ТАБЛИЦА 32

ИНТЕГРАЛЫ ВИДА

$$\int x^n \cos^m px\,dx;\quad \begin{matrix} m=1,\ 2,\ 3,\ \ldots, \\ n=0,\ 1,\ 2,\ \ldots \end{matrix}$$

32.1. $\int \cos px\,dx = \frac{1}{p}\sin px.$

32.2. $\int \cos^2 px\,dx = \frac{x}{2} + \frac{\sin 2px}{4p}.$

32.3. $\int \cos^3 px\,dx = \frac{1}{p}\sin px - \frac{\sin^3 px}{3p}.$

32.4 $\int \cos^m px\,dx = \frac{\sin px \cos^{m-1} px}{mp} + \frac{m-1}{m}\int \cos^{m-2} px\,dx.$

32.5. $\int x\cos px\,dx = \frac{1}{p^2}\cos px + \frac{x}{p}\sin px.$

32.6. $\int x\cos^2 px\,dx = \frac{x^2}{4} + \frac{x\sin 2px}{4p} + \frac{\cos 2px}{8p^2}.$

32.7. $\int x\cos^3 px\,dx = \frac{x\sin 3px}{12p} + \frac{\cos 3px}{36p^2} + \frac{3}{4p}x\sin px + \frac{3}{4p^2}\cos px.$

32.8. $\int x\cos^m px\,dx =$

$$= \frac{\cos^{m-1} x}{m^2p^2}[\cos px + pm\,x\sin px] + \frac{m-1}{m}\int x\cos^{m-2} px\,dx.$$

32.9. $\int x^2\cos px\,dx = \frac{2x\cos px}{p^2} + \frac{p^2x^2-2}{p^3}\sin px.$

32.10. $\int x^2\cos^2 px\,dx = \frac{x^3}{6} + \frac{2p^2x^2-1}{8p^3}\sin 2px + \frac{x\cos 2px}{4p^2}.$

32.11. $\int x^2\cos^m px\,dx = \frac{x\cos^{m-1} px}{m^2p^2}[2\cos px + mpx\sin px] +$

$$+\frac{m-1}{m}\int x^2\cos^{m-2} px\,dx - \frac{2}{m^2p^2}\int \cos^m px\,dx \quad \text{(см. 32.4).}$$

32.12. $\int x^3\cos px\,dx = \frac{3p^2x^2-6}{p^4}\cos px + \frac{p^2x^3-6x}{p^3}\sin px.$

32.13. $\int x^3\cos^2 px\,dx = \frac{x^4}{8} + \frac{2p^2x^3-3x}{8p^3}\sin 2px + \frac{6p^2x^2-3}{16p^4}\cos 2px.$

32.14. $\int x^3\cos^m px\,dx = \frac{x^2\cos^{m-1} px}{m^2p^2}[3\cos px + mpx\sin px] +$

$$+\frac{m-1}{m}\int x^3\cos^{m-2} px\,dx - \frac{6}{m^2p^2}\int x\cos^m px\,dx \quad \text{(см. 32.8).}$$

32.15. $\int x^4\cos px\,dx =$

$$= \frac{1}{p^4}(4p^2x^3-24x)\cos px + \frac{1}{p^5}(p^4x^4-12p^2x^2+24)\sin px.$$

32.16. $\int x^4\cos^2 px\,dx =$

$$= \frac{x^5}{10} + \frac{2p^4x^4-6p^2x^2+3}{8p^5}\sin 2px + \frac{2p^2x^3-3x}{4p^4}\cos 2px.$$

32.17. $\int x^4\cos^m px\,dx = \frac{x^3\cos^{m-1} px}{m^2p^2}(4\cos px + mpx\sin px) +$

$$+\frac{m-1}{m}\int x^4\cos^{m-2} px\,dx - \frac{12}{m^2p^2}\int x^2\cos^m px\,dx \quad \text{(см. 32.12).}$$

32.18. $\int x^n\cos px\,dx =$

$$= \frac{x^n}{p}\sin px + \frac{nx^{n-1}}{p^2}\cos px - \frac{n(n-1)}{p^2}\int x^{n-2}\cos px\,dx.$$

32.19. $\int P_n(x)\cos px\,dx =$

$$= \sin px\sum_{\nu=0}^{E_1}\frac{(-1)^\nu}{p^{2\nu}}P_n^{(2\nu)}(x) + \cos px\sum_{\nu=1}^{E_2}\frac{(-1)^{k-1}}{p^{2\nu}}P_n^{(2\nu-1)}(x),$$

где $E_1 = \frac{n}{2}$ и $E_2 = \frac{n}{2}$ при n четном;

$E_1 = \frac{n-1}{2}$ и $E_2 = \frac{n+1}{2}$ при n нечетном.

32.20. $\int x^n\cos^m px\,dx = \frac{x^{n-1}\cos^{m-1} px}{m^2p^2}(n\cos px + mpx\sin px) +$

$$+\frac{m-1}{m}\int x^n\cos^{m-2} px\,dx - \frac{n(n-1)}{m^2p^2}\int x^{n-2}\cos^m px\,dx.$$