ТАБЛИЦА 46

ИНТЕГРАЛЫ ВИДА

$$\int x^{\pm n} \ln^m (a+bx)\,dx; \quad \begin{matrix} n=0,\ 1,\ 2,\ \ldots, \\ m=1,\ 2,\ 3,\ \ldots \end{matrix}$$

46.1. $\int \lg_A x\,dx = \frac{1}{\ln A}(x \ln x - x) = x \lg_A \frac{x}{e} \quad (A>0,\ A \neq 1).$

46.2. $\int \ln(a+bx)\,dx = \frac{1}{b}(a+bx)\ln(a+bx) - x.$

46.3. $\int x \ln(a+bx)\,dx = \frac{b^2x^2-a^2}{2b^2}\ln(a+bx) - \frac{bx^2-2ax}{4b}.$

46.4. $\int x^2 \ln(a+bx)\,dx =$

$$= \frac{1}{3}\left(x^3 - \frac{a^3}{b^3}\right)\ln(a+bx) - \frac{1}{3}\left(\frac{x^3}{3} - \frac{ax^2}{2b} + \frac{a^2x}{b^2}\right).$$

46.5. $\int x^3 \ln(a+bx)\,dx =$

$$= \frac{1}{4}\left(x^4 - \frac{a^4}{b^4}\right)\ln(a+bx) - \frac{1}{4}\left(\frac{x^4}{4} - \frac{ax^3}{3b} + \frac{a^2x^2}{2b^2} - \frac{a^3x}{b^3}\right).$$

46.6. $\int x^n \ln(a+bx)\,dx = \frac{1}{n+1}\left(x^{n+1} - \frac{a^{n+1}}{b^{n+1}}\right)\ln(a+bx) +$

$$+ \frac{1}{n+1}\sum_{\nu=1}^{n+1} \frac{(-1)^\nu x^{n-\nu+2} a^{\nu-1}}{(m-\nu+2)\,b^{\nu-1}}.$$

46.7. $\int x^n \lg_A x\,dx = \frac{1}{\ln A}\left[\frac{x^{n+1}}{n+1}\ln x - \frac{x^{n+1}}{(n+1)^2}\right].$

46.8. $\int \ln^m (a+bx)\,dx = \frac{(a+bx)\ln^m(a+bx)}{b} - m\int \ln^{m-1}(a+bx)\,dx.$

46.9. $\int x \ln^m (a+bx)\,dx =$

$$= \frac{(a+bx)^2 \ln^m (a+bx)}{2b^2} - a\int \ln^m (a+bx)\,dx -$$

$$- \frac{m}{2b}\int (a+bx)\ln^{m-1}(a+bx)\,dx \quad \text{(см. 46.8).}$$

46.10. $\int x^2 \ln^m (a+bx)\,dx =$

$$= \left[\frac{(a+bx)^3}{3} - a(a+bx) + a^2(a+bx)\right]\frac{\ln^m(a+bx)}{b^3} -$$

$$- \frac{m}{b^3}\left(\frac{1}{3}\int u^2 \ln^{m-1} u\,du - \frac{1}{2}\int u \ln^{m-1} u\,du + \int \ln^{m-1} u\,du\right),$$

где $u = a+bx$.

46.11. $\int x^n \ln^m (a+bx)\,dx =$

$$= \frac{\ln^m(a+bx)}{b^{n+1}}\sum_{\nu=0}^{n} \frac{(-1)^{n+\nu}\,n!\,(a+bx)^{\nu+1}}{a^{\nu-n}(n-\nu)!\,\nu!\,(\nu+1)} -$$

$$- \frac{m}{b^{n+1}}\sum_{\nu=0}^{n} \frac{(-1)^{n+\nu}\,n!}{a^{\nu-n}(n-\nu)!\,\nu!\,(\nu+1)}\int u^\nu \ln^{m-1} u\,du,$$

где $u = a+bx$.

46.12.* $\int \frac{\ln x\,dx}{a+bx} = \frac{\ln x \ln(a+bx)}{b} - \frac{1}{b}\int \frac{\ln(a+bx)}{x}\,dx$ (см. 46.13).

46.13.* $\int \frac{\ln(a+bx)}{x}\,dx =$

$$= \begin{cases} \ln a \ln x + \frac{bx}{a} - \frac{b^2x^2}{2^2a^2} + \frac{b^3x^3}{3^2a^3} - \frac{b^4x^4}{4^2a^4} + \ldots & \text{при } b^2x^2 < a^2; \\ \frac{\ln^2 bx}{2} - \frac{a}{bx} + \frac{a^2}{2^2b^2x^2} - \frac{a^3}{3^2b^3x^3} + \frac{a^4}{4^2b^4x^4} - \ldots & \text{при } b^2x^2 > a^2. \end{cases}$$

46.14. $\int \frac{\ln(a+bx)}{x^2}\,dx = \frac{b}{a}\ln x - \frac{a+bx}{ax}\ln(a+bx).$

46.15. $\int \frac{\ln(a+bx)}{x^n}\,dx = -\frac{\ln(a+bx)}{(n-1)\,x^{n-1}} + \frac{b}{(n-1)}\int \frac{dx}{x^{n-1}(a+bx)}$

$(n \geqslant 2)$ (см. 2.19).

46.16.* $\int \frac{\ln^2(a+bx)}{x^2}\,dx =$

$$= -\left(\frac{1}{x} + \frac{1}{a}\right)\ln^2(a+bx) + \frac{2}{a}\int \frac{\ln(a+bx)}{x}\,dx \quad \text{(см. 46.13).}$$

46.17. $\int \frac{\ln^2(a+bx)}{x^n}\,dx = \frac{\ln^2(a+bx)}{(n-1)\,x^{n-1}} - \frac{b}{(n-1)}\int \frac{\ln(a+bx)}{x^{n-1}(a+bx)}\,dx.$

46.18. $\int \frac{\ln^m(a+bx)}{x^2}\,dx = -\frac{\ln^m(a+bx)}{x} + mb\int \frac{\ln^{m-1}(a+bx)}{x(a+bx)}\,dx.$