ТАБЛИЦА 48

ИНТЕГРАЛЫ ВИДА

$$\int x^n \ln|x^2 \pm a^2|\,dx,$$

$$\int x^{\pm n} \ln(x+\sqrt{x^2 \pm a^2})\,dx; \qquad n=0,\ 1,\ 2,\ \ldots$$

48.1. $\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) - 2x + 2a\operatorname{arctg}\frac{x}{a}$.

48.2. $\int x\ln(x^2+a^2)\,dx = \frac{1}{2}[(x^2+a^2)\ln(x^2+a^2) - x^2]$.

48.3. $\int x^2\ln(x^2+a^2)\,dx =$

$$= \frac{1}{3}\left[x^3\ln(x^2+a^2) - \frac{2}{3}x^3 + 2xa^2 - 2a^3\operatorname{arctg}\frac{x}{a}\right].$$

48.4. $\int x^n\ln(x^2+a^2)\,dx = \frac{x^{n+1}}{n+1}\ln(x^2+a^2) - \frac{2}{n+1}\int\frac{x^{n+2}\,dx}{x^2+a^2}$.

48.5. $\int \ln|x^2-a^2|\,dx = x\ln|x^2-a^2| - 2x + a\ln\left|\frac{x+a}{x-a}\right|$.

48.6. $\int x\ln|x^2-a^2|\,dx = \frac{1}{2}[(x^2-a^2)\ln|x^2-a^2| - x^2]$.

48.7. $\int x^2\ln|x^2-a^2|\,dx =$

$$= \frac{1}{3}\left[x^3\ln|x^2-a^2| - \frac{2}{3}x^3 - 2xa^2 + a^3\ln\left|\frac{x+a}{x-a}\right|\right].$$

48.8. $\int x^n\ln|x^2-a^2|\,dx = \frac{x^{n+1}}{n+1}\ln|x^2-a^2| - \frac{2}{n+1}\int\frac{x^{n+2}\,dx}{x^2-a^2}$.

48.9. $\int \ln(x+\sqrt{x^2\pm a^2})\,dx = -\sqrt{x^2\pm a^2} + x\ln(x+\sqrt{x^2\pm a^2})$

48.10. $\int x\ln(x+\sqrt{x^2\pm a^2})\,dx =$

$$= -\frac{x\sqrt{x^2\pm a^2}}{4} + \frac{2x^2\pm a^2}{4}\ln(x+\sqrt{x^2\pm a^2})$$

48.11. $\int x^2\ln(x+\sqrt{x^2\pm a^2})\,dx =$

$$= \pm\frac{a^2\sqrt{x^2\pm a^2}}{3} - \frac{\sqrt{(x^2\pm a^2)^3}}{9} + \frac{x^3}{3}\ln(x+\sqrt{x^2\pm a^2}).$$

48.12. $\int x^n\ln(x+\sqrt{x^2\pm a^2})\,dx =$

$$= \frac{x^{n+1}}{n+1}\ln(x+\sqrt{x^2\pm a^2}) - \frac{1}{n+1}\int\frac{x^{n+1}\,dx}{\sqrt{x^2\pm a^2}}.$$

48.13.* $\int\frac{\ln\left(\frac{x}{a}+\sqrt{\frac{x^2}{a^2}+1}\right)}{x}\,dx = \frac{x}{a} - \frac{1}{2\cdot3\cdot3}\left(\frac{x}{a}\right)^3 +$

$$+\frac{1\cdot3}{2\cdot4\cdot5\cdot5}\left(\frac{x}{a}\right)^5 - \frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot7\cdot7}\left(\frac{x}{a}\right)^7 + \ldots \text{ при } x^2 < a^2;$$

$$= \frac{1}{2}\left(\ln\frac{2x}{a}\right)^2 - \frac{1}{2^3}\left(\frac{a^2}{x}\right) +$$

$$+\frac{1\cdot3}{2\cdot4^3}\left(\frac{a}{x}\right)^4 - \frac{1\cdot3\cdot5}{2\cdot4\cdot6^3}\left(\frac{a}{x}\right)^6 + \ldots \text{ при } \frac{x}{a} > 1.$$

48.14.* $\int\frac{\ln\left(\frac{x}{a}+\sqrt{\frac{x^2}{a^2}-1}\right)}{x}\,dx = \frac{1}{2}\left(\ln\frac{2x}{a}\right)^2 + \frac{1}{2^3}\left(\frac{a}{x}\right)^2 +$

$$+\frac{1\cdot3}{2\cdot4^3}\left(\frac{a}{x}\right)^4 + \frac{1\cdot3\cdot5}{2\cdot4\cdot6^3}\left(\frac{a}{x}\right)^6 + \ldots \text{ при } \frac{x}{a} > 1.$$

48.15. $\int\frac{\ln(x+\sqrt{x^2+a^2})}{x^2}\,dx =$

$$= -\frac{\ln(x+\sqrt{x^2+a^2})}{x} - \frac{1}{a}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|.$$

48.16. $\int\frac{\ln x+(\sqrt{x^2-a^2})}{x^2}\,dx =$

$$= -\frac{\ln(x+\sqrt{x^2-a^2})}{x} + \frac{1}{a}\operatorname{arcsec}\left|\frac{x}{a}\right| \quad \left(0<\operatorname{arcsec}\left|\frac{x}{a}\right|<\frac{\pi}{2}\right).$$

48.17. $\int\frac{\ln(x+\sqrt{x^2\pm a^2})}{x^n}\,dx =$

$$= -\frac{\ln(x+\sqrt{x^2\pm a^2})}{(n-1)\,x^{n-1}} + \frac{1}{n-1}\int\frac{dx}{x^{n-1}\sqrt{x^2\pm a^2}} \quad (n\geqslant 2).$$