ТАБЛИЦА 37

ИНТЕГРАЛЫ ВИДА

$$\int F(x,\ \operatorname{tg} x,\ \operatorname{ctg} x)\,dx.$$

37.1. $\int \operatorname{tg} x\,dx = -\ln|\cos x|.$

37.2. $\int \operatorname{tg}^2 x\,dx = \operatorname{tg} x - x.$

37.3. $\int \operatorname{tg}^3 x\,dx = \frac{\operatorname{tg}^2 x}{2} + \ln|\cos x|.$

37.4. $\int \operatorname{tg}^n x\,dx = \frac{\operatorname{tg}^{n-1} x}{n-1} - \int \operatorname{tg}^{n-2} x\,dx \quad (n \geqslant 2).$

37.5. $\int \operatorname{ctg} x\,dx = \ln|\sin x|.$

37.6. $\int \operatorname{ctg}^2 x\,dx = -\operatorname{ctg} x - x.$

37.7. $\int \operatorname{ctg}^3 x\,dx = -\frac{\operatorname{ctg}^2 x}{2} - \ln|\sin x|.$

37.8. $\int \operatorname{ctg}^n x\,dx = -\frac{\operatorname{ctg}^{n-1} x}{n-1} - \int \operatorname{ctg}^{n-2} x\,dx \quad (n \geqslant 2).$

37.9. $\int \frac{dx}{\operatorname{tg} x \pm 1} = \pm\frac{x}{2} + \frac{1}{2}\ln|\sin x \pm \cos x|.$

37.10. $\int \frac{dx}{a + b\operatorname{tg} x} = \frac{1}{a^2+b^2}\left(b\ln|a + b\operatorname{tg} x| + b\ln|\cos x| + ax\right).$

37.11. $\int \frac{\operatorname{tg} x\,dx}{\operatorname{tg} x \pm 1} = \frac{x}{2} \mp \frac{1}{2}\ln|\sin x \pm \cos x|.$

37.12. $\int \frac{\operatorname{tg} x\,dx}{a + b\operatorname{tg} x} = \frac{1}{a^2+b^2}\left(bx - a\ln|a\cos x + b\sin x|\right).$

37.13. $\int \frac{dx}{1+\operatorname{tg}^2 x} = \frac{x}{2} + \frac{1}{4}\sin 2x.$

37.14. $\int \frac{dx}{a^2 + b^2\operatorname{tg}^2 x} = \frac{1}{a^2-b^2}\left[x - \left|\frac{b}{a}\right|\operatorname{arctg}\left(\left|\frac{b}{a}\right|\operatorname{tg} x\right)\right] \quad (a^2 \neq b^2)$

37.15. $\int \frac{dx}{a^2 - b^2\operatorname{tg}^2 x} = \frac{1}{a^2+b^2}\left(x + \frac{b}{2a}\ln\left|\frac{a + b\operatorname{tg} x}{a - b\operatorname{tg} x}\right|\right).$

37.16. $\int \frac{\operatorname{tg} x\,dx}{1+\operatorname{tg}^2 x} = -\frac{\cos^2 x}{2}.$

37.17. $\int \frac{\operatorname{tg} x\,dx}{1 + a^2\operatorname{tg}^2 x} = \frac{\ln(\cos^2 x + a^2\sin^2 x)}{2(a^2-1)} \quad (a \neq 1).$

37.18. $\int \frac{dx}{\operatorname{ctg} x \pm 1} = \frac{x}{2} \pm \frac{1}{2}\ln|\sin x \pm \cos x|.$

37.19. $\int \frac{dx}{a + b\operatorname{ctg} x} = \int \frac{\operatorname{tg} x\,dx}{a\operatorname{tg} x + b}$ (см. 37.12).

37.20. $\int \frac{\operatorname{ctg} x\,dx}{\operatorname{ctg} x \pm 2} = \pm\int \frac{dx}{\operatorname{tg} x \pm 1}$ (см. 37.9).

37.21. $\int \frac{\operatorname{ctg} x\,dx}{a + b\operatorname{ctg} x} = \int \frac{dx}{a\operatorname{tg} x + b}$ (см. 37.10).

37.22. $\int \frac{dx}{1+\operatorname{ctg}^2 x} = \frac{x}{2} - \frac{1}{4}\sin 2x.$

37.23. $\int \frac{dx}{a^2 + b^2\operatorname{ctg}^2 x} = \frac{1}{a^2-b^2}\left[x - \left|\frac{b}{a}\right|\operatorname{arctg}\left(-\left|\frac{b}{a}\right|\operatorname{ctg} x\right)\right]$
$(a^2 \neq b^2).$

37.24. $\int \frac{dx}{a^2 - b^2\operatorname{ctg}^2 x} = \frac{1}{a^2+b^2}\left(x + \frac{b}{2a}\ln\left|\frac{a - b\operatorname{ctg} x}{a + b\operatorname{ctg} x}\right|\right).$

37.25. $\int \frac{\operatorname{ctg} x\,dx}{1+\operatorname{ctg}^2 x} = \frac{\sin^2 x}{2}.$

37.26. $\int \frac{\operatorname{ctg} x\,dx}{1 + a^2\operatorname{ctg}^2 x} = \frac{1}{a^2}\int \frac{\operatorname{tg} x\,dx}{1 + \frac{1}{a^2}\operatorname{tg}^2 x}$ (см. 37.17).

37.27. $\int \frac{\operatorname{tg} x\,dx}{1 \pm \operatorname{ctg} x} = \mp\frac{x}{2} + \frac{1}{2}\ln\left|\frac{\sin x \pm \cos x}{\cos^2 x}\right|.$

37.28. $\int \frac{\operatorname{ctg} x\,dx}{1 \pm \operatorname{tg} x} = \mp\frac{x}{2} + \frac{1}{2}\ln|\sin^2 x(\sin x \pm \cos x)|.$

37.29.* $\int x\operatorname{tg} x\,dx = \frac{x^3}{3} + \frac{x^5}{15} + \ldots + \frac{2^{2n}(2^{2n}-1)B_n}{(2n+1)!}x^{2n+1} + \ldots$

37.30.* $\int x\operatorname{ctg} x\,dx = x - \frac{x^3}{9} - \frac{x^5}{225} - \ldots - \frac{2^{2n}B_n}{(2n+1)!}x^{2n+1} - \ldots$

37.31. $\int \frac{\operatorname{tg} x\,dx}{\sqrt{a + b\operatorname{tg}^2 x}} = \frac{1}{\sqrt{b-a}}\arccos\left(\frac{\sqrt{b-a}}{\sqrt{b}}\cos x\right).$