot3\cdot3x^3} + \frac{1\cdot3\cdot a^5}{2\cdot4\cdot5\cdot5x^5} + \frac{1\cdot3\cdot5\cdot a^7}{2\cdot4\cdot6\cdot7\cdot7x^7} + \ldots \quad \left(0<\operatorname{arcsec}\frac{x}{a}<\frac{\pi}{2}\right).$$

40.6. $$\int\frac{1}{x^2}\operatorname{arcsec}\frac{x}{a}\,dx = \frac{\sqrt{x^2-a^2}}{ax} - \frac{1}{x}\operatorname{arcsec}\frac{x}{a}\left(0<\operatorname{arcsec}\frac{x}{a}<\frac{\pi}{2}\right).$$

40.7. $$\int\frac{1}{x^3}\operatorname{arcsec}\frac{x}{a}\,dx = -\frac{1}{2x^2}\operatorname{arcsec}\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{4ax^2} + \frac{1}{4a^2}\arccos\left|\frac{a}{x}\right| \quad \left(0<\operatorname{arcsec}\frac{x}{a}<\frac{\pi}{2}\right).$$

40.8. $$\int\frac{1}{x^n}\operatorname{arcsec}\frac{x}{a}\,dx = \begin{cases} -\frac{1}{(n-1)x^{n-1}}\operatorname{arcsec}\frac{x}{a} + \frac{a}{n-1}\int\frac{dx}{x^n\sqrt{x^2-a^2}} \\ \qquad \text{при } 0<\operatorname{arcsec}\frac{x}{a}<\frac{\pi}{2}; \\ -\frac{1}{(n-1)x^{n-1}}\operatorname{arcsec}\frac{x}{a} - \frac{a}{n-1}\int\frac{dx}{x^n\sqrt{x^2-a^2}} \\ \qquad \text{при } \frac{\pi}{2}<\operatorname{arcsec}\frac{x}{a}<\pi \quad (n\geqslant 2) \quad (\text{см. табл. } 25). \end{cases}$$

40.9. $$\int\operatorname{arccosec}\frac{x}{a}\,dx = \begin{cases} x\operatorname{arccosec}\frac{x}{a} + a\ln\left|x+\sqrt{x^2-a^2}\right| \\ \qquad \text{при } 0<\operatorname{arccosec}\frac{x}{a}<\frac{\pi}{2}; \\ x\operatorname{arccosec}\frac{x}{a} - a\ln\left|x+\sqrt{x^2-a^2}\right| \\ \qquad \text{при } -\frac{\pi}{2}<\operatorname{arccosec}\frac{x}{a}<0. \end{cases}$$

40.10. $$\int x\operatorname{arccosec}\frac{x}{a}\,dx = \begin{cases} \frac{x^2}{2}\operatorname{arccosec}\frac{x}{a} - \frac{a}{2}\sqrt{x^2-a^2} & \text{при } -\frac{\pi}{2}<\operatorname{arccosec}\frac{x}{a}<0, \\ \frac{x^2}{2}\operatorname{arccosec}\frac{x}{a} + \frac{a}{2}\sqrt{x^2-a^2} & \text{при } 0<\operatorname{arccosec}\frac{x}{a}<\frac{\pi}{2}. \end{cases}$$

40.11.* $$\int\frac{1}{x}\operatorname{arccosec}\frac{x}{a}\,dx = -\frac{a}{x} - \frac{1}{2\cdot3}\frac{a^3}{x^3} - \frac{1\cdot3}{2\cdot4\cdot5}\frac{a^5}{x^5} - \frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot7}\frac{a^7}{x^7} - \ldots \quad \left(0<\operatorname{arccosec}\frac{x}{a}<\frac{\pi}{2}\right).$$

40.12. $$\int F(\operatorname{arccosec} x)\,dx = \int F\left(\frac{\pi}{2} - \operatorname{arcsec} x\right)dx.$$

ТАБЛИЦА 41

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n} e^{ax}\,dx, \quad \int x^n e^{\pm x^2}\,dx, \quad n=0,\ 1,\ 2,\ \ldots$$

41.1. $\int A^{ax+b}\,dx = \frac{1}{a \ln A} A^{ax+b} \quad (A>0,\ A\neq 1).$

41.2. $\int F(e^{ax})\,dx = \frac{1}{a}\int \frac{F(t)\,dt}{t}$, где $t = e^{ax}$.

41.3. $\int xe^{ax}\,dx = \frac{ax-1}{a^2} e^{ax}.$

41.4. $\int x^2 e^{ax}\,dx = \frac{a^2x^2-2ax+2}{a^3} e^{ax}.$

41.5. $\int x^3 e^{ax}\,dx = \frac{a^3x^3-3a^2x^2+6ax-6}{a^4} e^{ax}.$

41.6. $\int x^4 e^{ax}\,dx = \frac{a^4x^4-4a^3x^3+12a^2x^2-24ax+24}{a^5} e^{ax}.$

41.7. $\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax}\,dx;$

$$= e^{ax}\left[\frac{x^n}{a} - \frac{nx^{n-1}}{a^2} + \frac{n(n-1)x^{n-2}}{a^3} - \ldots + (-1)^{n-1}\frac{n!\,x}{a^n} + (-1)^n \frac{n!}{a^{n+1}}\right].$$

41.8. $\int P_n(x)\,e^{ax}\,dx = e^{ax}\left[\frac{P_n(x)}{a} - \frac{P_n'(x)}{a^2} + \ldots + (-1)^n \frac{P_n^{(n)}(x)}{a^{n+1}}\right].$

41.9.* $\int \frac{e^{ax}}{x}\,dx = \ln|x| + \frac{ax}{1!} + \frac{a^2x^2}{2\cdot 2!} + \frac{a^3x^3}{3\cdot 3!} + \frac{a^4x^4}{4\cdot 4!} + \ldots + \frac{a^nx^n}{n\cdot n!} + \ldots$

41.10.* $\int \frac{e^{ax}}{x^2}\,dx = -\frac{e^{ax}}{x} + a\int \frac{e^{ax}}{x}\,dx$ (см. 41.8).

41.11.* $\int \frac{e^{ax}}{x^3}\,dx = \frac{ax-1}{2x^2} e^{ax} + \frac{a^2}{2}\int \frac{e^{ax}}{x}\,dx$ (см. 41.8).

41.12.* $\int \frac{e^{ax}}{x^4}\,dx = -\frac{e^{ax}}{3x^3} - \frac{ae^{ax}}{6x^2} - \frac{a^2e^{ax}}{6x} + \frac{a^3}{6}\int \frac{e^{ax}}{x}\,dx$ (см. 41.8).

41.13.* $\int \frac{e^{ax}\,dx}{x^n} = -\frac{e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{e^{ax}}{x^{n-1}}\,dx \quad (n\geqslant 2).$

41.14.* $\int e^{\pm x^2}\,dx =$

$$= x \pm \frac{x^3}{1!\cdot 3} + \frac{x^5}{2!\cdot 5} \pm \frac{x^7}{3!\cdot 7} + \ldots + (-1)^n \frac{x^{2n+1}}{n!\,(2n+1)} + \ldots$$

41.15. $\int xe^{x^2}\,dx = \frac{1}{2} e^{x^2}.$

41.16.* $\int x^2 e^{x^2}\,dx = \frac{1}{2} xe^{x^2} - \frac{1}{2}\int e^{x^2}\,dx$ (см. 41.13).

41.17. $\int x^3 e^{x^2}\,dx = \frac{x^2-1}{2} e^{x^2}.$

41.18. $\int x^4 e^{x^2}\,dx = \frac{2x^3-3x}{4} e^{x^2} + \frac{3}{4}\int e^{x^2}\,dx$ (см. 41.13).

41.19. $\int x^{2k+1} e^{x^2}\,dx = \frac{e^{x^2}}{2}\left[x^{2k} - kx^{2k-2} + k(k-1)x^{2k-4} + \ldots + (-1)^{k-1} k!\,x^2 + (-1)^k k!\right].$

41.20.* $\int x^{2k} e^{x^2}\,dx = \frac{x^{2k}e^{x^2}}{2} - \frac{2k-1}{2}\int x^{2k-2} e^{x^2}\,dx.$

41.21. $\int xe^{-x^2}dx = -\frac{1}{2} e^{-x^2}.$

41.22. $\int x^n e^{-x^2}\,dx = -\frac{x^{n-1}e^{-x^2}}{2} + \frac{n-1}{2}\int x^{n-2} e^{-x^2}\,dx.$

ТАБЛИЦА 42

ИНТЕГРАЛЫ ВИДА

$$\int R(x)\,e^{ax}\,dx, \qquad \int R(x,\ \sqrt{x},\ e^{x},\ e^{\sqrt{x}})\,dx.$$

Обозначения: $u=\alpha+\beta x$, $t=\alpha-\beta x$, $s=\sqrt{x}$.

