ТАБЛИЦА 12

ИНТЕГРАЛЫ ВИДА

$$\int \frac{dx}{x^n (ax^2+bx+c)^m}, \quad b^2-4ac\neq 0; \quad n=1,\ 2,\ 3,\ \ldots, \quad m=1,\ 2,\ 3,\ldots$$

12.1. $$\int \frac{dx}{x(ax^2+bx+c)} = \begin{cases} \frac{1}{2c}\ln\frac{x^2}{|ax^2+bx+c|} - \frac{b}{c\sqrt{-\delta}}\operatorname{arctg}\frac{2ax+b}{\sqrt{-\delta}} & (\delta<0); \\ \frac{1}{2c}\ln\frac{x^2}{|ax^2+bx+c|} - \frac{b}{2c\sqrt{\delta}}\ln\left|\frac{2ax+b-\sqrt{\delta}}{2ax+b+\sqrt{\delta}}\right| & (\delta>0) \end{cases}$$

12.2. $$\int \frac{dx}{x(ax^2+bx+c)^2} = \frac{abx-2ac+b^2}{c\delta(ax^2+bx+c)} + \frac{1}{2c}\ln\frac{x^2}{|ax^2+bx+c|} + \frac{b(6ac-b^2)}{2c^2\delta}\int\frac{dx}{ax^2+bx+c} \quad \text{(см. 11.1).}$$

12.3. $$\int \frac{dx}{x(ax^2+bx+c)^3} = \frac{2ax^2+2bx+3c}{4c^2(ax^2+bx+c)^2} + \frac{1}{2c^3}\ln\frac{x^2}{|ax^2+bx+c|} - \frac{b}{2c^3}\int\frac{dx}{ax^2+bx+c} - \frac{b}{2c^2}\int\frac{dx}{(ax^2+bx+c)^2} - \frac{b}{2c}\int\frac{dx}{(ax^2+bx+c)}$$
(см. 11.1, 11.2 и 11.3)

12.4. $$\int \frac{dx}{x(ax^2+bx+c)^m} = -\frac{1}{c}\int\frac{t^{2m-3}\,dt}{(a+bt+ct^2)^{m-1}} + \frac{a}{c}\int\frac{t^{2m-3}\,dt}{(a+bt+ct^2)^m} + \frac{b}{c}\int\frac{t^{2m-2}\,dt}{(a+bt+ct^2)^m}, \text{ где } t=\frac{1}{x}$$
(см. 11.13).

12.5. $$\int \frac{dx}{x^2(ax^2+bx+c)} = -\frac{1}{cx} - \frac{b}{2c^2}\ln\frac{x^2}{|ax^2+bx+c|} + \frac{b^2-2ac}{2c^2}\int\frac{dx}{ax^2+bx+c} \quad \text{(см. 11.1).}$$

12.6. $$\int \frac{dx}{x^2(ax^2+bx+c)^2} = \frac{2a^2cx-ab^2x+3abc-b^3}{c^2\delta(ax^2+bx+c)} - \frac{1}{c^2x} - \frac{b}{c^2}\ln\frac{x^2}{|ax^2+bx+c|} + \frac{b^4-6ab^2c+6a^2c^2}{c^3\delta}\int\frac{dx}{ax^2+bx+c}$$
(см. 11.1).

12.7. $$\int \frac{dx}{x^2(ax^2+bx+c)^3} = -\frac{1}{cx(ax^2+bx+c)^2} - \frac{3b}{c}\int\frac{dx}{x(ax^2+bx+c)^3} - \frac{5a}{c}\int\frac{dx}{(ax^2+bx+c)^3} \quad \text{(см. 12.3 и 11.3).}$$

12.8. $$\int \frac{dx}{x^2(ax^2+bx+c)^m} = -\frac{1}{cx(ax^2+bx+c)^{m-1}} - \frac{mb}{c}\int\frac{dx}{x(ax^2+bx+c)^m} - \frac{(2m-1)a}{c}\int\frac{dx}{(ax^2+bx+c)^m}$$
(см. 12.4 и 11.4).

12.9. $$\int \frac{dx}{x^3(ax^2+bx+c)} = \frac{b^2-ac}{2c^3}\ln\frac{x^2}{|ax^2+bx+c|} + \frac{2bx-c}{2c^2x^2} + \frac{b(3c-b^2)}{2c^3}\int\frac{dx}{ax^2+bx+c} \quad \text{(см 11.1)}$$

12.10. $$\int \frac{dx}{x^3(ax^2+bx+c)^2} = \frac{3bx-c}{2c^2x^2(ax^2+bx+c)} + \frac{3b^2-2ac}{c^2}\int\frac{dx}{x(ax^2+bx+c)^2} + \frac{9ab}{2c^2}\int\frac{dx}{(ax^2+bx+c)}$$
(см. 12.2 и 11.2).

12.11. $$\int \frac{dx}{x^3(ax^2+bx+c)^3} = \frac{4bx-c}{2c^2x^2(ax^2+bx+c)^2} + \frac{6b^2-3ac}{c^2}\int\frac{dx}{x(ax^2+bx+c)^3} + \frac{10ab}{c^2}\int\frac{dx}{(ax^2+bx+c)^3}$$
(см. 12.3 и 11.3).

12.12. $$\int \frac{dx}{x^n(ax^2+bx+c)^m} = -\frac{1}{(n-1)cx^{n-1}(ax^2+bx+c)^{m-1}} - \frac{(n+m-2)b}{(n-1)c}\int\frac{dx}{x^{n-1}(ax^2+bx+c)^m} - \frac{(n+2m-3)a}{(n-1)c}\int\frac{dx}{x^{n-2}(ax^2+bx+c)^m} \quad (n\geq 2).$$