

**III. ТРИГОНОМЕТРИЧЕСКИЕ
И ОБРАТНЫЕ ТРИГОНОМЕТРИЧЕСКИЕ ФУНКЦИИ**

**ТАБЛИЦА 29
ИНТЕГРАЛЫ ВИДА**

$$\int x^n \sin^m px dx, \quad \begin{matrix} n=0, 1, 2, \dots \\ m=1, 2, 3, \dots \end{matrix}$$

$$29.1. \int \sin px dx = -\frac{1}{p} \cos px.$$

$$29.2. \int \sin^2 px dx = \frac{x}{2} - \frac{\sin 2px}{4p}.$$

$$29.3. \int \sin^3 px dx = \frac{\cos^2 px}{3p} - \frac{\cos px}{p}.$$

$$29.4. \int \sin^m px dx = -\frac{\sin^{m-1} px \cos px}{mp} + \frac{m-1}{m} \int \sin^{m-2} px dx.$$

$$29.5. \int x \sin px dx = \frac{1}{p^2} \sin px - \frac{x}{p} \cos px.$$

$$29.6. \int x \sin^2 px dx = \frac{x^2}{4} - \frac{x \sin 2px}{4p} - \frac{\cos 2px}{8p^2}.$$

$$29.7. \int x \sin^3 px dx = \frac{x \cos 3px}{12p} - \frac{\sin 3px}{36p^2} - \frac{3}{4p} x \cos px + \frac{3}{4p^2} \sin px.$$

$$29.8. \int x \sin^m px dx = \frac{\sin^{m-1} px}{m^2 p^2} [\sin px - mpx \cos px] + \frac{m-1}{m} \int x \sin^{m-2} px dx$$

$$29.9. \int x^2 \sin px dx = \frac{2x \sin px}{p^2} - \frac{p^2 x^2 - 2}{p^4} \cos px$$

$$29.10. \int x^2 \sin^3 px dx = \frac{x^3}{6} - \frac{2p^2 x^2 - 1}{8p^4} \sin 2px - \frac{x \cos 2px}{4p^2}.$$

$$29.11. \int x^4 \sin^m px dx = \frac{x \sin^{m-1} px}{m^2 p^2} [2 \sin px - mpx \cos px] + \\ + \frac{m-1}{m} \int x^2 \sin^{m-2} px dx - \frac{2}{m^2 p^2} \int \sin^m px dx \quad (\text{см. 29.4}).$$

$$29.12. \int x^3 \sin px dx = \frac{3p^2 x^2 - 6}{p^4} \sin px + \frac{6x - p^2 x^3}{p^3} \cos px.$$

$$29.13. \int x^3 \sin^2 px dx = \frac{x^4}{8} + \frac{3x - 2p^2 x^3}{8p^4} \sin 2px - \frac{6x^2 p^2 - 3}{16p^4} \cos 2px.$$

$$29.14. \int x^4 \sin^m px dx = \frac{x^4 \sin^{m-1} px}{m^2 p^2} [3 \sin px - mpx \cos px] + \\ + \frac{m-1}{m} \int x^3 \sin^{m-2} px dx - \frac{6}{m^2 p^2} \int x \sin^m px dx \quad (\text{см. 29.8}).$$

$$29.15. \int x^4 \sin px dx = \\ = \frac{1}{p^4} (4p^2 x^8 - 24x) \sin px - \frac{1}{p^4} (p^4 x^4 - 12p^2 x^2 + 24) \cos px.$$

$$29.16. \int x^n \sin px dx = \\ = -\frac{x^n}{p} \cos px + \frac{nx^{n-1}}{p^2} \sin px - \frac{n(n-1)}{p^2} \int x^{n-2} \sin px dx.$$

$$29.17. \int P_n(x) \sin px dx = \\ = -\cos px \sum_{v=0}^{E_1} \frac{(-1)^v}{p^{2v}} P_n^{(2v)}(x) + \sin px \sum_{v=1}^{E_2} \frac{(-1)^{v-1}}{p^{2v}} P_n^{(2v-1)}(x),$$

где $E_1 = \frac{n}{2}$ и $E_2 = \frac{n}{2}$ при n четном;

$E_1 = \frac{n-1}{2}$ и $E_2 = \frac{n+1}{2}$ при n нечетном.

$$29.18. \int x^n \sin^m px dx = \frac{x^{n-1} \sin^{m-1} px}{m^2 p^2} (n \sin px - mpx \cos px) + \\ + \frac{m-1}{m} \int x^n \sin^{m-2} px dx - \frac{n(n-1)}{m^2 p^2} \int x^{n-2} \sin^m px dx.$$