42.1.* $\displaystyle\int \frac{e^{ax}}{\alpha+\beta x}\,dx=\frac{e^{-\frac{a\alpha}{\beta}}}{\beta}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.2.* $\displaystyle\int \frac{xe^{ax}}{\alpha+\beta x}\,dx=\frac{e^{-ax}}{a\beta}-\frac{\alpha e^{-\frac{a\alpha}{\beta}}}{\beta^2}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.3.* $\displaystyle\int \frac{x^2e^{ax}}{\alpha+\beta x}\,dx=\frac{\beta^2(\beta x-\alpha)}{\alpha}e^{ax}+\frac{\alpha^2}{\beta^3}e^{-\frac{a\alpha}{\beta}}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.4.* $\displaystyle\int \frac{x^ne^{ax}}{\alpha+\beta x}\,dx=(-1)^n\frac{\alpha^n}{\beta^{n+1}}e^{-\frac{a\alpha}{\beta}}\sum_{\nu=0}^{n}\frac{(-1)^\nu C_n^\nu}{\alpha^\nu}\int u^{\nu-1}e^{\frac{a}{\beta}u}\,du$

(см. 41.7).

42.5. $\displaystyle\int \frac{e^{ax}\,dx}{(\alpha+\beta x)^2}=-\frac{e^{ax}}{\beta u}+\frac{ae^{-\frac{a\alpha}{\beta}}}{\beta^2}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.6. $\displaystyle\int \frac{xe^{ax}\,dx}{(\alpha+\beta x)^2}=\frac{\alpha e^{ax}}{\beta^2 u}+\frac{\beta-a\alpha}{\beta^3}e^{-\frac{a\alpha}{\beta}}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.7. $\displaystyle\int \frac{x^2e^{ax}}{(\alpha+\beta x)^2}\,dx=\frac{e^{ax}}{\beta^2}\left(\frac{\beta}{a}-\frac{\alpha^2}{u}\right)+\frac{a\alpha^2-2\alpha\beta}{\beta^4}e^{-\frac{a\alpha}{\beta}}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du.$

42.8. $\displaystyle\int \frac{x^ne^{ax}}{(\alpha+\beta x)^m}\,dx=(-1)^n\frac{\alpha^n}{\beta^{n+1}}e^{-\frac{a\alpha}{\beta}}\sum_{\nu=0}^{n}\frac{(-1)^\nu C_n^\nu}{\alpha^\nu}\int u^{\nu-m}e^{\frac{a}{\beta}u}\,du$

(см. 41.7 при $\nu-m\geqslant 0$ и 41.13 при $\nu-m<0$).

42.9* $\displaystyle\int \frac{e^{ax}}{\alpha^2-\beta^2x^2}\,dx=\frac{e^{-\frac{a\alpha}{\beta}}}{2\alpha\beta}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du-\frac{e^{\frac{a\alpha}{\beta}}}{2\beta a}\int \frac{e^{-\frac{a}{\beta}t}}{t}\,dt$ (см. 41.9).

42.10. $\displaystyle\int e^{\sqrt{x}}\,dx=2e^{\sqrt{x}}(\sqrt{x}-1).$

42.11. $\displaystyle\int \sqrt{x}\,e^{\sqrt{x}}\,dx=2e^{\sqrt{x}}(x-2\sqrt{x}+2).$

42.12. $\displaystyle\int xe^{\sqrt{x}}\,dx=2e^{\sqrt{x}}(x\sqrt{x}-3x+6\sqrt{x}-6).$

42.13. $\displaystyle\int x\sqrt{x}\,e^{\sqrt{x}}\,dx=2e^{\sqrt{x}}(x^2-4x\sqrt{x}+12x-24\sqrt{x}+24).$

42.14. $\displaystyle\int x^2e^{\sqrt{x}}\,dx=$

$$=2e^{\sqrt{x}}(x^2\sqrt{x}-5x^2+20x\sqrt{x}-60x+120\sqrt{x}-120).$$

42.15. $\displaystyle\int x^ne^{\sqrt{x}}\,dx=2e^{\sqrt{x}}[x^n\sqrt{x}-(2n+1)x^n+$

$$+(2n+1)2nx^{n-1}\sqrt{x}-\ldots+(2n+1)!\sqrt{x}-(2n+1)!]$$

42.16. $\displaystyle\int x^{n+\frac{1}{2}}e^{\sqrt{x}}\,dx=e^{\sqrt{x}}\left[x^{n+1}-(2n+2)x^{\frac{2n+1}{2}}\right]-$

$$-\frac{(2n+1)(2n+2)}{2}\int x^{n-\frac{1}{2}}e^{\sqrt{x}}\,dx.$$

42.17. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx=2e^{\sqrt{x}}.$

42.18.* $\displaystyle\int \frac{e^{\sqrt{x}}}{x}\,dx=2\int \frac{e^s}{s}\,ds$ (см. 41.9).

42.19.* $\displaystyle\int \frac{e^{\sqrt{x}}\,dx}{x\sqrt{x}}=-\frac{2e^{\sqrt{x}}}{\sqrt{x}}+\int \frac{e^{\sqrt{x}}}{x}\,dx$ (см. 42.18).

42.20.* $\displaystyle\int \frac{e^{\sqrt{x}}}{x^n}\,dx=2\int \frac{e^s}{s^{2n-1}}\,ds$ (см. 41.13).

42.21.* $\displaystyle\int \frac{e^{\sqrt{x}}}{x^{n+\frac{1}{2}}}\,dx=2\int \frac{e^s}{s^{2n}}\,ds$ (см. 41.13).

42.22.* $\displaystyle\int \sqrt{x}\,e^x\,dx=2\int s^2e^{s^2}\,ds$ (см. 41.16).

42.23.* $\displaystyle\int \frac{e^{\pm x}}{\sqrt{x}}\,dx=2\int e^{\pm s^2}\,ds$ (см. 41.14).

ТАБЛИЦА 43

ИНТЕГРАЛЫ ВИДА

$$\int \frac{e^{ax}\,dx}{(\alpha+\beta e^{ax})^m}, \quad m=1,\ 2,\ 3,\ \ldots$$

43.1. $\displaystyle\int \frac{dx}{\alpha+\beta e^x} = \frac{x}{\alpha} - \frac{1}{\alpha}\ln|\alpha+\beta e^x|.$

43.2. $\displaystyle\int \frac{dx}{(\alpha+\beta e^x)^2} = -\frac{1}{\alpha^2}\left[\ln\left|\frac{\alpha+\beta e^x}{e^x}\right| + \frac{\beta e^x}{\alpha+\beta e^x}\right].$

43.3 $\displaystyle\int \frac{dx}{(\alpha+\beta e^x)^3} = -\frac{1}{\alpha^3}\left[\ln\left|\frac{\alpha+\beta e^x}{e^x}\right| + \frac{2\beta e^x}{\alpha+\beta e^x} - \frac{\beta^2 e^{2x}}{2(\alpha+\beta e^x)^2}\right].$

43.4 $\displaystyle\int \frac{dx}{(\alpha+\beta e^x)^m} =$

$$= \frac{1}{\alpha^m}\left[\sum_{\nu=1}^{m-1} \frac{(-1)^\nu}{\nu} C_{m-1}^{\nu} \frac{\beta^\nu e^{\nu x}}{(\alpha+\beta e^x)^\nu} - \ln\left|\frac{\alpha+\beta e^x}{e^x}\right|\right].$$

43.5. $\displaystyle\int \frac{e^{ax}\,dx}{\alpha+\beta e^{ax}} = \frac{1}{a\beta}\ln|\alpha+\beta e^{ax}|.$

43.6. $\displaystyle\int \frac{e^{ax}\,dx}{(\alpha+\beta e^{ax})^2} = -\frac{1}{a\beta(\alpha+\beta e^{ax})}.$

43.7. $\displaystyle\int \frac{e^{ax}\,dx}{(\alpha+\beta e^{ax})^m} = -\frac{1}{(m-1)\,a\beta(\alpha+\beta e^{ax})^{m-1}} \qquad (m\geqslant 2).$

43.8. $\displaystyle\int \frac{e^{2ax}\,dx}{\alpha+\beta e^{ax}} = \frac{e^{ax}}{a\beta} - \frac{\alpha}{a\beta^2}\ln|\alpha+\beta e^{ax}|.$

43.9. $\displaystyle\int \frac{e^{2ax}\,dx}{(\alpha+\beta e^{ax})^2} = \frac{1}{a\beta^2}\left[\ln|\alpha+\beta e^{ax}| + \frac{\alpha}{\alpha+\beta e^{ax}}\right].$

43.10. $\displaystyle\int \frac{e^{2ax}\,dx}{(\alpha+\beta e^{ax})^m} =$

$$= \frac{\alpha}{(m-1)\,a\beta^2(\alpha+\beta e^{ax})^{m-1}} - \frac{1}{(m-2)\,a\beta^2(\alpha+\beta e^{ax})^{m-1}} \qquad (m>2).$$

43.11. $\displaystyle\int \frac{e^{3ax}\,dx}{\alpha+\beta e^{ax}} =$

$$= \frac{1}{a\beta^3}\left[\frac{(\alpha+\beta e^{ax})^2}{2} - 2\alpha(\alpha+\beta e^{ax}) + \alpha^2\ln|\alpha+\beta e^{ax}|\right].$$

43.12. $\displaystyle\int \frac{e^{nax}\,dx}{\alpha+\beta e^{ax}} = \frac{1}{a}\int \frac{t^{n-1}\,dt}{\alpha+\beta t}$, где $t=e^{ax}$ $(n\geqslant 1)$ (см. 1.20).

