ТАБЛИЦА 19

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^n\,dx}{\sqrt{(a^2+b^2x^2)^m}}; \quad a>0, \quad n=0,\ 1,\ 2,\ \ldots, \quad b>0, \quad m=1,\ 3,\ 5,\ \ldots$$

19.1. $$\int \frac{dx}{\sqrt{a^2+b^2x^2}} = \frac{1}{b}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.2. $$\int \frac{dx}{\sqrt{(a^2+b^2x^2)^3}} = \frac{x}{a^2\sqrt{a^2+b^2x^2}}.$$

19.3. $$\int \frac{dx}{\sqrt{(a^2+b^2x^2)^5}} = \frac{1}{a^4}\left[\frac{x}{\sqrt{a^2+b^2x^2}} - \frac{b^2x^3}{3\sqrt{(a^2+b^2x^2)^3}}\right].$$

19.4. $$\int \frac{dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m-1}}\sum_{\nu=0}^{\frac{m-3}{2}} \frac{(-1)^\nu C_{\frac{m-3}{2}}^{\nu} b^{2\nu}x^{2\nu+1}}{(2\nu+1)\sqrt{(a^2+b^2x^2)^{2\nu+1}}} \quad (m \geqslant 3).$$

19.5. $$\int \frac{x\,dx}{\sqrt{a^2+b^2x^2}} = \frac{1}{b^2}\sqrt{a^2+b^2x^2}.$$

19.6. $$\int \frac{x\,dx}{\sqrt{(a^2+b^2x^2)^3}} = -\frac{1}{b^2\sqrt{a^2+b^2x^2}}.$$

19.7. $$\int \frac{x\,dx}{\sqrt{(a^2+b^2x^2)^m}} = -\frac{1}{(m-2)\,b^2\sqrt{(a^2+b^2x^2)^{m-2}}} \quad (m \geqslant 3).$$

19.8. $$\int \frac{x^2\,dx}{\sqrt{a^2+b^2x^2}} = \frac{x\sqrt{a^2+b^2x^2}}{2b^2} - \frac{a^2}{2b^3}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.9. $$\int \frac{x^2\,dx}{\sqrt{(a^2+b^2x^2)^3}} = -\frac{x}{b^2\sqrt{a^2+b^2x^2}} + \frac{1}{b^3}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.10. $$\int \frac{x^2\,dx}{\sqrt{(a^2+b^2x^2)^5}} = \frac{1}{3a^2}\,\frac{x^3}{\sqrt{(a^2+b^2x^2)^3}}.$$

19.11. $$\int \frac{x^2\,dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m-3}}\sum_{\nu=0}^{\frac{m-5}{2}} \frac{(-1)^\nu C_{\frac{m-5}{2}}^{\nu} b^{2\nu}x^{2\nu+3}}{(2\nu+3)\sqrt{(a^2+b^2x^2)^{2\nu+3}}} \quad (m \geqslant 5).$$

19.12. $$\int \frac{x^3\,dx}{\sqrt{a^2+b^2x^2}} = \frac{\sqrt{(a^2+b^2x^2)^3}}{3b^4} - \frac{a^2}{b^4}\sqrt{a^2+b^2x^2}.$$

19.13. $$\int \frac{x^3\,dx}{\sqrt{(a^2+b^2x^2)^3}} = \frac{\sqrt{a^2+b^2x^2}}{b^4} + \frac{a^2}{b^4\sqrt{a^2-b^2x^2}}.$$

19.14. $$\int \frac{x^3\,dx}{\sqrt{(a^2+b^2x^2)^m}} = -\frac{1}{(m-4)\,b^4\sqrt{(a^2+b^2x^2)^{m-4}}} + \frac{a^2}{(m-2)\,b^4\sqrt{(a^2+b^2x^2)^{m-2}}}.$$

19.15. $$\int \frac{x^4\,dx}{\sqrt{a^2+b^2x^2}} = \frac{x^3\sqrt{a^2+b^2x^2}}{4b^2} - \frac{3a^2x}{8b^4}\sqrt{a^2+b^2x^2} + \frac{3a^4}{8b^5}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.16. $$\int \frac{x^4\,dx}{\sqrt{(a^2+b^2x^2)^3}} = \frac{x\sqrt{a^2+b^2x^2}}{2b^4} + \frac{a^2x}{b^4\sqrt{a^2+b^2x^2}} - \frac{3a^2}{2b^5}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.17. $$\int \frac{x^4\,dx}{\sqrt{(a^2+b^2x^2)^5}} = -\frac{3a^2x+4b^2x^3}{3b^4\sqrt{(a^2+b^2x^2)^3}} + \frac{1}{b^5}\ln\left|bx+\sqrt{a^2+b^2x^2}\right|.$$

19.18. $$\int \frac{x^4\,dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m-5}}\sum_{\nu=0}^{\frac{m-7}{2}} \frac{(-1)^\nu C_{\frac{m-7}{2}}^{\nu} b^{2\nu}x^{2\nu+5}}{(2\nu+5)\sqrt{(a^2+b^2x^2)^{2\nu+5}}} \quad (m \geqslant 7).$$

19.19. $$\int \frac{x^{2k+1}dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{2}\int \frac{t^k\,dt}{\sqrt{(a^2+b^2t)^m}}, \text{ где } t=x^2 \quad \text{(см. 16.11).}$$

19.20. $$\int \frac{x^{2k}\,dx}{\sqrt{(a^2+b^2x^2)^m}} = \frac{1}{a^{m-2k-1}}\sum_{\nu=0}^{\frac{m-2k-3}{2}} \frac{(-1)^\nu C_{\frac{m-2k-3}{2}}^{\nu} b^{2\nu}x^{2\nu+2k+1}}{(2\nu+2k+1)\sqrt{(a^2+b^2x^2)^{2\nu+2k+1}}} \quad (m \geqslant 2k+3).$$

19.21. $$\int \frac{x^n\,dx}{\sqrt{(a^2+b^2x^2)^m}} = -\frac{x^{n-1}}{(m-2)\,b^2\sqrt{(a^2+b^2x^2)^{m-2}}} + \frac{n-1}{(m-2)\,b^2}\int \frac{x^{n-2}\,dx}{\sqrt{(a^2+b^2x^2)^{m-2}}}.$$