ТАБЛИЦА 47

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{\ln^m (a+bx)}; \quad n=0,\ 1,\ 2,\ \ldots,\quad m=1,\ 2,\ 3,\ \ldots$$

Обозначение: $u=a+bx$.

47.1.* $$\int \frac{dx}{\ln(a+bx)}=\frac{1}{b}\left[\ln|\ln(a+bx)|+\ln(a+bx)+\frac{\ln^2(a+bx)}{2\cdot 2!}+\frac{\ln^3(a+bx)}{3\cdot 3!}+\ldots\right].$$

47.2.* $$\int \frac{x\,dx}{\ln(a+bx)}=\frac{1}{b^2}\left[\ln|\ln(a+bx)|+2\ln(a+bx)+\frac{2^2\ln^2(a+bx)}{2\cdot 2!}+\frac{2^3\cdot\ln^3(a+bx)}{3\cdot 3!}+\ldots-a\int\frac{du}{\ln u}\right]$$ (см. 47.1).

47.3.* $$\int \frac{x^n\,dx}{\ln x}=\ln|\ln x|+(n+1)\ln x+\frac{(n+1)^2\ln^2 x}{2\cdot 2!}+\ldots+\frac{(n+1)^\nu\ln^\nu x}{\nu\cdot\nu!}+\ldots$$

47.4.* $$\int \frac{x^n\,dx}{\ln(a+bx)}=\frac{1}{b^{n+1}}\sum_{\nu=0}^{n}\frac{(-1)^{\nu+n}\,n!}{a^{\nu-n}(n-\nu)!\,\nu!}\int\frac{u^\nu\,du}{\ln u}$$ (см. 47.3).

47.5.* $$\int \frac{dx}{\ln^2(a+bx)}=-\frac{a+bx}{b\ln(a+bx)}+\frac{1}{b}\int\frac{du}{\ln u}$$ (см. 47.1).

47.6.* $$\int \frac{x\,dx}{\ln^2(a+bx)}=-\frac{x(a+bx)}{b\ln(a+bx)}-\frac{a}{b^2}\int\frac{du}{\ln u}+\frac{2}{b^2}\int\frac{u\,du}{\ln u}$$ (см. 47.3).

47.7.* $$\int \frac{x^2\,dx}{\ln^2(a+bx)}=-\frac{x^2(a+bx)}{b\ln(a+bx)}+\frac{a^2}{b^3}\int\frac{du}{\ln u}-\frac{4a}{b^3}\int\frac{u\,du}{\ln u}+\frac{3}{b^3}\int\frac{u\,du}{\ln u}$$ (см. 47.3).

47.8.* $$\int \frac{dx}{\ln^m(a+bx)}=-\frac{a+bx}{b\ln^{m-1}(a+bx)}+\frac{1}{(m-1)\,b}\int\frac{du}{\ln^{m-1}u}.$$

47.9.* $$\int \frac{x\,dx}{\ln^m(a+bx)}=-\frac{x(a+bx)}{(m-1)\,b\ln^{m-1}(a+bx)}-\frac{a}{(m-1)\,b^2}\int\frac{du}{\ln^{m-1}u}+\frac{2}{(m-1)\,b^2}\int\frac{u\,du}{\ln^{m-1}u}$$ (см. 47.8).

47.10. $$\int \frac{x^2\,dx}{\ln^m(a+bx)}=-\frac{x^2(a+bx)}{mb\ln^m(a+b)}+\frac{a^2}{mb^3}\int\frac{du}{\ln u}-\frac{4a}{mb^3}\int\frac{u\,du}{\ln^m u}+\frac{3}{mb^3}\int\frac{u^2\,du}{\ln^m u}$$ (см. 47.8 и 47.9).

47.11.* $$\int \frac{x^n\,dx}{\ln^m(a+bx)}=-\frac{x^n(a+bx)}{(m-1)\,b\ln^{m-1}(a+bx)}+\frac{1}{(m-1)\,b^{n+1}}\sum_{\nu=0}^{n}\frac{(-1)^{n+\nu}(\nu+1)\,n!}{a^{\nu+n}(n-\nu)!\,\nu!}\int\frac{u^\nu\,du}{\ln^{m-1}u}.$$

47.12.* $$\int \frac{dx}{x\ln(a+bx)}=\sum_{\nu=0}^{\infty}a^\nu\left[\ln|\ln(a+b\nu)|-\nu\ln(a+bx)+\frac{\nu^2\ln^2(a+bx)}{2\cdot 2!}-\frac{\nu^3\ln^3(a+bx)}{3\cdot 3!}+\ldots\right].$$

47.13.* $$\int \frac{dx}{x^n\ln(a+bx)}=b^{n-1}\sum_{\nu=0}^{\infty}\frac{(n+\nu-1)!\,a^\nu}{(n-1)!\,\nu!}\left[\ln|\ln(a+bx)|-(n+\nu-1)\ln(a+bx)+\frac{(n+\nu-1)^2\ln^2(a+bx)}{2\cdot 2!}-\frac{(n+\nu-1)^3\ln^3(a+bx)}{3\cdot 3!}+\ldots\right].$$

47.14.* $$\int \frac{dx}{x\ln^2(a+bx)}=-\frac{a+bx}{bx\ln(a+bx)}-\sum_{\nu=0}^{\infty}\nu a^\nu\int\frac{du}{u^{\nu+1}\ln u}$$ (см. 47. 13).

47.15. $$\int \frac{dx}{x^n\ln^2(a+bx)}=-\frac{a+bx}{bx^n\ln(a+bx)}-b^{n-1}\sum_{\nu=0}^{\infty}\frac{(n+\nu-1)!\,(n+\nu-1)\,a^\nu}{(n-1)!\,\nu!}\int\frac{du}{u^{n+\nu}\ln u}.$$

47.16. $$\int \frac{dx}{x\ln^m(a+bx)}=-\frac{a+bx}{(m-1)\,bx\ln^{m-1}(a+bx)}-\frac{1}{m-1}\sum_{\nu=0}^{\infty}\nu a^\nu\int\frac{du}{u^{\nu+1}\ln^{m-1}u}$$ (см. 46.13).

47.17. $$\int \frac{dx}{x^n\ln^m(a+bx)}=-\frac{a+bx}{(m-1)\,bx^n\ln^{m-1}(a+bx)}-\frac{b^{n-1}}{m-1}\sum_{\nu=0}^{\infty}\frac{(n+\nu-1)!\,(n+\nu-1)\,a^\nu}{(n-1)!\,\nu!}\int\frac{du}{u^{n+\nu}\ln^{m-1}u}.$$