43.13. $\displaystyle\int \frac{e^{nax}\,dx}{(\alpha+\beta e^{ax})^m} =$

$$= \frac{ae^{(n-1)ax}}{(n-m)\,\beta(\alpha+\beta e^{ax})^{m-1}} - \frac{n\alpha}{(n-m)\,\beta}\int \frac{e^{(n-1)ax}\,dx}{(\alpha+\beta e^{ax})^m} \qquad (n\neq m).$$

43.14. $\displaystyle\int \frac{dx}{e^{ax}(\alpha+\beta e^{ax})} = \frac{1}{a\beta}\left(-\frac{\alpha+\beta e^{ax}}{\beta e^{ax}} + \ln|\alpha+\beta e^{ax}|\right) - \frac{x}{\alpha}.$

43.15. $\displaystyle\int \frac{dx}{e^{ax}(\alpha+\beta e^{ax})^2} = -\frac{1}{a\beta}\left[\frac{\alpha^2(\alpha+2\beta e^{ax})}{\beta e^{ax}(\alpha+\beta e^x)} - 2\ln\left|\frac{\alpha+\beta e^{ax}}{\beta e^{ax}}\right|\right].$

43.16. $\displaystyle\int \frac{dx}{e^{ax}(\alpha+\beta e^{ax})^3} =$

$$= -\frac{1}{a\beta}\left[\frac{2-6\left(\frac{\beta}{\alpha}\right)^2 e^{2ax} - 3\left(\frac{\beta}{\alpha}\right)^3 e^{3ax}}{2\frac{\beta}{\alpha}e^{ax}\left(1+\frac{\beta}{\alpha}e^{ax}\right)^2} - 2\ln\left|\frac{\alpha+\beta e^{ax}}{\beta e^{ax}}\right|\right].$$

43.17. $\displaystyle\int \frac{dx}{e^{ax}(\alpha+\beta e^{ax})^m} = -\frac{\alpha^2}{a\beta^2 e^{ax}(\alpha+\beta e^{ax})} - \frac{m\alpha^m}{a\beta}\int \frac{ds}{(\alpha+\beta e^s)^m}$,
где $s=ax$ (см. 43.4).

ТАБЛИЦА 44

ИНТЕГРАЛЫ ВИДА

$\int e^{ax}\sqrt{(\alpha\pm\beta e^{ax})^{\pm m}}\,dx$; $a\geqslant 0$, $\alpha>0$, $\beta>0$, $m=1, 3, 5, \ldots$

44.1 $$\int\frac{dx}{\sqrt{\alpha\pm\beta e^{ax}}}=\frac{1}{a\sqrt{\alpha}}\ln\left|\frac{\sqrt{\alpha\pm\beta e^{ax}}-\sqrt{\alpha}}{\sqrt{\alpha\pm\beta e^{ax}}+\sqrt{\alpha}}\right|.$$

44.2 $$\int\frac{dx}{\sqrt{(\alpha\pm\beta e^{ax})^3}}=\frac{2}{a\alpha\sqrt{\alpha\pm\beta e^{ax}}}-\frac{1}{a\alpha\sqrt{\alpha}}\ln\left|\frac{\sqrt{\alpha\pm\beta e^{ax}}-\sqrt{\alpha}}{\sqrt{\alpha\pm\beta e^{ax}}+\sqrt{\alpha}}\right|.$$

44.3 $$\int\frac{dx}{\sqrt{(\alpha\pm\beta e^{ax})^m}}=$$
$$=\frac{1}{a}\left[\frac{2}{(m-2)\,\alpha\sqrt{(\alpha\pm\beta e^{ax})^{m-2}}}+\frac{1}{\alpha}\int\frac{dx}{\sqrt{(\alpha\pm\beta e^{ax})^{m-2}}}\right]\quad(m\geqslant 3).$$

44.4. $$\int\frac{dx}{\sqrt{e^x-1}}=2\operatorname{arctg}\sqrt{e^x-1}\,.$$

44.5. $$\int\frac{e^x\,dx}{\sqrt{1\pm e^x}}=\pm 2\sqrt{1\pm e^x}\,.$$

44.6. $$\int\frac{e^x\,dx}{\sqrt{(1\pm e^x)^3}}=\mp\frac{2}{\sqrt{1\pm e^x}}\,.$$

44.7 $$\int\frac{e^{ax}\,dx}{\sqrt{(\alpha\pm\beta e^{ax})^m}}=\mp\frac{2}{a\beta\sqrt{(\alpha\pm\beta e^{ax})^{m-2}}}\,.$$

44.8. $$\int\frac{dx}{e^x\sqrt{1\pm e^x}}=-\frac{\sqrt{1\pm e^x}}{e^x}-\frac{1}{2}\ln\left|\frac{\sqrt{1\pm e^x}-1}{\sqrt{1\pm e^x}+1}\right|.$$

44.9. $$\int\frac{dx}{e^x\sqrt{(1\pm e^x)^3}}=$$
$$=-\frac{1}{e^x\sqrt{1\pm e^x}}\mp\frac{3}{\sqrt{1\pm e^x}}-\frac{3}{2}\ln\left|\frac{\sqrt{1\pm e^x}-1}{\sqrt{1\pm e^x}+1}\right|.$$

44.10. $$\int\frac{dx}{e^x\sqrt{(\alpha\pm\beta e^{ax})^m}}=$$
$$=\frac{-1}{a}\left[\frac{1}{e^{ax}\sqrt{(\alpha\pm\beta e^{ax})^m}}+\frac{m}{2}\int\frac{dx}{\sqrt{(\alpha\pm\beta e^{ax})^{m+2}}}\right]\quad\text{(см. 44.3).}$$

44.11. $$\int\sqrt{\alpha\pm\beta e^{ax}}\,dx=\frac{\sqrt{\alpha}}{a}\ln\left|\frac{\sqrt{\alpha\pm\beta e^{ax}}-\sqrt{\alpha}}{\sqrt{\alpha\pm\beta e^{ax}}+\sqrt{\alpha}}\right|+\frac{2}{a}\sqrt{\alpha\pm\beta e^{ax}}\,.$$

44.12. $$\int\sqrt{(\alpha\pm\beta e^{ax})^3}\,dx=\frac{2}{3a}\sqrt{(\alpha\pm\beta e^{ax})^3}+$$
$$+\frac{2\alpha}{a}\sqrt{\alpha\pm\beta e^{ax}}+\frac{\alpha^2}{a}\int\frac{dt}{t\sqrt{\alpha\pm\beta t}},\quad\text{где } t=e^{ax}\quad\text{(см. 16.12).}$$

44.13. $$\int\sqrt{(\alpha\pm\beta e^{ax})^m}\,dx=\frac{1}{a}\left[2\sqrt{\alpha\pm\beta e^{ax}}\sum_{\nu=0}^{\frac{m-3}{2}}\frac{(\alpha\pm\beta e^{ax})^{\frac{m-1}{2}-\nu}}{m-2\nu}\alpha^\nu+\right.$$
$$\left.+\alpha^{\frac{m-1}{2}}\sqrt{\alpha\pm\beta e^{ax}}+\alpha^{\frac{m+1}{2}}\ln\left|\frac{\sqrt{\alpha\pm\beta e^{ax}}-\sqrt{\alpha}}{\sqrt{\alpha\pm\beta e^{ax}}+\sqrt{\alpha}}\right|\right].$$

