ТАБЛИЦА 39

ИНТЕГРАЛЫ ВИДА

$$\int F\left(x,\ \operatorname{arctg}\frac{x}{a}\right)dx,\qquad \int F\left(x,\ \operatorname{arcctg}\frac{x}{a}\right)dx.$$

39.1. $\int \operatorname{arctg}\frac{x}{a}\,dx = x\operatorname{arctg}\frac{x}{a} - \frac{a}{2}\ln(a^2+x^2).$

39.2. $\int\left(\operatorname{arctg}\frac{x}{a}\right)^2 dx = x\left(\operatorname{arctg}\frac{x}{a}\right)^2 - 2a\int\frac{x\operatorname{arctg}\frac{x}{a}}{a^2+x^2}\,dx$ (см. 39.11).

39.3. $\int x\operatorname{arctg}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\operatorname{arctg}\frac{x}{a} - \frac{ax}{2}.$

39.4. $\int x^2\operatorname{arctg}\frac{x}{a}\,dx = \frac{x^3}{3}\operatorname{arctg}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(a^2+x^2).$

39.5. $\int x^3\operatorname{arctg}\frac{x}{a}\,dx = \frac{1}{4}(x^4-a^4)\operatorname{arctg}\frac{x}{a} - \frac{ax^3}{12} + \frac{a^3x}{4}.$

39.6. $\int x^n\operatorname{arctg}\frac{x}{a}\,dx = \frac{x^{n+1}}{n+1}\operatorname{arctg}\frac{x}{a} - \frac{a}{n+1}\int\frac{x^{n+1}}{a^2+x^2}\,dx$ (см. 5.15)

39.7.* $\int\frac{1}{x}\operatorname{arctg}\frac{x}{a}\,dx = \sum_{\nu=0}^{\infty}(-1)^\nu\frac{x^{2\nu+1}}{(2\nu+1)^2a^{2\nu+1}}.$

39.8. $\int\frac{1}{x^2}\operatorname{arctg}\frac{x}{a} = -\frac{1}{x}\operatorname{arctg}\frac{x}{a} - \frac{1}{2a}\ln\frac{a^2+x^2}{x^2}.$

39.9. $\int\frac{1}{x^3}\operatorname{arctg}\frac{x}{a}\,dx = -\frac{a^2+x^2}{2a^2x^2}\operatorname{arctg}\frac{x}{a} - \frac{1}{2ax}.$

39.10. $\int\frac{1}{x^n}\operatorname{arctg}\frac{x}{a}\,dx = -\frac{1}{(n-1)x^{n-1}}\operatorname{arctg}\frac{x}{a} + \frac{a}{n-1}\int\frac{dx}{x^{n-1}(a^2+x^2)}$ $(n\geqslant 2)$ (см. 5.23).

39.11.* $\int\frac{x\operatorname{arctg}x}{1+x^2}\,dx = \frac{1}{2}\operatorname{arctg}x\ln(1+x^2) - \frac{x^3}{6} - \frac{x^5}{60} - \frac{x^7}{325} - \ldots - \frac{2^{n-1}(2^{2n}-1)B_n}{n(2n+1)!}x^{2n+1} - \ldots$

39.12. $\int\frac{x^2\operatorname{arctg}x}{1+x^2}\,dx = x\operatorname{arctg}x - \frac{1}{2}\ln(1-x^2) - \frac{1}{2}(\operatorname{arctg}x)^2.$

39.13.* $\int\frac{x^3\operatorname{arctg}x}{1+x^2}\,dx = -\frac{1}{2}x + \frac{1}{2}(1+x^2)\operatorname{arctg}x - \int\frac{x\operatorname{arctg}x}{1+x^2}\,dx$ (см. 39.11.)

39.14. $\int\frac{x^4\operatorname{arctg}x}{1+x^2}\,dx = -\frac{1}{6}x^2 + \frac{2}{3}\ln(1+x^2) + \left(\frac{x^3}{6} - x\right)\operatorname{arctg}x + \frac{1}{2}(\operatorname{arctg}x)^2.$

39.15. $\int\frac{x\operatorname{arctg}x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}\operatorname{arctg}x + \sqrt{2}\operatorname{arctg}\frac{x\sqrt{2}}{\sqrt{1-x^2}} - \arcsin x.$

39.16. $\int\frac{\operatorname{arctg}x}{(\alpha+\beta x)^2}\,dx = \frac{1}{\alpha^2+\beta^2}\left[\ln\left|\frac{\alpha+\beta x}{\sqrt{1+x^2}}\right| - \frac{\beta-\alpha x}{\alpha+\beta x}\operatorname{arctg}x\right].$

39.17. $\int\operatorname{arcctg}\frac{x}{a}\,dx = x\operatorname{arcctg}\frac{x}{a} + \frac{a}{2}\ln(a^2+x^2).$

39.18. $\int x^n\operatorname{arcctg}\frac{x}{a}\,dx = \frac{x^{n+1}}{n+1}\operatorname{arcctg}\frac{x}{a} + \frac{a}{n+1}\int\frac{x^{n+1}}{a^2+x^2}\,dx$ (см. 5.15).

39.19.* $\int\frac{1}{x}\operatorname{arcctg}\frac{x}{a}\,dx = \frac{\pi}{2}\ln x - \sum_{\nu=0}^{\infty}(-1)^\nu\frac{x^{2\nu+1}}{(2\nu+1)a^{2\nu+1}}.$

39.20. $\int\frac{1}{x^n}\operatorname{arcctg}\frac{x}{a}\,dx = -\frac{1}{(n-1)x^{n-1}}\operatorname{arcctg}\frac{x}{a} - \frac{a}{n+1}\int\frac{dx}{x^{n-1}(a^2+x^2)}$ $(n\geqslant 2)$ (см. 5.23).

39.21.* $\int\frac{x\operatorname{arcctg}x}{1+x^2}\,dx = \frac{\pi}{4}\ln(1+x^2) - \int\frac{x\operatorname{arctg}x}{1+x^2}\,dx$ (см. 39.11).

39.22. $\int F(\operatorname{arcctg}x)\,dx = \int F\left(\frac{\pi}{2} - \operatorname{arctg}x\right)dx.$