ТАБЛИЦА 9

ИНТЕГРАЛЫ ВИДА

$$\int \frac{x^{\pm n}\,dx}{(a+bx^4)^m}; \quad n=0,\ 1,\ 2,\ \ldots,\quad m=1,\ 2,\ 3,\ \ldots$$

9.1. $$\int \frac{dx}{a+bx^4} = \begin{cases} \dfrac{\eta}{4\sqrt{2}\,a}\left[\ln\dfrac{x^2+\sqrt{2}\,\eta x+\eta^2}{x^2-\sqrt{2}\,\eta x+\eta^2}+2\operatorname{arctg}\dfrac{\sqrt{2}\,\eta x}{\eta^2-x^2}\right] & \text{при } ab>0; \\ -\dfrac{\eta}{4a}\left[\ln\left|\dfrac{x+\eta}{x-\eta}\right|+2\operatorname{arctg}\dfrac{x}{\eta}\right] & \text{при } ab<0. \end{cases}$$

9.2. $$\int \frac{dx}{(a+bx^4)^2} = \frac{x}{4a\,(a+bx^4)}+\frac{3}{4a}\int\frac{dx}{a+bx^4} \quad \text{(см. 9.1).}$$

9.3. $$\int \frac{dx}{(a+bx^4)^3} = \frac{11ax+7bx^5}{32a^2\,(a+bx^4)^2}+\frac{21}{32a^2}\int\frac{dx}{a+bx^4} \quad \text{(см. 9.1).}$$

9.4. $$\int \frac{dx}{(a+bx^4)^m} = \frac{x}{4\,(m-1)\,a\,(a+bx^4)^{m-1}}+ \\ +\frac{4m-5}{4\,(m-1)\,a}\int\frac{dx}{(a+bx^4)^{m-1}} \quad (m\geqslant 2).$$

9.5. $$\int \frac{x\,dx}{a+bx^4} = \begin{cases} \dfrac{1}{2b\eta^2}\operatorname{arctg}\dfrac{x^2}{\eta^2} & \text{при } ab>0; \\ -\dfrac{1}{4\sqrt{|ab|}}\ln\left|\dfrac{a-x^2\sqrt{|ab|}}{a+x^2\sqrt{|ab|}}\right| & \text{при } ab<0. \end{cases}$$

9.6. $$\int \frac{x\,dx}{(a+bx^4)^2} = \frac{x^2}{4a\,(a+bx^4)}+\frac{1}{2a}\int\frac{x\,dx}{a+bx^4} \quad \text{(см. 9.5).}$$

9.7. $$\int \frac{x\,dx}{(a+bx^4)^3} = \frac{5ax^2+3bx^6}{16a^2\,(a+bx^4)^2}+\frac{3}{8a^2}\int\frac{x\,dx}{a+bx^4} \quad \text{(см. 9.5).}$$

9.8 $$\int \frac{x\,dx}{(a+bx^4)^m} = \frac{x^2}{4\,(m-1)\,a\,(a+bx^4)^{m-1}}+ \\ +\frac{2m-3}{2\,(m-1)\,a}\int\frac{x\,dx}{(a+bx^4)^{m-1}} \quad (m\geqslant 2).$$

9.9. $$\int \frac{x^2\,dx}{a+bx^4} = \begin{cases} \dfrac{-1}{4\sqrt{2}\,b\eta}\left[\ln\dfrac{x^2+\sqrt{2}\,\eta x+\eta^2}{x^2-\sqrt{2}\,\eta x+\eta^2}-2\operatorname{arctg}\dfrac{\sqrt{2}\,\eta x}{\eta^2-x^2}\right] & \text{при } ab>0; \\ -\dfrac{1}{4b\eta}\left[\ln\left|\dfrac{x+\eta}{x-\eta}\right|-2\operatorname{arctg}\dfrac{x}{\eta}\right] & \text{при } ab<0. \end{cases}$$

9.10. $$\int \frac{x^2\,dx}{(a+bx^4)^2} = \frac{x^3}{4a\,(a+bx^4)}+\frac{1}{4a}\int\frac{x^2\,dx}{a+bx^4} \quad \text{(см. 9.9).}$$

9.11. $$\int \frac{x^2\,dx}{(a+bx^4)^3} = \frac{9ax^3+5bx^7}{32a^2\,(a+bx^4)^2}+\frac{5}{32a^2}\int\frac{x^2\,dx}{a+bx^4} \quad \text{(см. 9.9).}$$

9.12. $$\int \frac{x^n\,dx}{(a+bx^4)^m} = \frac{x^{n+1}}{4\,(m-1)\,a\,(a+bx^4)^{m-1}}+ \\ +\frac{4m-n-5}{4\,(m-1)\,a}\int\frac{x^n\,dx}{(a+bx^4)^{m-1}} \quad (m\geqslant 2); \\ =\frac{x^{n-3}}{(n+1-4m)\,b\,(a+bx^4)^{m-1}}- \\ -\frac{(n-3)\,a}{(n+1-4m)\,b}\int\frac{x^{n-4}\,dx}{(a+bx^4)^m} \quad (n\neq 4m-1).$$

9.13. $$\int \frac{dx}{x\,(a+bx^4)} = \frac{1}{4a}\ln\left|\frac{x^4}{a+bx^4}\right|.$$

9.14. $$\int \frac{dx}{x\,(a+bx^4)^2} = \frac{1}{4a\,(a+bx^4)}+\frac{1}{4a^2}\ln\left|\frac{x^4}{a+bx^4}\right|.$$

9.15. $$\int \frac{dx}{x\,(a+bx^4)^3} = \frac{3+2bx^4}{8a\,(a+bx^4)^2}+\frac{1}{4a^3}\ln\left|\frac{x^4}{a+bx^4}\right|.$$

9.16. $$\int \frac{dx}{x^2\,(a+bx^4)} = -\frac{1}{ax}-\frac{b}{a}\int\frac{x^2\,dx}{a+bx^4} \quad \text{(см. 9.9).}$$

9.17. $$\int \frac{dx}{x^2\,(a+bx^4)^2} = -\frac{1}{a^2x}-\frac{bx^3}{4a^2\,(a+bx^4)}-\frac{5b}{4a^2}\int\frac{x^2\,dx}{a+bx^4} \quad \text{(см. 9.9).}$$

9.18. $$\int \frac{dx}{x^2\,(a+bx^4)^3} = -\frac{1}{ax\,(a+bx^4)^2}-\frac{9b}{a}\int\frac{x^2\,dx}{(a+bx^4)^3} \quad \text{(см. 9.11).}$$

9.19. $$\int \frac{dx}{x^n\,(a+bx^4)^m} = -\frac{1}{(n-1)\,ax^{n-1}\,(a+bx^4)^{m-1}}- \\ -\frac{(4m+n-5)\,b}{(n-1)\,a}\int\frac{dx}{x^{n-4}\,(a+bx^4)^m} \quad (n\geqslant 2); \\ =\frac{1}{a}\int\frac{dx}{x^n\,(a+bx^4)^{m-1}}-\frac{b}{a}\int\frac{dx}{x^{n-4}\,(a+bx^4)^m}.$$