44.14. $$\int e^x\sqrt{1\pm e^x}\,dx=\pm\frac{2\sqrt{(1\pm e^x)^3}}{3}\,.$$

44.15. $$\int e^x\sqrt{(1\pm e^x)^3}\,dx=\pm\frac{2\sqrt{(1\pm e^x)^5}}{5}\,.$$

44.16. $$\int e^{ax}\sqrt{(\alpha\pm\beta e^{ax})^m}\,dx=\frac{2\sqrt{\alpha\pm\beta e^{ax}}}{(m+1)\,\beta}\,.$$

44.17 $$\int\frac{\sqrt{\alpha\pm\beta e^{ax}}}{e^{ax}}\,dx=-\frac{\sqrt{\alpha\pm\beta e^{ax}}}{ae^{ax}}\pm\frac{\beta}{a\sqrt{\alpha}}\ln\left|\frac{\sqrt{\alpha\pm\beta e^{ax}}-\sqrt{\alpha}}{\sqrt{\alpha\pm\beta e^{ax}}+\sqrt{\alpha}}\right|.$$

44.18. $$\int\frac{\sqrt{(\alpha\pm\beta e^{ax})^5}}{e^{ax}}\,dx=-\frac{\sqrt{(\alpha\pm\beta e^{ax})^5}}{a\alpha e^{ax}}+$$
$$+\frac{3\beta}{2a\alpha}\left[\frac{2\sqrt{\alpha\pm\beta e^{ax}}}{3}+2\alpha\sqrt{\alpha\pm\beta e^{ax}}+\alpha^2\int\frac{dt}{t\sqrt{\alpha\pm\beta t}}\right],$$
где $t=e^{ax}$ (см. 16.12).

44.19. $$\int\frac{\sqrt{(\alpha\pm\beta e^{ax})^m}}{e^{ax}}\,dx=\frac{1}{a}\int\frac{\sqrt{(\alpha\pm\beta t)^m}\,dt}{t^2},\quad\text{где } t=e^{ax}\quad\text{(см. 17.16).}$$

ТАБЛИЦА 45

ИНТЕГРАЛЫ ВИДА

$$\int e^{ax}\sin^n px\,dx,\quad \int e^{ax}\cos^n px\,dx;$$

$$\int e^{ax}\sin^m x\cos^n x\,dx;\quad m, n = 1, 2, 3, \ldots$$

45.1. $$\int e^{ax}\sin px\,dx = \frac{e^{ax}(a\sin px - p\cos px)}{a^2+p^2}.$$

45.2. $$\int e^{ax}\sin^2 px\,dx = \frac{e^{ax}}{a^2+4p^2}\left(a\sin^2 px - 2p\cos px\sin px + \frac{2p^2}{a}\right).$$

45.3. $$\int e^{ax}\sin^3 px\,dx = \frac{e^{ax}}{a^2+9p^2}\left[a\sin^3 px - 3p\cos px\cdot\sin^2 px + \frac{6p^2}{a^2+p^2}(a\sin px - p\cos px)\right]$$

45.4. $$\int e^{ax}\sin^n px\,dx = \frac{e^{ax}\sin^{n-1} px}{a^2+n^2p^2}\left[a\sin px - np\cos px\right] + \frac{n(n-1)p^2}{a^2+n^2p^2}\int e^{ax}\sin^{n-2} px\,dx$$

45.5. $$\int e^{ax}\cos px\,dx = \frac{e^{ax}(a\cos px + p\sin px)}{a^2+p^2}.$$

45.6. $$\int e^{ax}\cos^2 px\,dx = \frac{e^{ax}}{a^2+4p^2}\left(a\cos^2 px - 2p\cos px\sin px + \frac{2p^2}{a}\right).$$

45.7. $$\int e^{ax}\cos^3 px\,dx = \frac{e^{ax}}{a^2+9p^2}\left[a\cos^3 px + 3p\sin px\cos^2 px + \frac{6p^2}{a^2+p^2}(a\cos px + p\sin px)\right].$$

45.8. $$\int e^{ax}\cos^n px\,dx = \frac{e^{ax}\cos^{n-1} px}{a^2+n^2p^2}[a\cos px + np\sin px] + \frac{n(n-1)p^2}{a^2+n^2p^2}\int e^{ax}\cos^{n-2} px\,dx.$$

45.9. $$\int e^{ax}\sin px\cos qx\,dx = \frac{ae^{ax}}{2}\left[\frac{\sin(p+q)x}{a^2+(p+q)^2} + \frac{\sin(p-q)x}{a^2+(p-q)^2}\right] - \frac{e^{ax}}{2}\left[\frac{(p+q)\cos(p+q)x}{a^2+(p+q)^2} + \frac{(p-q)\cos(p-q)x}{a^2+(p-q)^2}\right].$$

45.10. $$\int e^{ax}\sin px\cos px\,dx = \frac{e^{ax}}{2a^2+8p^2}(a\sin 2px - 2p\cos 2px).$$

45.11. $$\int e^{ax}\sin^2 px\cos px\,dx = \frac{e^{ax}}{a^2+9p^2}\left[\frac{a^2+3p^2}{a^2+p^2}(a\cos px + p\sin px) - \cos^2 px\,(a\cos px + 3p\sin px)\right].$$

45.12. $$\int e^{ax}\sin^2 px\cos^2 px\,dx = \frac{1}{4}\int e^{ax}\sin^2 2px\,dx \quad (\text{см. } 45.2).$$

45.13. $$\int e^{ax}\sin px\cos^2 px\,dx = \frac{e^{ax}}{a^2+9p^2}\left[\frac{a^2+3p^2}{a^2+p^2}(a\sin px - p\cos px) - \sin^2 px\,(a\sin px + 3p\cos px)\right].$$

45.14. $$\int e^{ax}\sin^m x\cos^n x\,dx =$$

$$= \frac{1}{(m+n)^2+a^2}\left\{e^{ax}\sin^m x\cos^{n-1} x\,[a\cos x + (m+n)\sin x] - ma\int e^{ax}\sin^{m-1} x\cos^{n-1} x\,dx + (n-1)(m+n)\int e^{ax}\sin^m x\cos^{n-2} x\,dx\right\};$$

$$= \frac{1}{(m+n)^2+a^2}\left\{e^{ax}\sin^{m-1} x\cos^n x\,[a\sin x - (m+n)\cos x] + na\int e^{ax}\sin^{m-1} x\cos^{n-1} x\,dx + (m-1)(m+n)\int e^{ax}\sin^{m-2} x\cos^n x\,dx\right\}.$$

ТАБЛИЦА 46

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\ln^m(a+bx)\,dx;\quad \begin{matrix} n=0,\ 1,\ 2,\ \ldots,\\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

46.1. $\int \lg_A x\,dx=\frac{1}{\ln A}(x\ln x-x)=x\lg_A\frac{x}{e}\quad (A>0,\ A\neq 1).$

46.2. $\int \ln(a+bx)\,dx=\frac{1}{b}(a+bx)\ln(a+bx)-x.$

46.3. $\int x\ln(a+bx)\,dx=\frac{b^2x^2-a^2}{2b^2}\ln(a+bx)-\frac{bx^2-2ax}{4b}.$

46.4. $\int x^2\ln(a+bx)\,dx=$

$$=\frac{1}{3}\left(x^3-\frac{a^3}{b^3}\right)\ln(a+bx)-\frac{1}{3}\left(\frac{x^3}{3}-\frac{ax^2}{2b}+\frac{a^2x}{b^2}\right).$$

46.5. $\int x^3\ln(a+bx)\,dx=$

$$=\frac{1}{4}\left(x^4-\frac{a^4}{b^4}\right)\ln(a+bx)-\frac{1}{4}\left(\frac{x^4}{4}-\frac{ax^3}{3b}+\frac{a^2x^2}{2b^2}-\frac{a^3x}{b^3}\right).$$

46.6. $\int x^n\ln(a+bx)\,dx=\frac{1}{n+1}\left(x^{n+1}-\frac{a^{n+1}}{b^{n+1}}\right)\ln(a+bx)+$

$$+\frac{1}{n+1}\sum_{\nu=1}^{n+1}\frac{(-1)^\nu x^{n-\nu+2}a^{\nu-1}}{(m-\nu+2)\,b^{\nu-1}}.$$

46.7. $\int x^n\lg_A x\,dx=\frac{1}{\ln A}\left[\frac{x^{n+1}}{n+1}\ln x-\frac{x^{n+1}}{(n+1)^2}\right].$

46.8. $\int \ln^m(a+bx)\,dx=\frac{(a+bx)\ln^m(a+bx)}{b}-m\int \ln^{m-1}(a+bx)\,dx.$

46.9. $\int x\ln^m(a+bx)\,dx=$

$$=\frac{(a+bx)^2\ln^m(a+bx)}{2b^2}-a\int \ln^m(a+bx)\,dx-$$

$$-\frac{m}{2b}\int (a+bx)\ln^{m-1}(a+bx)\,dx \quad (\text{см. } 46.8).$$

46.10. $\int x^2\ln^m(a+bx)\,dx=$

$$=\left[\frac{(a+bx)^3}{3}-a(a+bx)+a^2(a+bx)\right]\frac{\ln^m(a+bx)}{b^3}-$$

$$-\frac{m}{b^3}\left(\frac{1}{3}\int u^2\ln^{m-1}u\,du-\frac{1}{2}\int u\ln^{m-1}u\,du+\int \ln^{m-1}u\,du\right),$$

где $u=a+bx$.

