ТАБЛИЦА 26

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{\sqrt{(ax^2+bx+c)^m}}; \qquad n=0,\ 1,\ 2,\ \ldots,\quad m=1,\ 3,\ 5,\ \ldots$$

26.1. $$\int \frac{dx}{\sqrt{x^2+px+q}} = \ln\left|2x+p+2\sqrt{x^2+px+q}\right|.$$

26.2. $$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| & \text{при } a>0,\ b^2\neq 4ac; \\ \dfrac{1}{\sqrt{a}}\ln|2ax+b| & \text{при } a>0,\ b^2=4ac; \\ -\dfrac{1}{\sqrt{-a}}\arcsin\dfrac{2ax+b}{\sqrt{b^2-4ac}} & \text{при } a<0,\ b^2>4ac. \end{cases}$$

26.3. $$\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{4ax+2b}{(4ac-b^2)\sqrt{ax^2+bx+c}}.$$

26.4. $$\int \frac{dx}{\sqrt{(ax^2+bx+c)^5}} = \frac{4ax+2b}{3(4ac-b^2)\sqrt{ax^2+bx+c}}\left(\frac{1}{\sqrt{ax^2+bx+c}}+\frac{8a}{4ac-b^2}\right).$$

26.5. $$\int \frac{dx}{\sqrt{(ax^2+bx+c)^m}} = \frac{4ax+2b}{(m-2)(4ac-b^2)\sqrt{(ax^2+bx+c)^{m-2}}} + \frac{4(m-3)a}{(m-2)(4ac-b^2)}\int \frac{dx}{\sqrt{(ax^2+bx+c)^{m-2}}} \quad (m\geqslant 3).$$

26.6. $$\int \frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{ax^2+bx+c}} \quad \text{(см. 26.2).}$$

26.7. $$\int \frac{x\,dx}{\sqrt{(ax^2+bx+c)^3}} = -\frac{2bx+4c}{(4ac-b^2)\sqrt{ax^2+bx+c}}.$$

26.8. $$\int \frac{x\,dx}{\sqrt{(ax^2+bx+c)^m}} = -\frac{1}{(m-2)a\sqrt{(ax^2+bx+c)^{m-2}}} - \frac{b}{2a}\int \frac{dx}{\sqrt{(ax^2+bx+c)^m}} \quad \text{(см. 26.5).}$$

26.9. $$\int \frac{x^2\,dx}{\sqrt{ax^2+bx+c}} = \frac{2ax-3b}{4a^2}\sqrt{ax^2+bx+c} + \frac{3b^2-4ac}{8a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}} \quad \text{(см. 26.2).}$$

26.10. $$\int \frac{x^3\,dx}{\sqrt{ax^2+bx+c}} = \frac{8a^2x^2-10abx+15b^2-8ac}{24a^3}\sqrt{ax^2+bx+c} - \frac{5b^3-12bc}{16a^3}\int \frac{dx}{\sqrt{ax^2+bx+c}} \quad \text{(см. 26.2).}$$

26.11. $$\int \frac{x^n\,dx}{\sqrt{(ax^2+bx+c)^m}} = \frac{1}{a}\int \frac{x^{n-2}\,dx}{\sqrt{(ax^2+bx+c)^{m-2}}} - \frac{c}{a}\int \frac{x^{n-2}\,dx}{\sqrt{(ax^2+bx+c)^m}} - \frac{b}{a}\int \frac{x^{n-1}\,dx}{\sqrt{(ax^2+bx+c)^m}}.$$

26.12. $$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = \begin{cases} -\dfrac{1}{\sqrt{c}}\ln\left|\dfrac{bx+2c+2\sqrt{c}\sqrt{ax^2+bx+c}}{x}\right| & \text{при } c>0,\ b^2\neq 4ac; \\ -\dfrac{1}{\sqrt{c}}\ln\left|\dfrac{bx+2c}{x}\right| & \text{при } c>0,\ b^2=4ac; \\ \dfrac{1}{\sqrt{-c}}\arcsin\dfrac{bx+2c}{x\sqrt{b^2-4ac}} & \text{при } c<0,\ b^2>4ac. \end{cases}$$

26.13. $$\int \frac{dx}{x\sqrt{(ax^2+bx+c)^m}} = \frac{1}{(m-2)c\sqrt{(ax^2+bx+c)^{m-2}}} + \frac{a}{c}\int \frac{dx}{x\sqrt{(ax^2+bx+c)^{m-2}}} - \frac{b}{2c}\int \frac{dx}{\sqrt{(ax^2+bx+c)^m}} \quad \text{(см. 26.5).}$$

26.14. $$\int \frac{dx}{x^2\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{ax} - \frac{b}{2c}\int \frac{dx}{x\sqrt{ax^2+bx+c}} \quad \text{(см. 26.12).}$$

26.15. $$\int \frac{dx}{x^n\sqrt{(ax^2+bx+c)^m}} = -\frac{1}{(n-1)cx^{n-1}\sqrt{(ax^2+bx+c)^{m-2}}} - \frac{(2n+m-4)b}{2(n-1)c}\int \frac{dx}{x^{n-1}\sqrt{(ax^2+bx+c)^m}} - \frac{(n+m-3)a}{(n-1)c}\int \frac{dx}{x^{n-2}\sqrt{(ax^2+bx+c)^m}} \quad (n\geqslant 2).$$