

ТАБЛИЦА 13
ИНТЕГРАЛЫ ВИДА

$$\int \frac{(ax+\beta)^{\pm n} dx}{(ax^2+bx+c)^m}, \quad b^2-4ac \neq 0; \quad \begin{cases} n=1, 2, 3, \dots, \\ m=1, 2, 3, \dots, \end{cases}$$

$$13.1. \int \frac{ax+\beta}{ax^2+bx+c} dx =$$

$$= \begin{cases} \frac{\alpha}{2a} \ln |ax^2+bx+c| + \frac{2a\beta-b\alpha}{a\sqrt{-\delta}} \operatorname{arctg} \frac{2ax+b}{\sqrt{-\delta}} & (\delta < 0); \\ \frac{\alpha}{2a} \ln |ax^2+bx+c| + \frac{2a\beta-b\alpha}{2a\sqrt{\delta}} \ln \left| \frac{2ax+b-\sqrt{\delta}}{2ax+b+\sqrt{\delta}} \right| & (\delta > 0). \end{cases}$$

$$13.2. \int \frac{(ax+\beta) dx}{(ax^2+bx+c)^2} = \frac{b\alpha x + 2\alpha a}{\delta(ax^2+bx+c)} + \frac{\alpha b}{\delta} \int \frac{dx}{ax^2+bx+c} +$$

$$+ \beta \int \frac{dx}{(ax^2+bx+c)^2} \quad (\text{см. 11.1 и 11.2}).$$

$$13.3. \int \frac{(ax+\beta) dx}{(ax^2+bx+c)^3} = -\frac{(b\alpha+2\alpha\beta)x + b\beta+2\alpha a}{2\delta(ax^2+bx+c)^2} -$$

$$-\frac{6\alpha\beta-3\alpha b}{2\delta} \int \frac{dx}{(ax^2+bx+c)^2} \quad (\text{см. 11.2}).$$

$$13.4. \int \frac{(ax+\beta)^2 dx}{ax^2+bx+c} = \frac{\alpha^2}{a} x + \frac{a\beta^2-\alpha^2}{a} \int \frac{dx}{ax^2+bx+c} +$$

$$+ \frac{\alpha}{a} (2a\beta-b\alpha) \int \frac{x dx}{ax^2+bx+c} \quad (\text{см. 11.1 и 11.5}).$$

$$13.5. \int \frac{(ax+\beta)^n dx}{(ax^2+bx+c)^m} = \frac{\alpha(ax+\beta)^{n-1}}{(n-2m+1)a(ax^2+bx+c)^{m-1}} -$$

$$-\frac{(n-1)(\alpha^2-b\alpha\beta+a\beta^2)}{(n-2m+1)a} \int \frac{(ax+\beta)^{n-2}}{(ax^2+bx+c)^m} dx -$$

$$-\frac{(n-m)(b\alpha-2a\beta)}{(n-2m+1)a} \int \frac{(ax+\beta)^{n-1}}{(ax^2+bx+c)^m} dx \quad (n \neq 2m-1);$$

$$= \frac{(2a\beta-b\alpha-2ax)(ax+\beta)^n}{(m-1)\delta(ax^2+bx+c)^{m-1}} +$$

$$+ \frac{n(b\alpha-2a\beta)}{(m-1)\delta} \int \frac{(ax+\beta)^{n-1} dx}{(ax^2+bx+c)^{m-1}} +$$

$$+ \frac{2(n-2m+3)a}{(m-1)\delta} \int \frac{(ax+\beta)^{n-1} dx}{(ax^2+bx+c)^{m-1}} \quad (m \neq 1).$$

$$13.6. \int \frac{dx}{(ax+\beta)(ax^2+bx+c)} =$$

$$= \begin{cases} \frac{\alpha}{2(\alpha a^2-b\alpha\beta+a\beta^2)} \ln \left| \frac{(ax+\beta)^2}{ax^2+bx+c} \right| - \\ \frac{b\alpha-2a\beta}{(a\alpha^2-b\alpha\beta+a\beta^2)\sqrt{-\delta}} \operatorname{arctg} \frac{2ax+b}{\sqrt{-\delta}} & (\delta < 0); \\ \frac{\alpha}{2(\alpha a^2-b\alpha\beta+a\beta^2)} \ln \left| \frac{(ax+\beta)^2}{ax^2+bx+c} \right| - \\ -\frac{b\alpha-2a\beta}{2(\alpha a^2-b\alpha\beta+a\beta^2)\sqrt{\delta}} \ln \left| \frac{2ax+b-\sqrt{\delta}}{2ax+b+\sqrt{\delta}} \right| & (\delta > 0). \end{cases}$$

$$13.7. \int \frac{dx}{(ax+\beta)(ax^2+bx+c)^2} = \frac{1}{2(\alpha a^2-b\alpha\beta+a\beta^2)} \left[\frac{\alpha}{ax^2+bx+c} - \right.$$

$$\left. - 2a^2 \int \frac{dx}{(ax+\beta)(ax^2+bx+c)} - (b\alpha-2a\beta) \int \frac{dx}{(ax^2+bx+c)^2} \right] \quad (\text{см. 13.6 и 11.2}).$$

$$13.8. \int \frac{dx}{(ax+\beta)^2(ax^2+bx+c)} = -\frac{1}{\alpha a^2-b\alpha\beta+a\beta^2} \left[\frac{\alpha}{ax+\beta} + \right.$$

$$\left. + (ab-2a\beta) \int \frac{dx}{(ax+\beta)(ax^2+bx+c)} + a \int \frac{dx}{ax^2+bx+c} \right] \quad (\text{см. 13.6 и 11.1}).$$

$$13.9. \int \frac{dx}{(ax+\beta)^n(ax^2+bx+c)^m} =$$

$$= \frac{\alpha}{(n-1)(\alpha a^2-b\alpha\beta+a\beta^2)(ax+\beta)^{n-1}(ax^2+bx+c)^{m-1}} -$$

$$-\frac{(n+2m-3)a}{2(n-1)(\alpha a^2-b\alpha\beta+a\beta^2)} \int \frac{dx}{(ax+\beta)^{n-1}(ax^2+bx+c)^m} -$$

$$-\frac{(n+m-2)(b\alpha-2a\beta)}{(n-1)(\alpha a^2-b\alpha\beta+a\beta^2)} \int \frac{dx}{(ax+\beta)^{n-1}(ax^2+bx+c)^m} \quad (n \geq 2).$$

$$13.10. \int \frac{dx}{(ax+\beta)^n(ax^2+bx+c)^m} =$$

$$= \frac{\alpha}{2(m-1)(\alpha a^2-b\alpha\beta+a\beta^2)(ax+\beta)^{n-1}(ax^2+bx+c)^{m-1}} -$$

$$-\frac{b\alpha-2a\beta}{2(\alpha a^2-b\alpha\beta+a\beta^2)} \int \frac{dx}{(ax+\beta)^{n-1}(ax^2+bx+c)^m} +$$

$$+\frac{(n+2m-3)\alpha^2}{2(m-1)(\alpha a^2-b\alpha\beta+a\beta^2)} \int \frac{dx}{(ax+\beta)^n(ax^2+bx+c)^{m-1}} \quad (m \geq 2).$$