46.11. $\int x^n\ln^m(a+bx)\,dx=$

$$=\frac{\ln^m(a+bx)}{b^{n+1}}\sum_{\nu=0}^{n}\frac{(-1)^{n+\nu}n!\,(a+bx)^{\nu+1}}{a^{\nu-n}(n-\nu)!\,\nu!\,(\nu+1)}-$$

$$-\frac{m}{b^{n+1}}\sum_{\nu=0}^{n}\frac{(-1)^{n+\nu}n!}{a^{\nu-n}(n-\nu)!\,\nu!\,(\nu+1)}\int u^\nu\ln^{m-1}u\,du,$$

где $u=a+bx$.

46.12.* $\int \frac{\ln x\,dx}{a+bx}=\frac{\ln x\ln(a+bx)}{b}-\frac{1}{b}\int \frac{\ln(a+bx)}{x}\,dx$ (см. 46.13).

46.13.* $\int \frac{\ln(a+bx)}{x}\,dx=$

$$=\begin{cases}\ln a\ln x+\frac{bx}{a}-\frac{b^2x^2}{2^2a^2}+\frac{b^3x^3}{3^2a^3}-\frac{b^4x^4}{4^2a^4}+\ldots & \text{при } b^2x^2<a^2;\\ \frac{\ln^2 bx}{2}-\frac{a}{bx}+\frac{a^2}{2^2b^2x^2}-\frac{a^3}{3^2b^3x^3}+\frac{a^4}{4^2b^4x^4}-\ldots & \text{при } b^2x^2>a^2.\end{cases}$$

46.14. $\int \frac{\ln(a+bx)}{x^2}\,dx=\frac{b}{a}\ln x-\frac{a+bx}{ax}\ln(a+bx).$

46.15. $\int \frac{\ln(a+bx)}{x^n}\,dx=-\frac{\ln(a+bx)}{(n-1)\,x^{n-1}}+\frac{b}{(n-1)}\int \frac{dx}{x^{n-1}(a+bx)}$

$(n\geqslant 2)$ (см. 2.19).

46.16.* $\int \frac{\ln^2(a+bx)}{x^2}\,dx=$

$$=-\left(\frac{1}{x}+\frac{1}{a}\right)\ln^2(a+bx)+\frac{2}{a}\int \frac{\ln(a+bx)}{x}\,dx \quad (\text{см. } 46.13).$$

46.17. $\int \frac{\ln^2(a+bx)}{x^n}\,dx=\frac{\ln^2(a+bx)}{(n-1)\,x^{n-1}}-\frac{b}{(n-1)}\int \frac{\ln(a+bx)}{x^{n-1}(a+bx)}\,dx.$

46.18. $\int \frac{\ln^m(a+bx)}{x^2}\,dx=-\frac{\ln^m(a+bx)}{x}+mb\int \frac{\ln^{m-1}(a+bx)}{x(a+bx)}\,dx.$

ТАБЛИЦА 47

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{\ln^m (a+bx)}; \quad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

Обозначение: $u=a+bx$.

47.1.* $$\int \frac{dx}{\ln (a+bx)}=\frac{1}{b}\left[\ln|\ln (a+bx)|+\ln (a+bx)+\frac{\ln^2 (a+bx)}{2\cdot 2!}+\frac{\ln^3 (a+bx)}{3\cdot 3!}+\ldots\right].$$

47.2.* $$\int \frac{x\,dx}{\ln (a+bx)}=\frac{1}{b^2}\left[\ln|\ln (a+bx)|+2\ln (a+bx)+\frac{2^2\ln^2 (a+bx)}{2\cdot 2!}+\frac{2^3\cdot\ln^3 (a+bx)}{3\cdot 3!}+\ldots-a\int\frac{du}{\ln u}\right]$$ (см. 47.1).

47.3.* $$\int \frac{x^n\,dx}{\ln x}=\ln|\ln x|+(n+1)\ln x+\frac{(n+1)^2\ln^2 x}{2\cdot 2!}+\ldots+\frac{(n+1)^\nu\ln^\nu x}{\nu\cdot\nu!}+\ldots$$

47.4.* $$\int \frac{x^n\,dx}{\ln (a+bx)}=\frac{1}{b^{n+1}}\sum_{\nu=0}^{n}\frac{(-1)^{\nu+n}\,n!}{a^{\nu-n}\,(n-\nu)!\,\nu!}\int\frac{u^\nu\,du}{\ln u}$$ (см. 47.3).

47.5.* $$\int \frac{dx}{\ln^2 (a+bx)}=-\frac{a+bx}{b\ln (a+bx)}+\frac{1}{b}\int\frac{du}{\ln u}$$ (см. 47.1).

47.6.* $$\int \frac{x\,dx}{\ln^2 (a+bx)}=-\frac{x(a+bx)}{b\ln (a+bx)}-\frac{a}{b^2}\int\frac{du}{\ln u}+\frac{2}{b^2}\int\frac{u\,du}{\ln u}$$ (см. 47.3).

47.7.* $$\int \frac{x^2\,dx}{\ln^2 (a+bx)}=-\frac{x^2(a+bx)}{b\ln (a+bx)}+\frac{a^2}{b^3}\int\frac{du}{\ln u}-\frac{4a}{b^3}\int\frac{u\,du}{\ln u}+\frac{3}{b^3}\int\frac{u\,du}{\ln u}$$ (см. 47.3).

47.8.* $$\int \frac{dx}{\ln^m (a+bx)}=-\frac{a+bx}{b\ln^{m-1}(a+bx)}+\frac{1}{(m-1)\,b}\int\frac{du}{\ln^{m-1}u}.$$

47.9.* $$\int \frac{x\,dx}{\ln^m (a+bx)}=-\frac{x(a+bx)}{(m-1)\,b\ln^{m-1}(a+bx)}-\frac{a}{(m-1)\,b^2}\int\frac{du}{\ln^{m-1}u}+\frac{2}{(m-1)\,b^2}\int\frac{u\,du}{\ln^{m-1}u}$$ (см. 47.8).

47.10. $$\int \frac{x^2\,dx}{\ln^m (a+bx)}=-\frac{x^2(a+bx)}{mb\ln^m (a+b)}+\frac{a^2}{mb^3}\int\frac{du}{\ln u}-\frac{4a}{mb^3}\int\frac{u\,du}{\ln^m u}+\frac{3}{mb^3}\int\frac{u^2\,du}{\ln^m u}$$ (см. 47.8 и 47.9).

47.11.* $$\int \frac{x^n\,dx}{\ln^m (a+bx)}=-\frac{x^n(a+bx)}{(m-1)\,b\ln^{m-1}(a+bx)}+\frac{1}{(m-1)\,b^{n+1}}\sum_{\nu=0}^{n}\frac{(-1)^{n+\nu}(\nu+1)\,n!}{a^{\nu+n}\,(n-\nu)!\,\nu!}\int\frac{u^\nu\,du}{\ln^{m-1}u}.$$

47.12.* $$\int \frac{dx}{x\ln (a+bx)}=\sum_{\nu=0}^{\infty}a^\nu\left[\ln|\ln (a+b\nu)|-\nu\ln (a+bx)+\frac{\nu^2\ln^2 (a+bx)}{2\cdot 2!}-\frac{\nu^3\ln^3 (a+bx)}{3\cdot 3!}+\ldots\right].$$

47.13.* $$\int \frac{dx}{x^n\ln (a+bx)}=b^{n-1}\sum_{\nu=0}^{\infty}\frac{(n+\nu-1)!\,a^\nu}{(n-1)!\,\nu!}\left[\ln|\ln (a+bx)|-(n+\nu-1)\ln (a+bx)+\frac{(n+\nu-1)^2\ln^2 (a+bx)}{2\cdot 2!}-\frac{(n+\nu-1)^3\ln^3 (a+bx)}{3\cdot 3!}+\ldots\right].$$

47.14.* $$\int \frac{dx}{x\ln^2 (a+bx)}=-\frac{a+bx}{bx\ln (a+bx)}-\sum_{\nu=0}^{\infty}\nu a^\nu\int\frac{du}{u^{\nu+1}\ln u}$$ (см. 47.13).

