

ТАБЛИЦА 10
ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n dx}{a+bx^m}, \quad n=0, 1, 2, \dots, \quad m=5, 6, 7, \dots$$

$$10.1. \int \frac{dx}{1+x^{2k}} = -\frac{1}{2k} \sum_{v=0}^{k-1} \ln \left(x^2 - 2x \cos \frac{2v+1}{2k} \pi + 1 \right) \cos \frac{2v+1}{2k} \pi + \\ + \frac{1}{k} \sum_{v=0}^{k-1} \operatorname{arctg} \left(\frac{x \sin \frac{2v+1}{2k} \pi}{1-x \cos \frac{2v+1}{2k} \pi} \right) \sin \frac{2v+1}{2k} \pi.$$

$$10.2. \int \frac{dx}{1+x^{2k+1}} = \frac{1}{2k+1} \ln |1+x| - \\ - \frac{1}{2k+1} \sum_{v=0}^{k-1} \ln \left(x^2 - 2x \cos \frac{2v+1}{2k+1} \pi + 1 \right) \cos \frac{2v+1}{2k+1} \pi + \\ + \frac{2}{2k+1} \sum_{v=0}^{k-1} \operatorname{arctg} \left(\frac{x \sin \frac{2v+1}{2k+1} \pi}{1-x \cos \frac{2v+1}{2k+1} \pi} \right) \sin \frac{2v+1}{2k+1} \pi.$$

$$10.3. \int \frac{dx}{1-x^{2k}} = \frac{1}{2k} \ln \left| \frac{1+x}{1-x} \right| - \\ - \frac{1}{2k} \sum_{v=0}^{k-1} \ln \left(x^2 + 2x \cos \frac{2v+1}{2k} \pi + 1 \right) \cos \frac{v}{k} \pi + \\ + \frac{1}{k} \sum_{v=0}^{k-1} \operatorname{arctg} \left(\frac{x \sin \frac{2v+1}{2k} \pi}{1+x \cos \frac{2v+1}{2k} \pi} \right) \sin \frac{v}{k} \pi.$$

$$10.4. \int \frac{dx}{1-x^{2k+1}} = -\frac{1}{2k+1} \ln |1-x| + \\ + \frac{1}{2k+1} \sum_{v=0}^{k-1} \ln \left(x^2 + 2x \cos \frac{2v+1}{2k+1} \pi + 1 \right) \cos \frac{2v+1}{2k+1} \pi + \\ + \frac{2}{2k+1} \sum_{v=0}^{k-1} \operatorname{arctg} \left(\frac{x \sin \frac{2v+1}{2k+1} \pi}{1+x \cos \frac{2v+1}{2k+1} \pi} \right) \sin \frac{2v+1}{2k+1} \pi.$$

$$10.5. \int \frac{x^n dx}{1+x^m} = \frac{1}{k} \sum_{v=1}^k \operatorname{arctg} \frac{x - \cos \frac{2v-1}{2k} \pi}{\sin \frac{2v-1}{2k} \pi} \cos \frac{(n+1)(2v-1)}{2k} \pi -$$

$$-\frac{1}{2k} \sum_{v=1}^k \ln \left(x^2 - 2x \cos \frac{2v-1}{2k} \pi + 1 \right) \cos \frac{(n+1)(2v-1)}{2k} \pi.$$

$$10.6. \int \frac{x^n dx}{1+x^{2k+1}} = (-1)^n \frac{\ln |1+x|}{2k+1} - \\ - \frac{1}{2k+1} \sum_{v=1}^{2k+1} \ln \left(x^2 - 2x \cos \frac{2v-1}{2k+1} \pi + 1 \right) \cos \frac{(n+1)(2v-1)}{2k+1} \pi + \\ + \frac{2}{2k+1} \sum_{v=1}^{2k+1} \operatorname{arctg} \frac{x - \cos \frac{2v-1}{2k+1} \pi}{\sin \frac{2v-1}{2k+1} \pi} \sin \frac{(n+1)(2v-1)}{2k+1} \pi.$$

$$10.7. \int \frac{x^n dx}{1-x^{2k}} = \frac{1}{2k} \{ (-1)^n [\ln |1+x|] - \ln |1-x| \} + \\ + (-1)^n \frac{1}{2k} \sum_{v=1}^{k-1} \ln \left(x^2 + 2x \cos \frac{v}{k} \pi + 1 \right) \cos \frac{v(n+1)}{k} \pi + \\ + (-1)^n \frac{1}{k} \sum_{v=1}^{k-1} \operatorname{arctg} \frac{x + \cos \frac{v}{k} \pi}{\sin \frac{v}{k} \pi} \sin \frac{v(n+1)}{k} \pi.$$

$$10.8. \int \frac{x^n dx}{1-x^{2k+1}} = -\frac{1}{2k+1} \ln |1-x| + \\ + \frac{(-1)^n}{2k+1} \sum_{v=1}^{2k+1} \ln \left(x^2 + 2x \cos \frac{2v-1}{2k+1} \pi + 1 \right) \cos \frac{(n+1)(2v-1)}{2k+1} \pi + \\ + (-1)^n \frac{2}{2k+1} \sum_{v=1}^{2k+1} \operatorname{arctg} \frac{x + \cos \frac{2v-1}{2k+1} \pi}{\sin \frac{2v-1}{2k+1} \pi} \sin \frac{(n+1)(2v-1)}{2k+1} \pi.$$

$$10.9. \int \frac{x^n dx}{a+bx^m} = \\ = \begin{cases} \frac{1}{a} \sqrt[m]{\left(\frac{a}{b}\right)^{m+1}} \int \frac{t^n dt}{1+t^m}, & \text{где } t = \sqrt[m]{\frac{b}{a}} x \quad (ab > 0); \\ \frac{1}{a} \sqrt[m]{\left(-\frac{a}{b}\right)^{m+1}} \int \frac{t^n dt}{1-t^m}, & \text{где } t = \sqrt[m]{-\frac{b}{a}} x \quad (ab < 0). \end{cases}$$