## ТАБЛИЦА 43

### ИНТЕГРАЛЫ ВИДА

$$\int \frac{e^{ax}\,dx}{(\alpha+\beta e^{ax})^m}, \quad m=1,\ 2,\ 3,\ \ldots$$

**43.1.** $\displaystyle\int \frac{dx}{\alpha+\beta e^x} = \frac{x}{\alpha} - \frac{1}{\alpha}\ln|\alpha+\beta e^x|.$

**43.2.** $\displaystyle\int \frac{dx}{(\alpha+\beta e^x)^2} = -\frac{1}{\alpha^2}\left[\ln\left|\frac{\alpha+\beta e^x}{e^x}\right| + \frac{\beta e^x}{\alpha+\beta e^x}\right].$

**43.3** $\displaystyle\int \frac{dx}{(\alpha+\beta e^x)^3} = -\frac{1}{\alpha^3}\left[\ln\left|\frac{\alpha+\beta e^x}{e^x}\right| + \frac{2\beta e^x}{\alpha+\beta e^x} - \frac{\beta^2 e^{2x}}{2(\alpha+\beta e^x)^2}\right].$

**43.4.** $\displaystyle\int \frac{dx}{(\alpha+\beta e^x)^m} =$

$$= \frac{1}{\alpha^m}\left[\sum_{\nu=1}^{m-1} \frac{(-1)^\nu}{\nu} C_{m-1}^{\nu} \frac{\beta^\nu e^{\nu x}}{(\alpha+\beta e^x)^\nu} - \ln\left|\frac{\alpha+\beta e^x}{e^x}\right|\right].$$

**43.5.** $\displaystyle\int \frac{e^{ax}\,dx}{\alpha+\beta e^{ax}} = \frac{1}{a\beta}\ln|\alpha+\beta e^{ax}|.$

**43.6.** $\displaystyle\int \frac{e^{ax}\,dx}{(\alpha+\beta e^{ax})^2} = -\frac{1}{a\beta(\alpha+\beta e^{ax})}.$

**43.7.** $\displaystyle\int \frac{e^{ax}\,dx}{(\alpha+\beta e^{ax})^m} = -\frac{1}{(m-1)\,a\beta(\alpha+\beta e^{ax})^{m-1}} \qquad (m\geqslant 2).$

**43.8.** $\displaystyle\int \frac{e^{2ax}\,dx}{\alpha+\beta e^{ax}} = \frac{e^{ax}}{a\beta} - \frac{\alpha}{a\beta^2}\ln|\alpha+\beta e^{ax}|.$

**43.9.** $\displaystyle\int \frac{e^{2ax}\,dx}{(\alpha+\beta e^{ax})^2} = \frac{1}{a\beta^2}\left[\ln|\alpha+\beta e^{ax}| + \frac{\alpha}{\alpha+\beta e^{ax}}\right].$



**43.10.** $\displaystyle\int \frac{e^{2ax}\,dx}{(\alpha+\beta e^{ax})^m} =$

$$= \frac{\alpha}{(m-1)\,a\beta^2(\alpha+\beta e^{ax})^{m-1}} - \frac{1}{(m-2)\,a\beta^2(\alpha+\beta e^{ax})^{m-1}} \qquad (m>2).$$

**43.11.** $\displaystyle\int \frac{e^{3ax}\,dx}{\alpha+\beta e^{ax}} =$

$$= \frac{1}{a\beta^3}\left[\frac{(\alpha+\beta e^{ax})^2}{2} - 2\alpha(\alpha+\beta e^{ax}) + \alpha^2\ln|\alpha+\beta e^{ax}|\right].$$

**43.12.** $\displaystyle\int \frac{e^{nax}\,dx}{\alpha+\beta e^{ax}} = \frac{1}{a}\int \frac{t^{n-1}\,dt}{\alpha+\beta t}$, где $t=e^{ax}$ $(n\geqslant 1)$ (см. 1.20).

**43.13.** $\displaystyle\int \frac{e^{nax}\,dx}{(\alpha+\beta e^{ax})^m} =$

$$= \frac{ae^{(n-1)ax}}{(n-m)\,\beta(\alpha+\beta e^{ax})^{m-1}} - \frac{n\alpha}{(n-m)\,\beta}\int \frac{e^{(n-1)ax}\,dx}{(\alpha+\beta e^{ax})^m} \qquad (n\neq m).$$

**43.14.** $\displaystyle\int \frac{dx}{e^{ax}(\alpha+\beta e^{ax})} = \frac{1}{a\beta}\left(-\frac{\alpha+\beta e^{ax}}{\beta e^{ax}} + \ln|\alpha+\beta e^{ax}|\right) - \frac{x}{\alpha}.$

**43.15.** $\displaystyle\int \frac{dx}{e^{ax}(\alpha+\beta e^{ax})^2} = -\frac{1}{a\beta}\left[\frac{\alpha^2(\alpha+2\beta e^{ax})}{\beta e^{ax}(\alpha+\beta e^x)} - 2\ln\left|\frac{\alpha+\beta e^{ax}}{\beta e^{ax}}\right|\right].$

**43.16.** $\displaystyle\int \frac{dx}{e^{ax}(\alpha+\beta e^{ax})^3} =$

$$= -\frac{1}{a\beta}\left[\frac{2-6\left(\frac{\beta}{\alpha}\right)^2 e^{2ax} - 3\left(\frac{\beta}{\alpha}\right)^3 e^{3ax}}{2\frac{\beta}{\alpha}e^{ax}\left(1+\frac{\beta}{\alpha}e^{ax}\right)^2} - 2\ln\left|\frac{\alpha+\beta e^{ax}}{\beta e^{ax}}\right|\right].$$

**43.17.** $\displaystyle\int \frac{dx}{e^{ax}(\alpha+\beta e^{ax})^m} = -\frac{\alpha^2}{a\beta^2 e^{ax}(\alpha+\beta e^{ax})} - \frac{m\alpha^m}{a\beta}\int \frac{ds}{(\alpha+\beta e^s)^m},$ где $s=ax$ (см. 43.4).