47.15. $$\int \frac{dx}{x^n\ln^2 (a+bx)}=-\frac{a+bx}{bx^n\ln (a+bx)}-b^{n-1}\sum_{\nu=0}^{\infty}\frac{(n+\nu-1)!\,(n+\nu-1)\,a^\nu}{(n-1)!\,\nu!}\int\frac{du}{u^{n+\nu}\ln u}.$$

47.16. $$\int \frac{dx}{x\ln^m (a+bx)}=-\frac{a+bx}{(m-1)\,bx\ln^{m-1}(a+bx)}-\frac{1}{m-1}\sum_{\nu=0}^{\infty}\nu a^\nu\int\frac{du}{u^{\nu+1}\ln^{m-1}u}$$ (см. 46.13).

47.17. $$\int \frac{dx}{x^n\ln^m (a+bx)}=-\frac{a+bx}{(m-1)\,bx^n\ln^{m-1}(a+bx)}-\frac{b^{n-1}}{m-1}\sum_{\nu=0}^{\infty}\frac{(n+\nu-1)!\,(n+\nu-1)\,a^\nu}{(n-1)!\,\nu!}\int\frac{du}{u^{n+\nu}\ln^{m-1}u}.$$

ТАБЛИЦА 48

ИНТЕГРАЛЫ ВИДА

$$\int x^n \ln|x^2 \pm a^2|\,dx,$$

$$\int x^{\pm n} \ln(x+\sqrt{x^2 \pm a^2})\,dx; \qquad n=0,\ 1,\ 2,\ \ldots$$

48.1. $\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) - 2x + 2a\operatorname{arctg}\frac{x}{a}$.

48.2. $\int x\ln(x^2+a^2)\,dx = \frac{1}{2}[(x^2+a^2)\ln(x^2+a^2) - x^2]$.

48.3. $\int x^2\ln(x^2+a^2)\,dx =$

$$= \frac{1}{3}\left[x^3\ln(x^2+a^2) - \frac{2}{3}x^3 + 2xa^2 - 2a^3\operatorname{arctg}\frac{x}{a}\right].$$

48.4. $\int x^n\ln(x^2+a^2)\,dx = \frac{x^{n+1}}{n+1}\ln(x^2+a^2) - \frac{2}{n+1}\int\frac{x^{n+2}\,dx}{x^2+a^2}$.

48.5. $\int \ln|x^2-a^2|\,dx = x\ln|x^2-a^2| - 2x + a\ln\left|\frac{x+a}{x-a}\right|$.

48.6. $\int x\ln|x^2-a^2|\,dx = \frac{1}{2}[(x^2-a^2)\ln|x^2-a^2| - x^2]$.

48.7. $\int x^3\ln|x^2-a^2|\,dx =$

$$= \frac{1}{3}\left[x^3\ln|x^2-a^2| - \frac{2}{3}x^3 - 2xa^2 + a^3\ln\left|\frac{x+a}{x-a}\right|\right].$$

48.8. $\int x^n\ln|x^2-a^2|\,dx = \frac{x^{n+1}}{n+1}\ln|x^2-a^2| - \frac{2}{n+1}\int\frac{x^{n+2}\,dx}{x^2-a^2}$.

48.9. $\int \ln(x+\sqrt{x^2\pm a^2})\,dx = -\sqrt{x^2\pm a^2} + x\ln(x+\sqrt{x^2\pm a^2})$

48.10. $\int x\ln(x+\sqrt{x^2\pm a^2})\,dx =$

$$= -\frac{x\sqrt{x^2\pm a^2}}{4} + \frac{2x^2\pm a^2}{4}\ln(x+\sqrt{x^2\pm a^2})$$

48.11. $\int x^2\ln(x+\sqrt{x^2\pm a^2})\,dx =$

$$= \pm\frac{a^2\sqrt{x^2\pm a^2}}{3} - \frac{\sqrt{(x^2\pm a^2)^3}}{9} + \frac{x^3}{3}\ln(x+\sqrt{x^2\pm a^2}).$$

48.12. $\int x^n\ln(x+\sqrt{x^2\pm a^2})\,dx =$

$$= \frac{x^{n+1}}{n+1}\ln(x+\sqrt{x^2\pm a^2}) - \frac{1}{n+1}\int\frac{x^{n+1}\,dx}{\sqrt{x^2\pm a^2}}.$$

48.13.* $\int\frac{\ln\left(\frac{x}{a}+\sqrt{\frac{x^2}{a^2}+1}\right)}{x}\,dx = \frac{x}{a} - \frac{1}{2\cdot3\cdot3}\left(\frac{x}{a}\right)^3 +$

$$+\frac{1\cdot3}{2\cdot4\cdot5\cdot5}\left(\frac{x}{a}\right)^5 - \frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot7\cdot7}\left(\frac{x}{a}\right)^7 + \ldots \text{ при } x^2 < a^2;$$

$$= \frac{1}{2}\left(\ln\frac{2x}{a}\right)^2 - \frac{1}{2^3}\left(\frac{a^2}{x}\right) +$$

$$+\frac{1\cdot3}{2\cdot4^3}\left(\frac{a}{x}\right)^4 - \frac{1\cdot3\cdot5}{2\cdot4\cdot6^3}\left(\frac{a}{x}\right)^6 + \ldots \text{ при } \frac{x}{a} > 1.$$

48.14.* $\int\frac{\ln\left(\frac{x}{a}+\sqrt{\frac{x^2}{a^2}-1}\right)}{x}\,dx = \frac{1}{2}\left(\ln\frac{2x}{a}\right)^2 + \frac{1}{2^3}\left(\frac{a}{x}\right)^2 +$

$$+\frac{1\cdot3}{2\cdot4^3}\left(\frac{a}{x}\right)^4 + \frac{1\cdot3\cdot5}{2\cdot4\cdot6^3}\left(\frac{a}{x}\right)^6 + \ldots \text{ при } \frac{x}{a} > 1.$$

48.15. $\int\frac{\ln(x+\sqrt{x^2+a^2})}{x^2}\,dx =$

$$= -\frac{\ln(x+\sqrt{x^2+a^2})}{x} - \frac{1}{a}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|.$$

48.16. $\int\frac{\ln x+(\sqrt{x^2-a^2})}{x^2}\,dx =$

$$= -\frac{\ln(x+\sqrt{x^2-a^2})}{x} + \frac{1}{a}\operatorname{arcsec}\left|\frac{x}{a}\right| \quad \left(0<\operatorname{arcsec}\left|\frac{x}{a}\right|<\frac{\pi}{2}\right).$$

48.17. $\int\frac{\ln(x+\sqrt{x^2\pm a^2})}{x^n}\,dx =$

$$= -\frac{\ln(x+\sqrt{x^2\pm a^2})}{(n-1)\,x^{n-1}} + \frac{1}{n-1}\int\frac{dx}{x^{n-1}\sqrt{x^2\pm a^2}} \quad (n\geqslant 2).$$

ТАБЛИЦА 49

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\,\mathrm{sh}^{\pm m}\,px\,dx;\quad \begin{matrix} n=0,\ 1,\ 2,\ldots, \\ m=1,\ 2,\ 3,\ldots \end{matrix}$$

49.1. $\int \mathrm{sh}\,px\,dx=\frac{1}{p}\,\mathrm{ch}\,px.$

49.2 $\int \mathrm{sh}^2\,px\,dx=\frac{\mathrm{sh}\,2px}{4p}-\frac{x}{2}.$

49.3. $\int \mathrm{sh}^3\,px\,dx=\frac{\mathrm{ch}\,px}{3p}\,(\mathrm{ch}^2\,px-3).$

49.4. $\int \mathrm{sh}^4\,px\,dx=\frac{1}{8p}\left(\frac{1}{4}\,\mathrm{sh}\,4px-2\,\mathrm{sh}\,2px+3x\right).$

49.5 $\int \mathrm{sh}^m\,px\,dx=\frac{\mathrm{sh}^{m-1}\,px\,\mathrm{ch}\,px}{mp}-\frac{m-1}{m}\int \mathrm{sh}^{m-2}\,px\,dx.$

49.6. $\int x\,\mathrm{sh}\,px\,dx=\frac{x}{p}\,\mathrm{ch}\,px-\frac{1}{p^2}\,\mathrm{sh}\,px.$

49.7. $\int x\,\mathrm{sh}^2\,px\,dx=\frac{x\,\mathrm{sh}\,2px}{4p}-\frac{\mathrm{ch}\,2px}{8p^2}-\frac{x^2}{4}.$

