ТАБЛИЦА 42

ИНТЕГРАЛЫ ВИДА

$$\int R(x)\,e^{ax}\,dx, \qquad \int R\left(x,\ \sqrt{x},\ e^{x},\ e^{\sqrt{x}}\right)dx.$$

Обозначения: $u=\alpha+\beta x$, $t=\alpha-\beta x$, $s=\sqrt{x}$.

42.1.* $\displaystyle\int \frac{e^{ax}}{\alpha+\beta x}\,dx=\frac{e^{-\frac{a\alpha}{\beta}}}{\beta}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.2.* $\displaystyle\int \frac{xe^{ax}}{\alpha+\beta x}\,dx=\frac{e^{-ax}}{a\beta}-\frac{\alpha e^{-\frac{a\alpha}{\beta}}}{\beta^2}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.3.* $\displaystyle\int \frac{x^2e^{ax}}{\alpha+\beta x}\,dx=\frac{\beta^2(\beta x-\alpha)}{\alpha}\,e^{ax}+\frac{\alpha^2}{\beta^3}\,e^{-\frac{a\alpha}{\beta}}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.4.* $\displaystyle\int \frac{x^ne^{ax}}{\alpha+\beta x}\,dx=(-1)^n\,\frac{\alpha^n}{\beta^{n+1}}\,e^{-\frac{a\alpha}{\beta}}\sum_{\nu=0}^{n}\frac{(-1)^\nu C_n^\nu}{\alpha^\nu}\int u^{\nu-1}e^{\frac{a}{\beta}u}\,du$

(см. 41.7).

42.5. $\displaystyle\int \frac{e^{ax}\,dx}{(\alpha+\beta x)^2}=-\frac{e^{ax}}{\beta u}+\frac{ae^{-\frac{\alpha a}{\beta}}}{\beta^2}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.6. $\displaystyle\int \frac{xe^{ax}\,dx}{(\alpha+\beta x)^2}=\frac{\alpha e^{ax}}{\beta^2u}+\frac{\beta-a\alpha}{\beta^3}\,e^{-\frac{a\alpha}{\beta}}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du$ (см. 41.9).

42.7. $\displaystyle\int \frac{x^2e^{ax}}{(\alpha+\beta x)^2}\,dx=\frac{e^{ax}}{\beta^3}\left(\frac{\beta}{a}-\frac{\alpha^2}{u}\right)+\frac{a\alpha^2-2\alpha\beta}{\beta^4}\,e^{-\frac{a\alpha}{\beta}}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du.$

42.8. $\displaystyle\int \frac{x^ne^{ax}}{(\alpha+\beta x)^m}\,dx=(-1)^n\,\frac{\alpha^n}{\beta^{n+1}}\,e^{-\frac{a\alpha}{\beta}}\sum_{\nu=0}^{n}\frac{(-1)^\nu C_n^\nu}{\alpha^\nu}\int u^{\nu-m}e^{\frac{a}{\beta}u}\,du$

(см. 41.7 при $\nu-m\geqslant 0$ и 41.13 при $\nu-m<0$).

42.9* $\displaystyle\int \frac{e^{ax}}{\alpha^2-\beta^2x^2}\,dx=\frac{e^{-\frac{a\alpha}{\beta}}}{2\alpha\beta}\int \frac{e^{\frac{a}{\beta}u}}{u}\,du-\frac{e^{\frac{a\alpha}{\beta}}}{2\beta a}\int \frac{e^{-\frac{a}{\beta}t}}{t}\,dt$ (см. 41.9).

42.10. $\displaystyle\int e^{\sqrt{x}}\,dx=2e^{\sqrt{x}}\left(\sqrt{x}-1\right).$

42.11. $\displaystyle\int \sqrt{x}\,e^{\sqrt{x}}\,dx=2e^{\sqrt{x}}\left(x-2\sqrt{x}+2\right).$

42.12. $\displaystyle\int xe^{\sqrt{x}}\,dx=2e^{\sqrt{x}}\left(x\sqrt{x}-3x+6\sqrt{x}-6\right).$

42.13. $\displaystyle\int x\sqrt{x}\,e^{\sqrt{x}}\,dx=2e^{\sqrt{x}}\left(x^2-4x\sqrt{x}+12x-24\sqrt{x}+24\right).$

42.14. $\displaystyle\int x^2e^{\sqrt{x}}\,dx=$

$$=2e^{\sqrt{x}}\left(x^2\sqrt{x}-5x^2+20x\sqrt{x}-60x+120\sqrt{x}-120\right).$$

42.15. $\displaystyle\int x^ne^{\sqrt{x}}\,dx=2e^{\sqrt{x}}\left[x^n\sqrt{x}-(2n+1)\,x^n+\right.$

$$\left.+(2n+1)\,2nx^{n-1}\sqrt{x}-\ldots+(2n+1)!\,\sqrt{x}-(2n+1)!\right]$$

42.16. $\displaystyle\int x^{n+\frac{1}{2}}e^{\sqrt{x}}\,dx=e^{\sqrt{x}}\left[x^{n+1}-(2n+2)\,x^{\frac{2n+1}{2}}\right]-$

$$-\frac{(2n+1)(2n+2)}{2}\int x^{n-\frac{1}{2}}e^{\sqrt{x}}\,dx.$$

42.17. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx=2e^{\sqrt{x}}.$

42.18.* $\displaystyle\int \frac{e^{\sqrt{x}}}{x}\,dx=2\int \frac{e^s}{s}\,ds$ (см. 41.9).

42.19.* $\displaystyle\int \frac{e^{\sqrt{x}}\,dx}{x\sqrt{x}}=-\frac{2e^{\sqrt{x}}}{\sqrt{x}}+\int \frac{e^{\sqrt{x}}}{x}\,dx$ (см. 42.18).

42.20.* $\displaystyle\int \frac{e^{\sqrt{x}}}{x^n}\,dx=2\int \frac{e^s}{s^{2n-1}}\,ds$ (см. 41.13).

42.21.* $\displaystyle\int \frac{e^{\sqrt{x}}}{x^{n+\frac{1}{2}}}\,dx=2\int \frac{e^s}{s^{2n}}\,ds$ (см. 41.13).

42.22.* $\displaystyle\int \sqrt{x}\,e^x\,dx=2\int s^2e^{s^2}\,ds$ (см. 41.16).

42.23.* $\displaystyle\int \frac{e^{\pm x}}{\sqrt{x}}\,dx=2\int e^{\pm s^2}\,ds$ (см. 41.14).