ТАБЛИЦА 33

ИНТЕГРАЛЫ ВИДА

$$\int \frac{\cos^m px}{x^n}\,dx, \quad \int \frac{x^n\,dx}{\cos^m px}, \quad \int \frac{x^n \cos^r x}{(a+b\cos x)^m}\,dx;$$

$$m=1,\ 2,\ 3,\ \ldots,\quad n=0,\ 1,\ 2,\ \ldots,\quad r=0,\ 1.$$

33.1.* $\int \frac{\cos x}{x}\,dx = \ln x - \frac{x^2}{2\cdot 2!} + \frac{x^4}{4\cdot 4!} - \frac{x^6}{6\cdot 6!} + \frac{x^8}{8\cdot 8!} - \ldots$

$$\ldots + (-1)^k \frac{x^{2k}}{2k\,(2k!)} + \ldots$$

33.2.* $\int \frac{\cos px}{x^n}\,dx = -\frac{\cos px}{(n-1)\,x^{n-1}} - \frac{p^{n-1}}{n-1}\int \frac{\sin t\,dt}{t^{n-1}}$, где $t=px$

$(n \geqslant 2)$ (см. 30.2)

33.3. $\int \frac{\cos^m px}{x^n}\,dx = -\frac{\cos^{m-1} px\,[(n-2)\cos px - mpx\sin px]}{(n-1)(n-2)\,x^{m-1}} -$

$$-\frac{m^2p^2}{(n-1)(n-2)}\int \frac{\cos^m px\,dx}{x^{n-2}} + \frac{m(m-1)\,p^2}{(n-1)(n-2)}\int \frac{\cos^{m-2} px\,dx}{x^{n-2}} \quad (n \geqslant 3)$$

33.4. $\int \frac{dx}{\cos x} = \ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|.$

33.5 $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x.$

33.6. $\int \frac{dx}{\cos^3 x} = \frac{\sin x}{2\cos^2 x} + \frac{1}{2}\ln\left|\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|.$

33.7. $\int \frac{dx}{\cos^m px} = \frac{\sin px}{(m-1)\,p\cos^{m-1} px} + \frac{m-2}{m-1}\int \frac{dx}{\cos^{m-2} px} \quad (m \geqslant 2).$

33.8.* $\int \frac{x\,dx}{\cos x} = \frac{x^2}{2} + \frac{x^4}{4\cdot 2!} + \frac{5x^6}{6\cdot 4!} + \frac{61x^8}{8\cdot 6!} + \ldots + \frac{E_n x^{2n+2}}{(2n+2)\,(2n)!} + \ldots$

33.9. $\int \frac{x\,dx}{\cos^2 x} = x\operatorname{tg} x + \ln|\cos x|.$

33.10.* $\int \frac{x\,dx}{\cos^3 x} = \frac{x\sin x}{2\cos^2 x} - \frac{1}{2\cos x} + \frac{1}{2}\int \frac{x\,dx}{\cos x}$ (см. 33.8).

33.11. $\int \frac{x\,dx}{\cos^m px} = \frac{x\sin px}{p\,(m-1)\cos^{m-1} px} -$

$$-\frac{1}{(m-1)(m-2)\,p^2\cos^{m-2} px} + \frac{m-2}{m-1}\int \frac{x\,dx}{\cos^{m-2} px} \quad (m \geqslant 3).$$

33.12. $\int \frac{x^n\,dx}{\cos^m px} =$

$$= -\frac{x^{n-1}}{(m-1)(m-2)\,p^2\cos^{m-1} px}\{n\cos px - (m-2)\,px\sin px\} +$$

$$+\frac{m-2}{m-1}\int \frac{x^n\,dx}{\cos^{m-2} px} + \frac{n(n-1)}{(m-1)(m-2)\,p^2}\int \frac{x^{n-2}\,dx}{\cos^{m-2} px} \quad (m \geqslant 3).$$

33.13. $\int \frac{dx}{1\pm\cos x} = \pm\operatorname{tg}\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4}-\frac{x}{2}\right)\right].$

33.14. $\int \frac{dx}{a+b\cos x} =$

$$= \begin{cases} \dfrac{2}{\sqrt{a^2-b^2}}\operatorname{arctg}\dfrac{(a-b)\operatorname{tg}\frac{x}{2}}{\sqrt{a^2-b^2}} & \text{при } a^2 > b^2; \\[2ex] \dfrac{1}{\sqrt{b^2-a^2}}\ln\left|\dfrac{(b-a)\operatorname{tg}\frac{x}{2}+\sqrt{b^2-a^2}}{(b-a)\operatorname{tg}\frac{x}{2}-\sqrt{b^2-a^2}}\right| & \text{при } a^2 < b^2. \end{cases}$$

33.15. $\int \frac{dx}{(1+\cos x)^2} = \frac{1}{2}\operatorname{tg}\frac{x}{2} + \frac{1}{6}\operatorname{tg}^3\frac{x}{2}.$

33.16. $\int \frac{dx}{(1-\cos x)^2} = -\frac{1}{2}\operatorname{ctg}\frac{x}{2} - \frac{1}{6}\operatorname{ctg}^3\frac{x}{2}.$

33.17. $\int \frac{dx}{(a+b\cos x)^2} = \frac{b\sin x}{(b^2-a^2)(a+b\cos x)} - \frac{a}{b^2-a^2}\int \frac{dx}{a+b\cos x}$

$(a^2 \neq b^2)$ (см. 33.14).

33.18. $\int \frac{x\,dx}{1\pm\cos x} =$

$$= \pm x\operatorname{tg}\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4}-\frac{x}{2}\right)\right] + 2\ln\left|\cos\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4}-\frac{x}{2}\right)\right]\right|.$$

33.19. $\int \frac{\cos x\,dx}{1\pm\cos x} = \pm x \mp \operatorname{tg}\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4}-\frac{x}{2}\right)\right].$

33.20. $\int \frac{\cos x\,dx}{a+b\cos x} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+b\cos x}$ $(a^2 \neq b^2)$ (см. 33.14).

33.21. $\int \frac{\cos x\,dx}{(1\pm\cos x)^2} =$

$$= \frac{1}{2}\operatorname{tg}\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4}-\frac{x}{2}\right)\right] - \frac{1}{6}\operatorname{tg}^3\left[\frac{\pi}{4} \mp \left(\frac{\pi}{4}-\frac{x}{2}\right)\right].$$

33.22. $\int \frac{\cos x\,dx}{(a+b\cos x)^2} = \frac{a\sin x}{(a^2-b^2)(a+b\cos x)} - \frac{b}{a^2-b^2}\int \frac{dx}{a+b\cos x}$

$(a^2 \neq b^2)$ (см. 33.14)