49.8. $\int x\,\mathrm{sh}^3\,px\,dx=\frac{x\,\mathrm{ch}\,px}{3p}\,(\mathrm{ch}^2\,px-3)-\frac{\mathrm{sh}\,px}{9p^2}\,(\mathrm{sh}^2\,px-6).$

49.9. $\int x^2\,\mathrm{sh}\,px\,dx=\frac{p^2x^2+2}{p^3}\,\mathrm{ch}\,px-\frac{2x}{p^2}\,\mathrm{sh}\,px.$

49.10. $\int x^2\,\mathrm{sh}^2\,px\,dx=\frac{(4p^2x^2+2)\,\mathrm{sh}\,2px}{p^3}-\frac{4x}{p^2}\,\mathrm{ch}\,2px-\frac{x^3}{6}.$

49.11. $\int x^2\,\mathrm{sh}^3\,px\,dx=\frac{\mathrm{ch}\,px}{27p^3}\,(p^2x^2\,\mathrm{ch}^2\,px-27p^2x^2-54)-$

$$-\frac{x\,\mathrm{sh}\,px}{9p^2}\,(\mathrm{sh}^2\,px-12).$$

49.12 $\int x^3\,\mathrm{sh}\,px\,dx=\frac{p^2x^2+6x}{p^3}\,\mathrm{ch}\,px-\frac{3p^2x^2+6}{p^4}\,\mathrm{sh}\,px.$

49.13. $\int x^n\,\mathrm{sh}\,px\,dx=\frac{x^n}{p}\,\mathrm{ch}\,px-\frac{n}{p}\int x^{n-1}\,\mathrm{ch}\,px\,dx.$

49.14. $\int x^n\,\mathrm{sh}^2\,px\,dx=\frac{x^n}{4p}\,\mathrm{sh}\,2px-\frac{x^{n+1}}{2\,(n+1)}-\frac{n}{2^{n+2}\,p^{n+1}}\int t^{n-1}\,\mathrm{sh}\,t\,dt,$

где $t=2px$ (см. 49.13).

49.15.* $\int \frac{\mathrm{sh}\,px}{x}\,dx=px+\frac{(px)^3}{3\cdot 3!}+\frac{(px)^5}{5\cdot 5!}+\frac{(px)^7}{7\cdot 7!}+\cdots$

49.16.* $\int \frac{\mathrm{sh}\,px}{x^2}\,dx=-\frac{\mathrm{sh}\,px}{x}+p\int \frac{\mathrm{ch}\,px}{x}\,dx$ (см. 50.15).

49.17.* $\int \frac{\mathrm{sh}^2\,px\,dx}{x}=-\frac{1}{2}\ln|x|+\frac{1}{2}\int \frac{\mathrm{ch}\,pt}{t}\,dt,$ где $t=2x$ (см. 50.15).

49.18. $\int \frac{dx}{\mathrm{sh}\,px}=\frac{1}{p}\ln\left|\mathrm{th}\,\frac{px}{2}\right|=-\frac{1}{2p}\ln\frac{\mathrm{ch}\,px+1}{\mathrm{ch}\,px-1}.$

49.19. $\int \frac{dx}{\mathrm{sh}^2\,px}=-\frac{1}{p}\,\mathrm{cth}\,px.$

49.20. $\int \frac{dx}{\mathrm{sh}^3\,px}=-\frac{1}{p}\left(\frac{\mathrm{ch}\,px}{2\,\mathrm{sh}^2\,px}-\frac{1}{2}\ln\left|\mathrm{th}\,\frac{px}{2}\right|\right).$

49.21. $\int \frac{dx}{\mathrm{sh}^m\,px}=-\frac{\mathrm{ch}\,px}{(m-1)\,p\,\mathrm{sh}^{m-1}\,px}-\frac{m-2}{m-1}\int \frac{dx}{\mathrm{sh}^{m-2}\,px}\quad (m\geqslant 2).$

49.22*. $\int \frac{x\,dx}{\mathrm{sh}\,px}=\frac{1}{p^2}\left[px-\frac{(px)^3}{3\cdot 3!}+\frac{7\,(px)^5}{3\cdot 5\cdot 5!}-\frac{31\,(px)^7}{3\cdot 7\cdot 7!}+\cdots\right.$

$$\left.\cdots+(-1)^n\,\frac{2\,(2^{2n-1}-1)}{(2n+1)!}\,B_n\,(px)^{2n+1}+\cdots\right]\quad (p^2x^2<\pi^2).$$

49.23. $\int \frac{x\,dx}{\mathrm{sh}^2\,px}=-\frac{1}{p^2}\,(px\,\mathrm{cth}\,px+\ln|\mathrm{sh}\,px|).$

49.24. $\int \mathrm{sh}\,px\,\mathrm{sh}\,qx\,dx=\frac{\mathrm{sh}\,(p+q)\,x}{2\,(p+q)}-\frac{\mathrm{sh}\,(p-q)\,x}{2\,(p-q)}\quad (p^2\neq q^2).$

ТАБЛИЦА 50

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\,\mathrm{ch}^{\pm m}\,px\,dx; \qquad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

50.1. $\int \mathrm{ch}\,px\,dx=\frac{1}{p}\,\mathrm{sh}\,px.$

50.2. $\int \mathrm{ch}^2 px\,dx=\frac{\mathrm{sh}\,2px}{4p}+\frac{x}{2}.$

50.3. $\int \mathrm{ch}^3 px\,dx=\frac{\mathrm{sh}\,px}{3p}(\mathrm{sh}^2 px+3).$

50.4. $\int \mathrm{ch}^4 px\,dx=\frac{1}{8p}\left(\frac{1}{4}\,\mathrm{sh}\,4px+2\,\mathrm{sh}\,2px+3x\right).$

50.5. $\int \mathrm{ch}^m px\,dx=\frac{1}{mp}\,\mathrm{sh}\,px\,\mathrm{ch}^{m-1} px-\frac{(m-1)}{m}\int \mathrm{ch}^{m-2} px\,dx.$

50.6. $\int x\,\mathrm{ch}\,px\,dx=\frac{x}{p}\,\mathrm{sh}\,px-\frac{1}{p^2}\,\mathrm{ch}\,px.$

50.7. $\int x\,\mathrm{ch}^2 px\,dx=\frac{x\,\mathrm{sh}\,2px}{4p}-\frac{\mathrm{ch}\,2px}{8p^2}+\frac{x^2}{4}.$

50.8. $\int x\,\mathrm{ch}^3 px\,dx=\frac{x\,\mathrm{sh}\,px}{3p}(\mathrm{sh}^2 px+3)-\frac{\mathrm{ch}\,px}{9p^2}(\mathrm{ch}^2 px+6).$

50.9. $\int x^2\,\mathrm{ch}\,px\,dx=\frac{p^2x^2+2}{p^3}\,\mathrm{sh}\,px-\frac{2x}{p^2}\,\mathrm{ch}\,px.$

50.10. $\int x^2\,\mathrm{ch}^2 px\,dx=\frac{p^2x^2+1}{4p^3}\,\mathrm{sh}\,2px-\frac{x\,\mathrm{ch}\,2px}{4p^2}+\frac{x^3}{6}.$

50.11. $\int x^2\,\mathrm{ch}^3 px\,dx=\frac{\mathrm{sh}\,px}{9p^2}(9p^2x^2\,\mathrm{ch}^2 px+p\,\mathrm{sh}^2 px+18-6p)-$
$-\frac{2x\,\mathrm{ch}\,px}{9p^2}(\mathrm{ch}^2 px+12).$

50.12. $\int x^3\,\mathrm{ch}\,px\,dx=\frac{(p^3x^3+6x)}{p^3}\,\mathrm{sh}\,px-\frac{(3p^2x^2+6)}{p^4}\,\mathrm{ch}\,px.$

50.13. $\int x^n\,\mathrm{ch}\,px\,dx=\frac{x^n}{p}\,\mathrm{sh}\,px-\frac{n}{p}\int x^{n-1}\,\mathrm{sh}\,px\,dx.$

50.14. $\int x^n\,\mathrm{ch}^2 px\,dx=\frac{x^n}{4p}\,\mathrm{sh}\,2px+\frac{x^{n+1}}{2(n+1)}-\frac{n}{2^{n+2}\,p^{n+1}}\int t^{n-1}\,\mathrm{sh}\,t\,dt,$
где $t=2px$ (см. 49.13).

