

ТАБЛИЦА 31  
ИНТЕГРАЛЫ ВИДА

$$\int R(\sin px, \cos qx, \sqrt{a^2 \pm b^2 \sin^2 x}) dx.$$


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31.1.  $\int \sin px \sin qx dx = \frac{\sin(q-p)x}{2(q-p)} - \frac{\sin(q+p)x}{2(q+p)} \quad (p^2 \neq q^2).$

31.2.  $\int \sin px \sin^n x dx = -\frac{\sin^n x \cos px}{p} + \frac{n}{2p} \int \sin^{n-1} x \cos(p-1)x dx + \frac{n}{2p} \int \sin^{n-1} x \cos(p+1)x dx \quad (\text{см. 35.8}).$

31.3.  $\int \frac{\sin x dx}{\sqrt{a^2 + b^2 \sin^2 x}} = -\frac{1}{b} \arcsin \frac{b \cos x}{\sqrt{a^2 + b^2}}.$

31.4.  $\int \frac{\sin x dx}{\sqrt{a^2 - b^2 \sin^2 x}} = -\frac{1}{b} \ln |b \cos x + \sqrt{a^2 - b^2 \sin^2 x}|.$

31.5.  $\int \sin x \sqrt{a^2 + b^2 \sin^2 x} dx = -\frac{\cos x}{2} \sqrt{a^2 + b^2 \sin^2 x} - \frac{a^2 + b^2}{2b} \arcsin \frac{b \cos x}{\sqrt{a^2 + b^2}}.$

31.6.  $\int \sin x \sqrt{a^2 - b^2 \sin^2 x} dx = -\frac{\cos x}{2} \sqrt{a^2 - b^2 \sin^2 x} - \frac{a^2 - b^2}{2b} \ln |b \cos x + \sqrt{a^2 - b^2 \sin^2 x}|.$

31.7.  $\int \frac{\sin^n x dx}{\sin(2k+1)x} = \frac{1}{2k+1} \sum_{v=0}^{2k} (-1)^{k+v} \cos^n \frac{2v+1}{2(2k+1)} \pi \ln \left| \frac{\sin \left[ \frac{(v-k)\pi}{2(2k+1)} + \frac{x}{2} \right]}{\sin \left[ \frac{v+k+1}{2(2k+1)} \pi - \frac{x}{2} \right]} \right|$

31.8.  $\int \frac{\sin^{2l} x dx}{\sin 2kx} = \frac{(-1)^k}{2k} \left[ \ln \cos x + \sum_{v=1}^{k-1} (-1)^v \cos^{2l} \frac{v\pi}{2k} \ln \left( \cos^2 x - \sin^2 \frac{v\pi}{2k} \right) \right]$

31.9.  $\int \frac{\sin^{2l+1} x}{\sin 2kx} dx = \frac{(-1)^k}{2k} \left\{ \ln \operatorname{tg} \left( \frac{\pi}{4} - \frac{x}{2} \right) + \sum_{v=1}^{k-1} (-1)^v \cos^{2l+1} \frac{v\pi}{2k} \ln \left[ \operatorname{tg} \left( \frac{k+v}{4k} \pi - \frac{x}{2} \right) \operatorname{tg} \left( \frac{k-v}{4k} \pi - \frac{x}{2} \right) \right] \right\}.$

31.10.  $\int \frac{\sin 2x}{\sin x} dx = 2 \sin x.$

31.11.  $\int \frac{\sin 2x}{\sin^2 x} dx = 2 \ln \sin x.$

31.12.  $\int \frac{\sin 2x}{\sin^3 x} dx = -\frac{2}{\sin x}.$

31.13.  $\int \frac{\sin 2x dx}{\sin^n x} = -\frac{2}{(n-2) \sin^{n-2} x} \quad (n \geq 3).$

31.14.  $\int \frac{\sin x}{\sin 2x} dx = \frac{1}{2} \ln \left| \operatorname{ctg} \left( \frac{x}{2} - \frac{\pi}{4} \right) \right|.$

31.15.  $\int \frac{\sin^2 x}{\sin 2x} dx = -\frac{1}{2} \ln |\cos x|.$

31.16.  $\int \frac{\sin^3 x}{\sin 2x} dx = -\frac{1}{2} \ln \left| \operatorname{ctg} \left( \frac{x}{2} - \frac{\pi}{4} \right) \right| - \frac{1}{2} \sin x.$

31.17.  $\int \frac{\sin^n x}{\sin 2x} dx = \frac{1}{2} \int \frac{t^{n-1} dt}{1-t^2}, \quad \text{где } t = \sin x \quad (n \geq 2) \quad (\text{см. 6.13}).$

31.18.  $\int \frac{\sin 3x}{\sin x} dx = x + \sin 2x.$

31.19.  $\int \frac{\sin 3x}{\sin^2 x} dx = 3 \ln \left| \operatorname{tg} \frac{x}{2} \right| + 4 \cos x.$

31.20.  $\int \frac{\sin 3x}{\sin^3 x} dx = -3 \operatorname{ctg} x - 4x.$

31.21.  $\int \frac{\sin kx}{\sin^n x} dx = \sum_{v=0}^E (-1)^v C_k^{2v+1} \int \frac{\cos^{k-2v-1} x}{\sin^{n-2v-1} x} dx,$

где  $E = \begin{cases} \frac{k}{2}, & \text{если } k \text{ четное,} \\ \frac{k+1}{2}, & \text{если } k \text{ нечетное.} \end{cases}$