

ТАБЛИЦА 11
ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n dx}{(ax^2 + bx + c)^m}, \quad b^2 - 4ac \neq 0; \quad \begin{cases} n = 0, 1, 2, \dots \\ m = 1, 2, 3, \dots \end{cases}$$

$$11.1. \int \frac{dx}{ax^2 + bx + c} = \begin{cases} \frac{2}{V-\delta} \operatorname{arctg} \frac{2ax+b}{V-\delta} & \text{при } b^2 < 4ac; \\ \frac{1}{V\delta} \ln \left| \frac{2ax+b-V\delta}{2ax+b+V\delta} \right| & \text{при } b^2 > 4ac. \end{cases}$$

$$11.2. \int \frac{dx}{(ax^2 + bx + c)^2} = \frac{-2ax-b}{\delta(ax^2 + bx + c)} - \frac{2a}{\delta} \int \frac{dx}{ax^2 + bx + c} \quad (\text{см. 11.1}).$$

$$11.3. \int \frac{dx}{(ax^2 + bx + c)^3} = \frac{-2ax-b}{2\delta(ax^2 + bx + c)^2} + \frac{3a(2ax+b)}{\delta^2(ax^2 + bx + c)} + \frac{6a^2}{\delta^2} \int \frac{dx}{ax^2 + bx + c} \quad (\text{см. 11.1}).$$

$$11.4. \int \frac{dx}{(ax^2 + bx + c)^m} = \frac{-2ax-b}{(m-1)\delta(ax^2 + bx + c)^{m-1}} - \frac{2(2m-3)a}{(m-1)\delta} \int \frac{dx}{(ax^2 + bx + c)^{m-1}} \quad (m \geq 2).$$

$$11.5. \int \frac{x dx}{ax^2 + bx + c} = \begin{cases} \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{aV-\delta} \operatorname{arctg} \frac{2ax+b}{V-\delta} & (\delta < 0); \\ \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{2aV\delta} \ln \left| \frac{2ax+b-V\delta}{2ax+b+V\delta} \right| & (\delta > 0) \end{cases}$$

$$11.6. \int \frac{x dx}{(ax^2 + bx + c)^2} = \frac{bx+2c}{\delta(ax^2 + bx + c)} + \frac{b}{\delta} \int \frac{dx}{ax^2 + bx + c} \quad (\text{см. 11.1}).$$

$$11.7. \int \frac{x dx}{(ax^2 + bx + c)^m} = -\frac{bx+2c}{(m-1)\delta(ax^2 + bx + c)^{m-1}} - \frac{(2m-3)b}{(m-1)\delta} \int \frac{dx}{(ax^2 + bx + c)^{m-1}} \quad (m \geq 2) \quad (\text{см. 11.4}).$$

$$11.8. \int \frac{x^2 dx}{ax^2 + bx + c} = \frac{x}{a} - \frac{b}{2a^2} \ln |ax^2 + bx + c| + \frac{b^2 - 2ac}{2a^2} \int \frac{dx}{ax^2 + bx + c} \quad (\text{см. 11.1}).$$

$$11.9. \int \frac{x^2 dx}{(ax^2 + bx + c)^2} = -\frac{(b^2 - 2ac)x + bc}{a\delta(ax^2 + bx + c)} - \frac{2c}{\delta} \int \frac{dx}{ax^2 + bx + c} \quad (\text{см. 11.1})$$

$$11.10. \int \frac{x^2 dx}{(ax^2 + bx + c)^m} = -\frac{(b^2 - 2ac)x + bc}{(m-1)a\delta(ax^2 + bx + c)^{m-1}} - \frac{(m-4)b^2 + 10ac}{(m+1)a\delta} \int \frac{dx}{(ax^2 + bx + c)^{m-1}} \quad (m \geq 2) \quad (\text{см. 11.4}).$$

$$11.11. \int \frac{x^3 dx}{ax^2 + bx + c} = \frac{ax^2 - bx}{2a^2} + \frac{b^2 - ac}{2a^2} \ln |ax^2 + bx + c| - \frac{b(b^2 - 3ac)}{2a^2} \int \frac{dx}{ax^2 + bx + c} \quad (\text{см. 11.1}).$$

$$11.12. \int \frac{x^n dx}{ax^2 + bx + c} = \frac{x^{n-1}}{(n-1)a} - \frac{c}{a} \int \frac{x^{n-2} dx}{ax^2 + bx + c} - \frac{b}{a} \int \frac{x^{n-1} dx}{ax^2 + bx + c} \quad (n \geq 2).$$

$$11.13. \int \frac{x^3 dx}{(ax^2 + bx + c)^m} = -\frac{x^2}{2(m-2)a(ax^2 + bx + c)^{m-1}} + \frac{c}{(m-2)a} \int \frac{x dx}{(ax^2 + bx + c)^m} + \frac{(m-3)b}{2(m-2)a} \int \frac{x^2 dx}{(ax^2 + bx + c)^m} \quad (m \neq 2).$$

$$11.14. \int \frac{x^n dx}{(ax^2 + bx + c)^m} = -\frac{x^{n-1}}{(2m-n-1)a(ax^2 + bx + c)^{m-1}} + \frac{(n-1)c}{(2m-n-1)a} \int \frac{x^{n-2} dx}{(ax^2 + bx + c)^m} + \frac{(m-n)b}{(2m-n-1)a} \int \frac{x^{n-1} dx}{(ax^2 + bx + c)^m} \quad (n \neq 2m-1).$$

$$11.15. \int \frac{x^{2m-1} dx}{(ax^2 + bx + c)^m} = \frac{1}{a} \int \frac{x^{2m-3} dx}{(ax^2 + bx + c)^{m-1}} - \frac{c}{a} \int \frac{x^{2m-3} dx}{(ax^2 + bx + c)^m} - \frac{b}{a} \int \frac{x^{2m-2} dx}{(ax^2 + bx + c)^m}.$$