ТАБЛИЦА 45

ИНТЕГРАЛЫ ВИДА

$$\int e^{ax}\sin^n px\,dx,\quad \int e^{ax}\cos^n px\,dx;$$

$$\int e^{ax}\sin^m x\cos^n x\,dx;\quad m, n = 1, 2, 3, \ldots$$

45.1. $$\int e^{ax}\sin px\,dx = \frac{e^{ax}(a\sin px - p\cos px)}{a^2+p^2}.$$

45.2. $$\int e^{ax}\sin^2 px\,dx = \frac{e^{ax}}{a^2+4p^2}\left(a\sin^2 px - 2p\cos px\sin px + \frac{2p^2}{a}\right).$$

45.3. $$\int e^{ax}\sin^3 px\,dx = \frac{e^{ax}}{a^2+9p^2}\left[a\sin^3 px - 3p\cos px\cdot\sin^2 px + \frac{6p^2}{a^2+p^2}(a\sin px - p\cos px)\right]$$

45.4. $$\int e^{ax}\sin^n px\,dx = \frac{e^{ax}\sin^{n-1} px}{a^2+n^2p^2}\left[a\sin px - np\cos px\right] + \frac{n(n-1)p^2}{a^2+n^2p^2}\int e^{ax}\sin^{n-2} px\,dx$$

45.5. $$\int e^{ax}\cos px\,dx = \frac{e^{ax}(a\cos px + p\sin px)}{a^2+p^2}.$$

45.6. $$\int e^{ax}\cos^2 px\,dx = \frac{e^{ax}}{a^2+4p^2}\left(a\cos^2 px - 2p\cos px\sin px + \frac{2p^2}{a}\right).$$

45.7. $$\int e^{ax}\cos^3 px\,dx = \frac{e^{ax}}{a^2+9p^2}\left[a\cos^3 px + 3p\sin px\cos^2 px + \frac{6p^2}{a^2+p^2}(a\cos px + p\sin px)\right].$$

45.8. $$\int e^{ax}\cos^n px\,dx = \frac{e^{ax}\cos^{n-1} px}{a^2+n^2p^2}[a\cos px + np\sin px] + \frac{n(n-1)p^2}{a^2+n^2p^2}\int e^{ax}\cos^{n-2} px\,dx.$$

45.9. $$\int e^{ax}\sin px\cos qx\,dx = \frac{ae^{ax}}{2}\left[\frac{\sin(p+q)x}{a^2+(p+q)^2} + \frac{\sin(p-q)x}{a^2+(p-q)^2}\right] - \frac{e^{ax}}{2}\left[\frac{(p+q)\cos(p+q)x}{a^2+(p+q)^2} + \frac{(p-q)\cos(p-q)x}{a^2+(p-q)^2}\right].$$

45.10. $$\int e^{ax}\sin px\cos px\,dx = \frac{e^{ax}}{2a^2+8p^2}(a\sin 2px - 2p\cos 2px).$$

45.11. $$\int e^{ax}\sin^2 px\cos px\,dx = \frac{e^{ax}}{a^2+9p^2}\left[\frac{a^2+3p^2}{a^2+p^2}(a\cos px + p\sin px) - \cos^2 px\,(a\cos px + 3p\sin px)\right].$$

45.12. $$\int e^{ax}\sin^2 px\cos^2 px\,dx = \frac{1}{4}\int e^{ax}\sin^2 2px\,dx \quad (\text{см. } 45.2).$$

45.13. $$\int e^{ax}\sin px\cos^2 px\,dx = \frac{e^{ax}}{a^2+9p^2}\left[\frac{a^2+3p^2}{a^2+p^2}(a\sin px - p\cos px) - \sin^2 px\,(a\sin px + 3p\cos px)\right].$$

45.14. $$\int e^{ax}\sin^m x\cos^n x\,dx = \frac{1}{(m+n)^2+a^2}\left\{e^{ax}\sin^m x\cos^{n-1} x\,[a\cos x + (m+n)\sin x] - ma\int e^{ax}\sin^{m-1} x\cos^{n-1} x\,dx + (n-1)(m+n)\int e^{ax}\sin^m x\cos^{n-2} x\,dx\right\};$$

$$= \frac{1}{(m+n)^2+a^2}\left\{e^{ax}\sin^{m-1} x\cos^n x\,[a\sin x - (m+n)\cos x] + na\int e^{ax}\sin^{m-1} x\cos^{n-1} x\,dx + (m-1)(m+n)\int e^{ax}\sin^{m-2} x\cos^n x\,dx\right\}.$$