## ТАБЛИЦА 34

### ИНТЕГРАЛЫ ВИДА

$$\int R\left(\cos px,\ \cos qx,\ \sqrt{a^2 \pm b^2 \sin^2 x}\right) dx.$$

**34.1.** $$\int \cos px \cos qx\, dx = \frac{\sin (p+q)\, x}{2\,(p+q)} + \frac{\sin (p-q)\, x}{2\,(p-q)} \quad (p^2 \neq q^2).$$

**34.2.** $$\int \cos px \cos^n x\, dx = \frac{\cos^n x \sin px}{p} + \frac{n}{2p} \int \cos^{n-1} x \cos (p-1)\, x\, dx - \frac{n}{2p} \int \cos^{n-1} x \cos (p+1)\, x\, dx.$$

**34.3.** $$\int \frac{\cos x\, dx}{\sqrt{a^2 + b^2 \cos^2 x}} = \frac{1}{b} \arcsin \frac{b \sin x}{\sqrt{a^2+b^2}}.$$

**34.4.** $$\int \frac{\cos x\, dx}{\sqrt{a^2 - b^2 \cos^2 x}} = \frac{1}{b} \ln \left| b \sin x + \sqrt{a^2 - b^2 \cos^2 x} \right|.$$

**34.5.** $$\int \cos x \sqrt{a^2 + b^2 \cos^2 x}\, dx = \frac{\sin x}{2} \sqrt{a^2 + b^2 \cos^2 x} + \frac{a^2+b^2}{2b} \arcsin \frac{b \sin x}{\sqrt{a^2+b^2}}.$$

**34.6.** $$\int \cos x \sqrt{a^2 - b^2 \cos^2 x}\, dx = \frac{\sin x}{2} \sqrt{a^2 - b^2 \cos^2 x} + \frac{a^2-b^2}{2b} \ln \left| b \sin x + \sqrt{a^2 - b^2 \cos^2 x} \right|.$$

**34.7.** $$\int \frac{\cos^n x}{\cos kx}\, dx = \frac{1}{k} \sum_{\nu=0}^{k-1} (-1)^\nu \cos^n \frac{2\nu+1}{2k} \pi \ln \left| \frac{\sin \left[\frac{2\nu+1}{4k}\pi + \frac{x}{2}\right]}{\sin \left[\frac{2\nu+1}{4k}\pi - \frac{x}{2}\right]} \right|.$$

**34.8.** $$\int \frac{\cos 2x}{\cos x}\, dx = 2 \sin x - \ln \left| \operatorname{tg} \left( \frac{\pi}{4} + \frac{x}{2} \right) \right|.$$

**34.9.** $$\int \frac{\cos 2x}{\cos^2 x}\, dx = 2x - \operatorname{tg} x.$$

**34.10.** $$\int \frac{\cos 2x}{\cos^3 x}\, dx = -\frac{\sin x}{2 \cos^2 x} + \frac{3}{2} \ln \left| \operatorname{tg} \left( \frac{\pi}{4} + \frac{x}{2} \right) \right|.$$



**34.11.** $$\int \frac{\cos 2x}{\cos^n x}\, dx = -\frac{\sin x}{(n-1) \cos^{n-1} x} + \frac{n}{n-1} \int \frac{dx}{\cos^{n-2} x} \quad (n \geqslant 2) \quad \text{(см. 33.7).}$$

**34.12.** $$\int \frac{\cos x}{\cos 2x}\, dx = \frac{1}{2\sqrt{2}} \ln \left| \frac{1 - \sqrt{2} \sin x}{1 + \sqrt{2} \sin x} \right|.$$

**34.13.** $$\int \frac{\cos^2 x\, dx}{\cos 2x} = \frac{x}{2} - \frac{1}{4} \ln \left| \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} \right|.$$

**34.14.** $$\int \frac{\cos^3 x\, dx}{\cos 2x} = \frac{1}{2} \sin x + \frac{1}{4\sqrt{2}} \ln \left| \frac{1 - \sqrt{2} \sin x}{1 + \sqrt{2} \sin x} \right|.$$

**34.15.** $$\int \frac{\cos^n x}{\cos 2x}\, dx = \frac{1}{2} \int \cos^{n-2} x\, dx + \frac{1}{2} \int \frac{\cos^{n-2} x}{\cos 2x}\, dx \quad (n \geqslant 2) \quad \text{(см. 32.4).}$$

**34.16.** $$\int \frac{\cos 3x}{\cos x}\, dx = \sin 2x - x.$$

**34.17.** $$\int \frac{\cos 3x}{\cos^2 x}\, dx = 4 \sin x - 3 \ln \left| \operatorname{tg} \left( \frac{\pi}{4} + \frac{x}{2} \right) \right|.$$

**34.18.** $$\int \frac{\cos 3x}{\cos^3 x}\, dx = 4x - 3 \operatorname{tg} x.$$

**34.19.** $$\int \frac{\cos 3x}{\cos^n x}\, dx = 4 \int \frac{dx}{\cos^{n-3} x} - 3 \int \frac{dx}{\cos^{n-1} x} \quad \text{(см. 33.7).}$$

**34.20.** $$\int \frac{\cos kx}{\cos^n x}\, dx = \sum_{\nu=0}^{E} (-1)^\nu \frac{k}{k-\nu} 2^{k-2\nu-1} C_{n-\nu}^{\nu} \int \cos^{k-2\nu-n} x\, dx,$$

$$\text{где } E = \begin{cases} \frac{k}{2}, & \text{если } k \text{ четное;} \\ \frac{k-1}{2}, & \text{если } k \text{ нечетное.} \end{cases}$$

**34.21.** $$\int \frac{\cos x}{\cos 3x}\, dx = \frac{\sqrt{3}}{6} \ln \left| \frac{\sin \left( \frac{x}{2} + \frac{\pi}{12} \right) \sin \left( \frac{x}{2} - \frac{5\pi}{12} \right)}{\sin \left( \frac{x}{2} - \frac{\pi}{12} \right) \sin \left( \frac{x}{2} + \frac{5\pi}{12} \right)} \right|.$$

**34.22.** $$\int \frac{\cos^2 x}{\cos 3x}\, dx = \frac{1}{4} \ln \left| \frac{\sin \left( \frac{x}{2} + \frac{\pi}{12} \right) \sin \left( \frac{x}{2} + \frac{5\pi}{12} \right)}{\sin \left( \frac{x}{2} - \frac{\pi}{12} \right) \sin \left( \frac{x}{2} - \frac{5\pi}{12} \right)} \right|.$$