50.15. * $\int \frac{\mathrm{ch}\,px}{x}\,dx=\ln|px|+\frac{(px)^2}{2\cdot 2!}+\frac{(px)^4}{4\cdot 4!}+\frac{(px)^6}{6\cdot 6!}+\ldots$

50.16. * $\int \frac{\mathrm{ch}\,px}{x^2}\,dx=-\frac{\mathrm{ch}\,px}{x}+p\int \frac{\mathrm{sh}\,px}{x}\,dx.$

50.17. * $\int \frac{\mathrm{ch}^2 px}{x}\,dx=\frac{1}{2}\ln|x|+\frac{1}{2}\int \frac{\mathrm{ch}\,pt}{t}\,t,$ где $t=2x$ (см. 50.15).

50.18. $\int \frac{dx}{\mathrm{ch}\,px}=\frac{1}{p}\,\mathrm{arctg}\,(\mathrm{sh}\,px).$

50.19. $\int \frac{dx}{\mathrm{ch}^2 px}=\frac{1}{p}\,\mathrm{th}\,px.$

50.20. $\int \frac{dx}{\mathrm{ch}^3 px}=\frac{1}{2p}\frac{\mathrm{sh}\,px}{\mathrm{ch}^2 px}+\frac{1}{2p}\,\mathrm{arctg}\,(\mathrm{sh}\,px).$

50.21. $\int \frac{dx}{\mathrm{ch}^m px}=\frac{\mathrm{sh}\,px}{(m-1)\,p\,\mathrm{ch}^{m-1} px}+\frac{m-2}{m-1}\int \frac{dx}{\mathrm{ch}^{m-2} px} \qquad (m\geqslant 2).$

50.22. * $\int \frac{x\,dx}{\mathrm{ch}\,px}=\frac{1}{p^2}\left[\frac{(px)^2}{2}-\frac{(px)^4}{4\cdot 2!}+\frac{5\,(px)^6}{6\cdot 4!}-\frac{61\,(px)^8}{8\cdot 6!}+\right.$
$\left.+\frac{138\,(px)^{10}}{10\cdot 8!}+\ldots+\frac{(-1)^n E_n}{(2n+2)\,(2n)!}(px)^{2n+2}+\ldots\right] \qquad (p^2x^2<\pi^2/4).$

50.23. $\int \frac{x\,dx}{\mathrm{ch}^2 px}=\frac{1}{p^2}(px\,\mathrm{th}\,px-\ln\mathrm{ch}\,px).$

50.24. $\int \mathrm{ch}\,px\,\mathrm{ch}\,qx\,dx=\frac{\mathrm{sh}\,(p+q)\,x}{2\,(p+q)}+\frac{\mathrm{sh}\,(p-q)\,x}{2\,(p-q)} \qquad (p^2\neq q^2).$

ТАБЛИЦА 51

ИНТЕГРАЛЫ ВИДА

$$\int \operatorname{sh}^{\pm m} x \operatorname{ch}^{\pm n} x\, dx; \quad \begin{matrix} n = 1, 2, 3, \ldots, \\ m = 1, 2, 3, \ldots \end{matrix}$$

51.1. $\int \operatorname{sh} x \operatorname{ch} x\, dx = \frac{\operatorname{ch}^2 x}{2}$.

51.2. $\int \operatorname{sh} px \operatorname{ch} qx\, dx = \frac{1}{p^2 - q^2}(p \operatorname{ch} px \operatorname{ch} qx - q \operatorname{sh} px \operatorname{sh} qx)$ $(p^2 \neq q^2)$.

51.3. $\int \operatorname{sh} x \operatorname{ch}^n x\, dx = \frac{\operatorname{ch}^{n+1} x}{n+1}$.

51.4. $\int \operatorname{sh}^m x \operatorname{ch} x\, dx = \frac{\operatorname{sh}^{m+1} x}{m+1}$.

51.5. $\int \operatorname{sh}^2 x \operatorname{ch}^2 x\, dx = \frac{\operatorname{sh} 4x}{32} - \frac{x}{8}$.

51.6. $\int \frac{dx}{\operatorname{sh} x \operatorname{ch} x} = \ln |\operatorname{th} x|$.

51.7. $\int \frac{dx}{\operatorname{sh} x \operatorname{ch}^2 x} = \frac{1}{\operatorname{ch} x} + \ln |\operatorname{th} x|$.

51.8. $\int \frac{dx}{\operatorname{sh} x \operatorname{ch}^3 x} = \frac{1}{2\operatorname{ch}^2 x} + \ln |\operatorname{th} x|$.

51.9. $\int \frac{dx}{\operatorname{sh}^2 x \operatorname{ch}^2 x} = -2 \operatorname{cth} 2x$.

51.10. $\int \frac{dx}{\operatorname{sh}^2 x \operatorname{ch} x} = -\frac{1}{\operatorname{sh} x} - \operatorname{arctg}(\operatorname{sh} x)$.

51.11. $\int \frac{dx}{\operatorname{sh}^3 x \operatorname{ch} x} = -\frac{1}{2\operatorname{sh}^2 x} - \ln |\operatorname{th} x|$.

51.12. $\int \frac{dx}{\operatorname{sh} x \operatorname{ch}^n x} = \frac{1}{(n-1)\operatorname{ch}^{n-1} x} + \int \frac{dx}{\operatorname{sh} x \operatorname{ch}^{n-2} x}$ $(n \geqslant 2)$.

51.13. $\int \frac{dx}{\operatorname{sh}^m x \operatorname{ch} x} = -\frac{1}{(m-1)\operatorname{sh}^{m-1} x} - \int \frac{dx}{\operatorname{sh}^{m-2} x \operatorname{ch} x}$ $(m \geqslant 2)$.

51.14. $\int \frac{\operatorname{sh} x}{\operatorname{ch} x} dx = \int \operatorname{th} x\, dx = \ln \operatorname{ch} x$.

51.15. $\int \frac{\operatorname{sh} x}{\operatorname{ch}^n x} dx = -\frac{1}{(n-1)\operatorname{ch}^{n-1} x}$ $(n \geqslant 2)$.

51.16. $\int \frac{\operatorname{ch} x\, dx}{\operatorname{sh} x} = \int \operatorname{cth} x\, dx = \ln |\operatorname{sh} x|$.

51.17. $\int \frac{\operatorname{ch} x}{\operatorname{sh}^m x} dx = -\frac{1}{(m-1)\operatorname{sh}^{m-1} x}$ $(m \geqslant 2)$.

51.18. $\int \frac{\operatorname{sh}^2 x}{\operatorname{ch}^2 x} dx = \int \operatorname{th}^2 x\, dx = x - \operatorname{th} x$.

51.19. $\int \frac{\operatorname{ch}^2 x}{\operatorname{sh}^2 x} dx = \int \operatorname{cth}^2 x\, dx = x - \operatorname{cth} x$.

51.20. $\int \frac{\operatorname{sh}^m x}{\operatorname{ch}^m x} dx = \int \operatorname{th}^m x\, dx = -\frac{\operatorname{th}^{m-1} x}{m-1} + \int \operatorname{th}^{m-2} x\, dx$ $(m \geqslant 2)$.

51.21. $\int \frac{\operatorname{ch}^m x}{\operatorname{sh}^m x} dx = \int \operatorname{cth}^m x\, dx = -\frac{\operatorname{cth}^{m-1} x}{m-1} + \int \operatorname{cth}^{m-2} x\, dx$ $(m \geqslant 2)$.

51.22. $\int \frac{\operatorname{sh}^m x}{\operatorname{ch} x} dx = \frac{\operatorname{sh}^{m-1} x}{m-1} - \int \frac{\operatorname{sh}^{m-2} x}{\operatorname{ch} x} dx$ $(m \geqslant 2)$.

51.23. $\int \frac{\operatorname{sh}^m x}{\operatorname{ch}^n x} dx = -\frac{\operatorname{sh}^{m-1} x}{(n-1)\operatorname{ch}^{n-1} x} + \frac{m-1}{n-1} \int \frac{\operatorname{sh}^{m-2} x}{\operatorname{ch}^{n-2} x} dx$ $(n \geqslant 2)$.

51.24. $\int \frac{\operatorname{ch}^n x}{\operatorname{sh} x} dx = \frac{\operatorname{ch}^{n-1} x}{n-1} + \int \frac{\operatorname{ch}^{n-2} x}{\operatorname{sh} x} dx$ $(n \geqslant 2)$.

51.25. $\int \frac{\operatorname{ch}^n x}{\operatorname{sh}^m x} dx = -\frac{\operatorname{ch}^{n-1} x}{(m-1)\operatorname{sh}^{m-1} x} + \frac{n-1}{m-1} \int \frac{\operatorname{ch}^{n-2} x}{\operatorname{sh}^{m-2} x} dx$ $(m \geqslant 2)$.