ТАБЛИЦА 3

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n}\frac{(a+bx)^m}{(c+fx)^k}\,dx, \quad af\neq bc;$$

$$n=0,\ 1,\ 2,\ \ldots,\ m=1,\ 2,\ 3,\ \ldots,\ k=1,\ 2,\ 3,\ \ldots$$

3.1. $$\int\frac{a+bx}{c+fx}\,dx=\frac{1}{f}\left(bx+\frac{\Delta}{f}\ln|c+fx|\right).$$

3.2. $$\int\frac{(a+bx)^2}{c+fx}\,dx=\frac{1}{f}\left[\frac{b^2}{2}x^2+b\left(a+\frac{\Delta}{f}\right)x+\frac{\Delta^2}{f^2}\ln|c+fx|\right].$$

3.3. $$\int\frac{(a+bx)^m}{c+fx}\,dx=$$
$$=\sum_{\nu=1}^{m}C_m^\nu a^{m-\nu}b^\nu\int\frac{x^\nu\,dx}{c+fx}+\frac{a^m}{f}\ln|c+fx| \qquad \text{(см. 1.20).}$$

3.4. $$\int\frac{a+bx}{(c+fx)^2}\,dx=-\frac{1}{f^2}\left(\frac{\Delta}{c+fx}-b\ln|c+fx|\right).$$

3.5. $$\int\left(\frac{a+bx}{c+fx}\right)^2dx=\frac{1}{f^2}\left[b^2x-\frac{\Delta}{f}\left(\frac{\Delta}{c+fx}-2b\ln|c+fx|\right)\right].$$

3.6. $$\int\left(\frac{a+bx}{c+fx}\right)^m dx=\frac{1}{f^m}\sum_{\nu=0}^{m}C_m^\nu\Delta^{m-\nu}b^\nu\int\frac{dx}{(c+fx)^{m-\nu}} \qquad \text{(см. 1.2).}$$

3.7. $$\int\frac{(a+bx)^m}{(c+fx)^k}\,dx=-\frac{(a+bx)^m}{(k-1)f(c+fx)^{k-1}}+$$
$$+\frac{mb}{(k-1)f}\int\frac{(a+bx)^{m-1}}{(c+fx)^{k-1}}\,dx \qquad (k\geqslant 2).$$

3.8. $$\int x\frac{a+bx}{c+fx}\,dx=\frac{1}{f}\left[\frac{b}{2}x^2+\frac{\Delta}{f}\left(x-\frac{c}{f}\ln|c+fx|\right)\right].$$

3.9. $$\int x\frac{(a+bx)^2}{c+fx}\,dx=$$
$$=\frac{1}{f}\left[\frac{b^2}{3}x^3+\frac{b}{2}\left(a+\frac{\Delta}{f}\right)x^2+\frac{\Delta^2}{f^2}\left(x-\frac{c}{f}\ln|c+fx|\right)\right].$$

3.10. $$\int x\frac{a+bx}{(c+fx)^2}\,dx=\frac{1}{f^2}\left(bx+\frac{c\Delta}{f(c+fx)}+\left(\frac{2\Delta}{f}-a\right)\ln|c+fx|\right].$$

3.11. $$\int x\left(\frac{a+bx}{c+fx}\right)^2dx=\frac{1}{f^2}\left\{\frac{b^2}{2}x^2+\frac{2b\Delta}{f}x+\right.$$
$$\left.+\frac{\Delta}{f^2}\left[\frac{c\Delta}{c+fx}+(\Delta-2bc)\ln|c+fx|\right]\right\}.$$

3.12. $$\int x\left(\frac{a+bx}{c+fx}\right)^m dx=\frac{(a+bx)^m}{2f^2(c+fx)^{m-2}}+\frac{c(a+bx)^m}{(m-1)f^2(c+fx)^{m-1}}-$$
$$-\frac{m[(m-1)\Delta+2bc]}{2(m-1)f^2}\int\left(\frac{a+bx}{c+fx}\right)^{m-1}dx \quad (m\geqslant 2) \quad \text{(см. 3.6).}$$

3.13. $$\int x^2\frac{a+bx}{c+fx}\,dx=\frac{1}{f}\left[\frac{b}{3}x^3+\frac{\Delta}{f}\left(\frac{x^2}{2}-\frac{c}{f}x+\frac{c^2}{f^2}\ln|c+fx|\right)\right].$$

3.14. $$\int x^2\frac{(a+bx)^2}{c+fx}\,dx=\frac{1}{f}\left[\frac{b^2}{4}x^4+\frac{b}{3}\left(a+\frac{\Delta}{f}\right)x^3+\right.$$
$$\left.+\frac{\Delta^2}{f^2}\left(\frac{x^2}{2}-\frac{c}{f}x+\frac{c^2}{f^2}\ln|c+fx|\right)\right].$$

3.15. $$\int x^2\frac{a+bx}{(c+fx)^2}\,dx=\frac{1}{f^2}\left[\frac{b}{2}x^2+\left(\frac{2\Delta}{f}-a\right)x-\right.$$
$$\left.-\frac{c^2\Delta}{f^2(c+fx)}-\frac{c}{f}\left(\frac{3\Delta}{f}-a\right)\ln|c+fx|\right].$$

3.16. $$\int x^2\left(\frac{a+bx}{c+fx}\right)^2dx=\frac{1}{f^2}\left\{\frac{b^2}{3}x^3+\frac{b\Delta}{f}x^2+\frac{c\Delta^2x}{f^4(c+fx)}+\right.$$
$$\left.+\frac{\Delta}{f^2}[(\Delta-2bc)(c+fx)-2c(\Delta-bc)\ln|c+fx|]\right\}.$$

3.17. $$\int\frac{a+bx}{x(c+fx)}\,dx=\frac{a}{c}\ln|x|-\frac{\Delta}{cf}\ln|c+fx|.$$

3.18. $$\int\frac{(a+bx)^2}{x(c+fx)}\,dx=\frac{b^2}{f}x+\frac{1}{c}\left(a^2\ln|x|-\frac{\Delta^2}{f^2}\ln|c+fx|\right).$$

3.19. $$\int\frac{a+bx}{x(c+fx)^2}\,dx=\frac{\Delta}{cf(c+fx)}-\frac{a}{c^2}\ln\left|\frac{c}{x}+f\right|.$$

3.20. $$\int\frac{1}{x}\left(\frac{a+bx}{c+fx}\right)^2dx=$$
$$=\frac{\Delta^2}{cf^2(c+fx)}+\frac{a^2}{c^2}\ln|x|-\left[\left(\frac{a}{c}\right)^2-\left(\frac{b}{f}\right)^2\right]\ln|c+fx|.$$